The Application of ARIMA and Mean-variance Models on Financial Market

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Abstract. This study centers on forecasting return and constructing proper portfolios with 5 typical assets rarely focused on the Chinese A-share market. This paper applies the fittest ARIMA models for each of the selected stocks to predict their trend of returns in the next 20 days. Besides, we create the efficient frontier by Monte Carlo simulation under Markowitz's Mean-Variance framework to focus on two portfolios, i.e., the maximum Sharpe ratio portfolio and the minimum volatility portfolio. The empirical results of the ARIMA model indicate a rational prediction of return for assets in the A-share market. The maximum Sharpe ratio portfolio and the minimum volatility portfolio show that stock of Foshan Haitian Flavouring and Food Company Ltd. and stock of China Merchants Bank Co., Ltd. account for the largest proportion in the two portfolios. Further empirical results show that returns for two portfolios are higher than the market index return, which illuminates the two portfolios outperform the market index. The results in this paper will surely benefit related investors in the financial market.

Keywords: Markowitz theory; Monte Carlo simulation; ARIMA model; return prediction; portfolio optimization; Chinese A-share market

1. Introduction

The construction of a proper portfolio is a necessary step for a successful investment. Good portfolio construction is always an attractive object for every investor [1]. And from Markowitz's [2] theory on obtaining the efficient frontier and Sharpe's method [3] on building an optimal portfolio which maximizes the Sharpe ratio using mean value and variance data of portfolio return being published, the exploration and investigation on portfolio construction had never stopped.

Focusing on mean-variance portfolio optimization, many other theories and methods have been developed based on Markowitz's [2] theory. For example, Guran, Ugurlu, and Tas [4] applied second-order stochastic dominance efficiency to construct the portfolio of energy stocks. Kiris and Ustun [5] developed Markowitz's [2] model using multi-criteria decision-making (MCDM) to solve portfolio selection problems. Thakur, Bhattacharyya, and Sarkar [6] used Dempster-Shafer's evidence theory to construct the portfolio. Besides these, the time series analyses on stocks are also a critical topic on portfolio construction. Autocorrelation, an important feature in time series, can be examined to help predict and select stocks to construct the ideal portfolio. Many papers focus on ARMA model and ARIMA model, which are the most famous and significant models in forecasting stock time series. For instance, the ARIMA model is applied in investigating S&P 500 by Rounaghi and Zadeh [7] and Challa et al. [8]. Prediction is carried out by using a hybrid ARIMA-neural network model by Kumar and Thenmozhi [9]. Despite so many essays on portfolio construction, the Chinese stock market is a potential market that is paid less focus than that of the US. There exist some investigations on the Chinese stock market, for example, Qi et al. [10] tried to remove simplifications in large-scale portfolio selection through software. Chen et al. [11] examined the Chinese stock market asset allocation. Still, few investigations focus on applying the ARIMA model to forecast the time series of Chinese stock and portfolio construction.

To the best of our knowledge, this paper makes the following contributions to the literature. First, we obtained the most suitable parameter p, d and q for the ARIMA(p,d,q) model for every stock we chose. And then, we estimated the ARIMA model for stocks and predicted their future returns. The

outcome show ARIMA model can forecast the time series of Chinese stocks. Our research expands the application field of the ARIMA model. Second, we used the mean-variance model and Monto Carlo simulation to obtain the efficient frontier of our 5 selected stocks and determined the maximizing Sharpe-ratio portfolio point and the minimizing volatility portfolio point. Third, we compared the performance of the maximizing Sharpe-ratio portfolio and the minimizing volatility portfolio with the Chinese A-share market index. The outcome shows the portfolios obtained from our ARMA model can outperform the benchmark.

This paper is organized as follows. Section 2 shows the data and methodologies used in this paper. Section 3 is the related empirical results. Section 4 concludes the paper.

2. Data And Methods

2.1 Data

In this paper, we collect our dataset of prices from Yahoo Finance (https://finance.yahoo.com). We set the sample period from Jan 4, 2017, to May 28, 2021, and transfer these daily prices to simple returns. Specifically, we choose five assets, China Merchants Bank Co., Ltd, Aier Eye Hospital Group Co., Ltd, Wuliangye Yibin Co., Foshan Haitian Flavoring and Food Company Ltd, and Shanghai M&G Stationery Inc, respectively. The reasons why we select these assets are as follows.

We select China Merchants Bank since bank stocks are especially steady stocks that can affect the market risk in the Chinese market with their high market value. China Merchants Bank is a representative bank in the Chinese financial industry with the stock code 600036.SS. As a famous company in the Chinese medical industry, Aier Eye Hospital with the stock code of 300015.SZ is gaining high influence these years with a great number of Chinese students facing eye problems, and currently, the numbers of the hospital belong to the company reach more than 500. Another representative company we select is Wuliangye Yibin Co., Ltd, with the stock code of 000858.SZ. As we all know, the wine industry suffers a bull market in the A-share market in recent years, and Wuliangye is one top wine in China with better financial characteristics. Foshan Haitian Flavoring and Food Company Ltd., with the stock code of 603288.SS, also belongs to the consumption industry, an important industry in the Chinese A-share market. The market is a representative company inside the industry. Shanghai M&G Stationery Inc, with the code of 603899.SS, has the largest number of users in China, as currently, the population of China is still the largest in the world, and M&G is one of the most famous Chinese stationery companies. Some basic information on these assets is shown in Table 1.

Table 1. Descriptive statistics of the selected assets

	Mean	**Volatility**	**Max**	**Min**
600036.SZ	0.138%	0.032%	9.989%	-6.441%
300015.SZ	0.254%	0.061%	10.966%	-9.582%
000858.SZ	0.245%	0.061%	10.003%	-10.098%
603288.SS	0.215%	0.047%	10.000%	-10.000%
603899.SS	0.173%	0.047%	9.141%	-10.000%

From Table 1, we could find out the following information about the simple daily return between the five chosen stocks. The values of the mean are not that similar, with the range being 0.116% which is almost one time of the mean of stock 600036.SZ, the smallest one with a value of 0.138%, while the differences between each other in order do not vary that much. Volatility has a pattern of three steps. Each has a difference of around 0.2% to the near step. The maximum simple returns are similar to each other, all-around 10%. For the minimum of return, values are close to around -10% except for 600036.SZ with a minimum return of 6.441%.

2.2 Methods

2.2.1 ARIMA Model

ARIMA model, the Autoregressive Integrated Moving Average model, which is also widely known as the Box-Jenkins method, is one of the most conventional models in predicting future trends of time series. Successful application of the ARIMA model in return prediction has been made in the S&P market [7,8] and the German Frankfurt stock market [12]. Since the prediction for stock return by the ARIMA model only depends on current and past return values without considering the market influence, we therefore also employ the ARIMA model to predict for the 5 selected Chinese A-share market stocks.

ARIMA model simulates the future values based on current and past values. Hence it could only handle the stationary time series. Once dealing with a non-stationary original time sequence, a d-times difference is needed to make it stationary. ARIMA model combines auto-regressive (AR) model and moving average (MA) model with appropriate AR order p and MA order q. Then the fittest ARIMA(p,d,q) model is construted to make the prediction. This model could be presented as the formula as follows

$$\begin{aligned} Y_t - \varphi_1 Y_{t-1} - \cdots - \varphi_p Y_{t-p} = \\ \alpha_t - \theta_1 \alpha_{t-1} - \cdots - \theta_q \alpha_{t-q} \end{aligned} \quad (1)$$

where Y_t is the auto-regressive moving average series, p is the auto-regressive order of the model, q is the moving average order of the model, α_t, α_{t-1}, … , α_{t-q} are stationary white noises, φ_1, φ_2, … , φ_p are coefficients of the moving average model, θ_1, θ_2, … , θ_q are the coefficients of the auto-regressive model. In this paper, all the parameters are selected based on AIC information criteria.

2.2.2 Return and Variance of the portfolio

According to Markowitz [2], the return of one portfolio can be calculated as follows,

$$R_t = x^T \omega x \quad (2)$$

where Rt is the return of the portfolio, x is the weight for assets, and μ is the expected return of assets. According to Markowitz [2], the risk of the one portfolio is represented by the expected return volatility. Specifically, the volatility can be achieved by the following equation,

$$V_t = D(R_t) = x^T \omega x \quad (3)$$

where V_t is the volatility of the expected return of the portfolio, ω is the variance-covariance of the asset return.

2.2.3 Monte Carlo simulation

We apply Monte Carlo simulations to obtain the efficient frontier for our selected five stocks. According to Siswanah [13], the method is used in portfolio construction regarding the Indonesian stock market. Thereby, in this paper, we also adopt the method for portfolio construction. The Monte Carlo simulation can be summarized as follows.

In this paper, according to the theoretical illustration of Monte Carlo simulation, we generate five positive values (X1,…,X5) and then standardized these values to the simulated asset weight by the following equation.

$$x_i = X_i / \sum X_i \quad (4)$$

Once we get these simulated weights, we can get the corresponding portfolio return and volatility.

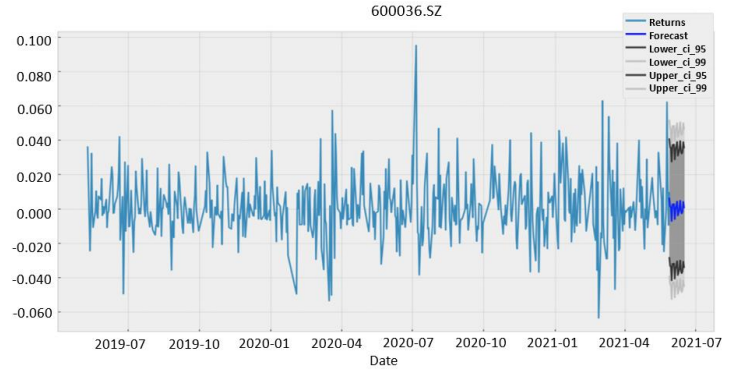
3. Results

By making the AIC comparison for different combinations of the AR order p and MA order q for each asset, the fittest models for time series forecast were chosen with the smallest AIC scores. The smallest AIC values and orders of the estimated ARIMA model are shown in Table 2.

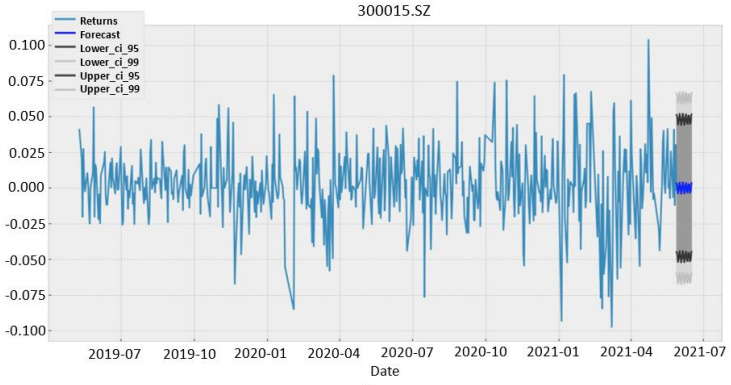
Table 2. Descriptive statistics of ARIMA model

	AIC	**(p,d,q)**
600036.SZ	-5571.514	(3,1,3)
300015.SZ	-4866.714	(3,1,2)
000858.SZ	-4857.063	(2,1,2)
603288.SS	-5136.663	(4,1,3)
603899.SS	-5148.757	(2,1,3)

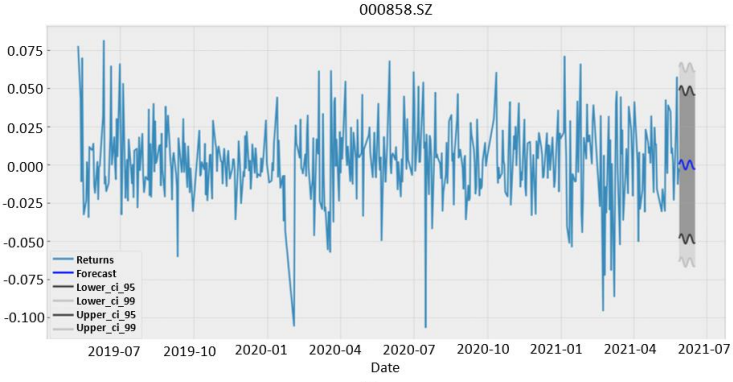
The prediction of the future 20-days returns of the 5 selected assets is depicted in Figure 1. The dark blue curves in the grey shadow portray the 20-days forecast trends of returns based on the holdback period from January 1st, 2017, to May 31st, 2021. They all demonstrate a steady trend with tiny fluctuation, indicate a more stationary feature than the original time sequences.



(a)



(b)



(c)

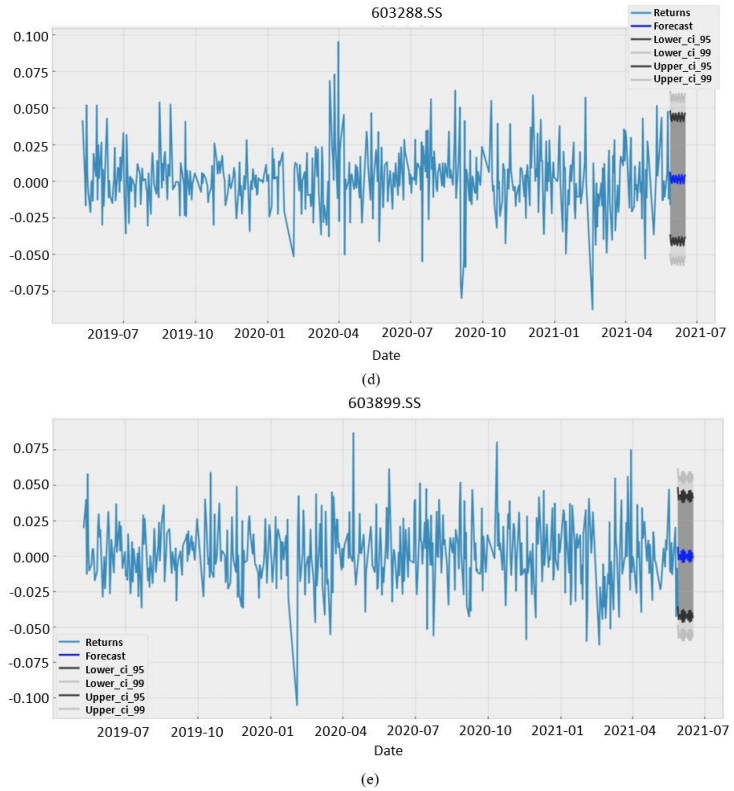


Figure 1. 20-day return forecast results for the selected assets. (a), (b), (c), (d), (e) are the actual return and 20-day forecast performance of stock 600036.SZ, 300015.SZ, 000858.SZ, 603288.SS and 603899.SS, respectively.

Then, we used 100000 times of Monte Carlo simulation and got 100000 different random portfolios constructed with our 5 selected stocks to obtain our outcome of the efficient frontier and the Sharpe Ratio maximizing portfolio and volatility minimizing portfolio. The results are shown as follows in Figure 2. And the Sharpe Ratio maximizing portfolio and the volatility minimizing portfolio are shown below in Table 3.

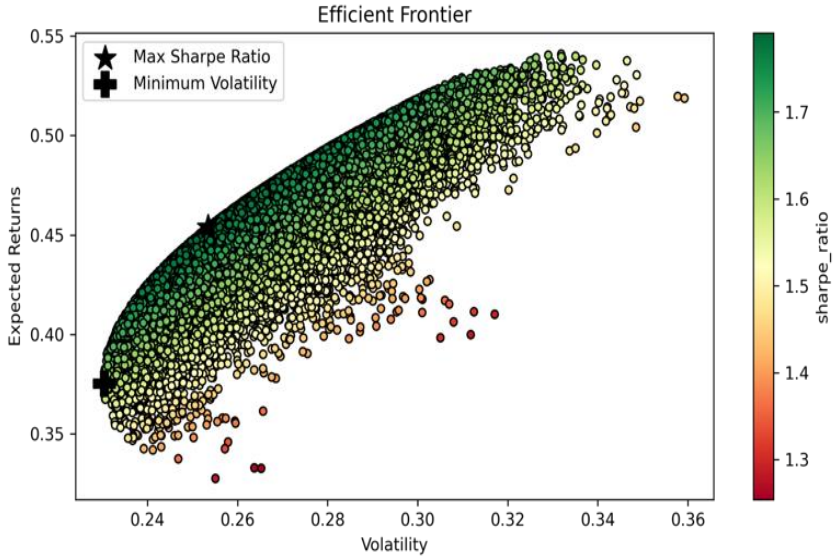


Figure 2. Efficient frontier, minimum variance, and maximum Sharpe ratio portfolio

Where the blue line is the efficient frontier, and the dots are randomly constructed portfolios.

Table 3. Financial characteristics of the two certain portfolios

Stock code	000858.SZ	300015.SZ	600036.SZ	603288.SS	603899.SS
Sharpe Ratio Maximizing	12.31%	25.43%	24.28%	26.25%	11.73%
Volatility Minimizing	1.70%	3.58%	49.87%	21.08%	23.77%
		Sharpe Ratio Maximizing		**Volatility Minimizing**	
Return		45.39%		37.51%	
Volatility		25.35%		23.05%	
Sharpe Ratio		179.07%		162.76%	

The table above displays that, in the Sharpe ratio maximizing portfolio, 300015.SZ, 600036.SZ and 603288.SS share a similar weight, all close to 25%, while the weight of 000858.SZ and 603899.SS are both slightly larger than 10%. In the volatility minimizing portfolio, 600036.SZ accounts for almost half, and the weight of 000858.SZ and 300015.SZ are both small, only 1.70% and 3.58%, respectively. The expected return of the Sharpe ratio maximizing portfolio is 45.39%, larger than the return of volatility minimizing portfolio 37.51%.

At last, we calculated the performance of our two constructed portfolios from 1st Jan 2021 to 2ed July 2021 and compared their performance with the Chinese A-share stock market index, the SSE Composite Index. The outcome is shown below in Table 4.

Table 4. Performance comparison of two portfolios and market index

	Sharpe Ratio Maximizing	Volatility Minimizing	SSE Composite Index
Return	12.66%	13.25%	0.96%

The return of the Sharpe ratio maximizing portfolio is 12.66% during the past half year, and that of the volatility minimizing portfolio is 13.25%. However, the market index only gains a 0.96% return. This implies our constructed portfolios perform much better than the market index.

4. Conclusion

To summarize, our research investigates the Chinese A-share stock market by mean-variance portfolio construction theory and obtained two portfolios using our 5 selected representative stocks. Unlike most of the existing research, our research focuses on the Chinese stock market, which is a seldomly analyzed but potential market, and applied the ARIMA model to investigate the time series of Chinese stocks.

The empirical results can be summarized as follow. Firstly, we used the ARIMA model to analyze the time series of our 5 selected stocks. The empirical results show that the ARIMA model is suitable for our prediction, and we obtained the predicted performance for the 5 chosen stocks. Secondly, we used the Monte Carlo simulation to construct portfolios, among which we obtained the efficient frontier and got two final portfolios, i.e., the Sharpe ratio maximizing portfolio and the volatility minimizing portfolio. Finally, we compared the performance of our two constructed portfolios and the market index return over the past half year. The result is that our portfolios can outperform the market. The results in this paper benefit related investors in financial markets. It is helpful for investors to construct their portfolios on the Chinese A-share stock market. Also, our model proved the suitability of the ARIMA model to predict, which extends the application of the ARIMA model and digs the possibility of the Chinese market.

Surely, our research also has some deficiencies. For example, our data only covers the last four and half years. Our ARIMA model analyzing and forecasting and further portfolio construction can be improved if we test the model in more periods with different lengths; there are various methods of stock prediction besides the ARIMA model. Thus, applying more different methods in stock forecasting will also improve our research.

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