Portfolio Selection Based on the Application of CAPM and FF3F Model

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Abstract. Portfolio optimization is the selection of the optimal portfolio from all the portfolios considered. This paper uses diversified data, including technology, medicine, real estate and so on, to make the data more referential. In the process, five stocks with better performance in the corresponding fields were selected. In this paper, the CAPM and FF3F model are used to select the optimal portfolio. This paper also uses Sharpe ratio and weight to measure whether the portfolio can achieve the optimal. The results show that except for 'VLO', the covariance of other assets is below 0.01, which can perform better in the minimization of variance. And 'MDT' has the highest weight in the CAPM model, followed by 'JLL', which can maximize the Sharpe ratio. But in the FF3F model, 'JLL' has the highest weight and 'WMT' has zero weight to maximize the Sharpe ratio. The results of this paper will enable investors in related industries to get a better portfolio paradigm.

Keywords: Portfolio Selection; FF3F model; CAPM model; Investment.

1. Introduction

In 1964, William Sharpe proposed a single-factor model using covariance matrix to simplify the estimation, which helps the portfolio to evolve from theory to practical application to a large extent, it can also be thought of as the famous capital asset pricing model proposed by Sharpe, Lint and Mawson. It not only provides a model framework for evaluating return-risk, but also lay an important theoretical foundation for portfolio analysis [1]. It is the choice of most investors to make a reasonable investment portfolio to obtain high returns and acceptable risks. Therefore, how to balance risk and return is the key for many investors to build a reasonable portfolio. Of course, a diversified portfolio can spread risk to a greater extent than a single asset. At the same time, there is a greater chance of higher returns [2].

In recent years, there are more and more research on portfolio selection and optimization. Researchers are also trying from various fields. For example, Golmakani and Fazel.'s paper studied about Constrained portfolio selection using particle swarm optimization [3]. Fisher and Sirmans applied of the study to real estate assets in a portfolio [4]. And Simonian studies multi-asset agricultural portfolio analysis [5]. Of course, there are many ways to build a portfolio involving different areas. This is undoubtedly the direction that many investors are considering. There are also some scholars who study the relationship between model tools and portfolio. Anghel MG and Paschia L. used the CAPM model to analyze the portfolio profitability of financial instruments [6].

Although portfolio construction, as discussed in this paper, is a hot topic at the moment. It is not common to directly use financial models to regulate Sharpe's proportion to construct portfolios. This paper focuses on the capital asset pricing model and Fama-French three-factor model and uses those two methods to optimize the Sharpe ratio, so as to build a risk-return balanced portfolio. At the same time, in order to better allocate the weight, we should try our best to diversify the selected assets so as to build diversified assets.

This paper mainly selected real estate Jones Lang LaSalle (JLL), pharmaceutical company Medtronic plc (MDT), technology company Foxconn, energy company Valero Energy Corporation (VLO) and retail service company Walmart Inc. (WMT). This paper uses covariance matrix to analyse the monthly return data of five assets from October 2017 to October 2022 and finds that they have good dispersion. In the process, the CAPM model and FF3F model are used for regression analysis. The beta coefficient and excess return alpha are compared and selected, and relatively good data are obtained. In order to make the selected portfolio more convincing, the CAPM model and

FF3F model are used to maximize the Sharpe ratio of assets. Compared with the single-factor capital asset pricing model, the Fama-French model considers many factors, which is the verification and supplement of the former. The data results of the two ways are slightly different, but the whole is the same, which better demonstrates the rationality of portfolio optimization under non-ideal conditions. And the Sharp ratio data of 11% and 14% were obtained in turn.

Here is a guide to the rest. The second part covers the data information and methods in the survey, the third part covers the specific results, and the fourth part summarizes the conclusions.

2. Data

The data used in this paper are from Investing (https://m.cn.investing.com/). This paper selects JLL, MDT, Foxconn, VLO, WMT five large listed companies. The historical monthly return of the whole five years from September 2017 to September 2022 is selected as the reference. In this paper, these data are selected mainly with reference to their higher Sharpe ratio. Through the data, it can be known that all of the stocks have a longer history and a more stable rate of return. By choosing companies with different industrial directions, the paper diversifies the investment assets to avoid excessive similarity. The chart and figure below show the historical data of the five companies more intuitively.

Table 1. Descriptive statistics of the five assets

	JLL	MDT	Foxconn	VLO	WMT
Variance	0.009676183	0.003328973	0.006687572	0.019847018	0.003383037
Arithmetic Avg	0.011135629	0.00597341	-0.00344219	0.019614355	0.012663237
CAPM weight	0.201235181	0.362247047	0.129718186	0.114077696	0.19272189
FF3F weight	0.355043176	0.162713427	0.236651733	0.245591664	0

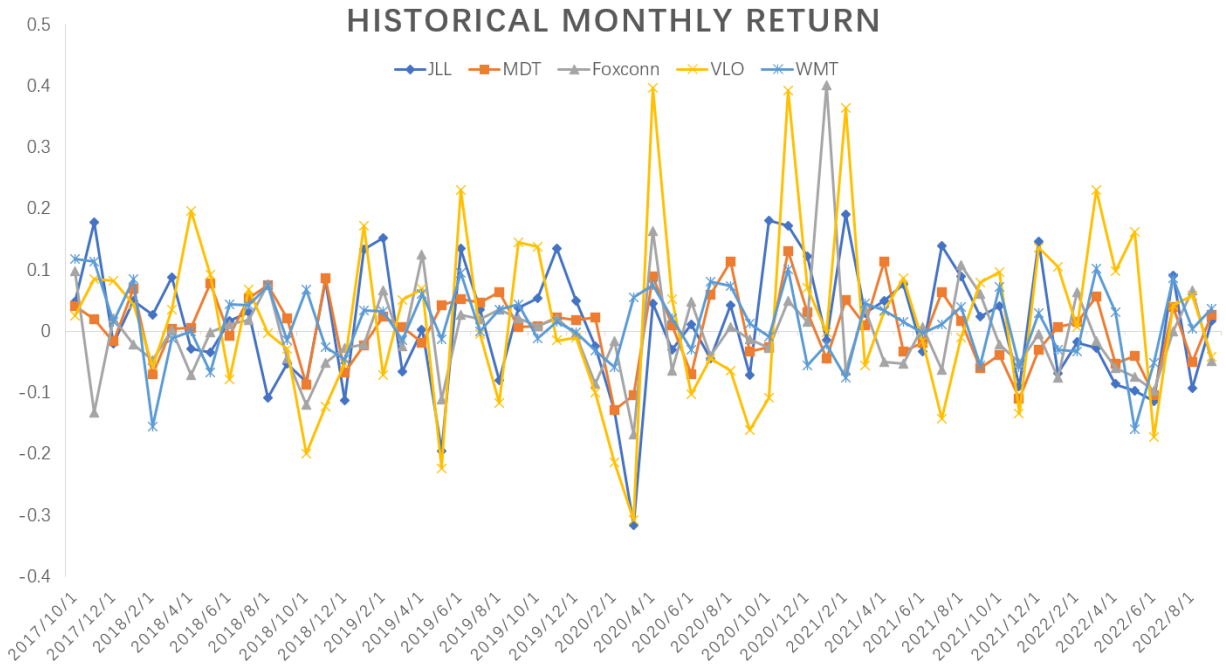

Figure 1. Historical monthly return of five assets

From the data shown in Table 1, VLO has the largest variance and MDT has the smallest variance of 0.003328973. Foxconn has a unique negative arithmetic mean. At the same time, the weights of JLL, Foxconn and VLO in FF3F model are higher than those in CAPM model. The zero-weight case occurs except for WMT.

For the data in the above figure 1, the floating space is between 0.2 and minus 0.2. VLO and Foxconn have obvious fluctuations, and the former fluctuates from negative 0.3 to positive 0.4 from April to June 2020. But on the whole, the fluctuations of the data are generally similar. This proves that they have a high degree of fitting. This can help to get better weights.

3. Methods

3.1 Covariance Matrix, Variance and Sharpe Ratio

When the selected assets are positively correlated, the return of the portfolio also shows a normal distribution. Olivier Ledoit and Michael Wolfs' paper uses two covariance matrices to average the optimal weights and estimate the stock returns [7]. The above article cited a good evident for the covariance matrix to calculate stock returns. The following is the covariance matrix for two assets.

$$\Sigma = \begin{bmatrix} \sigma(x,x) & \sigma(x,y) \\ \sigma(y,x) & \sigma(y,y) \end{bmatrix} \tag{1}$$

All are covariances, except for the diagonal data, which are variances. And the following formula can be used to calculate the variance of the portfolio consist of two assets.

$$\sigma^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{Covariance}(1,2) = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho \tag{2}$$

w_1w_2 represent the weight of asset one and asset two in turn. σ_1^2 is variance of first asset. σ_2^2 is variance of second asset. And $\text{Covariance}(1,2)$ means covariance of the two assets. It is also the result of multiplying the correlation coefficient (ρ) of two assets by the standard deviation.

Sharpe ratio is a commonly used indicator in portfolio construction. Sharpe ratio shows that the size of the risk determines the overall performance of the portfolio. Here is the formula.

$$\text{Sharpe ratio} = \frac{E(r_P) - r_f}{\sigma_P} \tag{3}$$

$E(r_P)$ is the expected rate of return for the year. on an investment portfolio. The standard deviation of the annualized rate of return of the portfolio is denoted by σ_P. And the annualized risk-free interest rate is expressed in r_f.

3.2 Capital Asset Pricing Model

The CAPM model mainly describe the portfolio and expected income in terms of a linear relationship and the risk-taking of investors when the financial market remains stable and balanced [8]. The CAPM model mainly determines the market rate of return of capital assets. In the selection of investment portfolio, CAPM model is also the majority choice [9]. It is more suitable for mature and rule-based market systems. As a single-factor model, its non-systematic risk can be offset by diversification, but the market risk will always exist. The relevant formula is as the following formula.

$$E(R_i) = R_f + \beta_{im}(E(R_m) - R_f) \tag{4}$$

There is a capital 'i' and a market 'm'. $E(r_i)$ is referred to as the expected return on an asset. $E(R_m)$ represents the expected market rate of return. β_{im} is the coefficient of systemic risk and beta of assets. r_f represents the risk-free interest rate. And the market risk premium is denoted by $E(R_m) - R_f$. The beta coefficient in the model can be used to assess the magnitude of the fluctuation a portfolio's risk relative to the market. Generally speaking, a portfolio with a large beta will be more volatile than the market.

Through this model, it is concluded that the greater the risk of investing in stocks, the higher the return.

3.3 Fama-French Three-factor Model

In 1993, Fama and France proposed the Fama-French Three-Factor Model, which includes SMB size factor and HML book-to-market value factor in addition to market beta [10]. This model is an extension of the CAPM model, adding two new factors that are potentially major risks. It is also frequently used to test Capital Asset Pricing Model in academic research and portfolios [11]. This model is usually represented by the following formula:

$$E(R_{it} - R_{tf}) = \beta_i(E(R_{mt}) - R_{ft}) + s_i\big(E(SMB_t)\big) + h_i(E(HML_t)) \quad (5)$$

Denote time by t and asset by i. R_{tf} represents the risk-free rate of return. R_{it} means the rate of return of the asset which is in t period of time. R_{mt} means the market rate of return. $E(R_{mt}) - R_{ft}$ is used to represent the market risk premium. SMB_t and HML_t are the simulated portfolio returns of the market value factor and the book-to-market value ratio factor. β_i, s_i and h_i are coefficients of three factors.

4. Results

In this paper, it uses covariance matrix and correlation function to analyse the degree of dispersion of assets, and then build portfolio. The covariance of every two stocks is less than 0.01, and the correlation coefficient is less than 0.5 (See Table 2). From Table 3 and Table 4, there is no negative correlation between assets. Therefore, there is no absolute causality (substitution). And there are no zeros in the correlation data, meaning that any two assets are correlated with each other. In Table 4, the correlation between asset 'JLL' and asset 'MDT' is about 0.4, which is relatively poor compared with other assets.

Table 2. Covariance matrix of five assets

	JLL return	MDT return	Foxconn return	VLO return	WMT return
JLL return	0.009676183	0.002451855	0.001693287	0.006629082	0.001183989
MDT return	0.002451855	0.003328973	0.000650696	0.003247308	0.001404189
Foxconn return	0.001693287	0.000650696	0.006687572	0.002868823	0.000616259
VLO return	0.006629082	0.003247308	0.002868823	0.019847018	0.001362463
WMT return	0.001183989	0.001404189	0.000616259	0.001362463	0.003383037

Table 3. Correlation coefficient of five assets

	JLL return	MDT return	Foxconn return	VLO return	WMT return
JLL return	1				
MDT return	0.432003895	1			
Foxconn return	0.210496226	0.137907744	1		
VLO return	0.478358775	0.399503634	0.249012941	1	
WMT return	0.206938919	0.418424429	0.129561506	0.166273697	1

As a theoretical model, CAPM usually considers the situation that alpha equals zero. It can also be said that the excess return equals zero. However, considering that the market is not completely competitive in the real world, expected rate of return is calculated separately with and without alpha in Table 4.

Table 4. CAPM model regression data

	JLL	MDT	Foxconn	VLO	WMT
MKT RP	0.00676791	0.00676791	0.00676791	0.00676791	0.00676791
RF	0.002663573	0.002663573	0.002663573	0.002663573	0.002663573
CAPM alpha	-0.000380037	-0.001704259	-0.00963955	0.002593461	0.006551826
CAPM beta	1.225052875	0.735368557	0.573186996	1.640454375	0.495633827
CAPM E(R)	0.010954621	0.007640481	0.006542851	0.013766021	0.006017978
E(R) with alpha	0.010574583	0.005936222	-0.003096699	0.016359481	0.012569804

Beta is related to the stock market and represents systemic risk. When the whole stock market rises sharply, the greater the correlation between a single stock and the stock market, the more it rises. As shown in the table, 'VLO' has the largest beta and 'WMT' has the smallest beta. In real life, excellent entrepreneurs create (alpha) excess returns to outperform the market. As shown in the table, the alpha values of VLO and WMT are greater than zero and the remaining assets are less than zero. It means that the actual performance of the two assets is better than the performance benchmark. Therefore, the actual expected rate of return considering alpha will be different from the expected rate of return of CAPM model. As the chart shows, Foxconn has the only negative expected return.

The CAPM model can be used to optimize the weight distribution and obtain the maximum Sharpe ratio by synthesize the above data information. See Table 5 for the data.

Table 5. The largest Sharpe ratio result of CAPM model

	JLL	MDT	Foxconn	VLO	WMT
CAPM weight	0.201235181	0.362247047	0.129718186	0.114077696	0.19272189
Expected Return	0.008551116				
Variance	0.002868435				
Standard deviation	0.053557776				
Sharp ratio	0.109928808				

On the whole, the weight distribution is relatively balanced, and there is no zero weight. The asset MDT has a maximum weight of 0.362247047. And that VLO has a minimum weight of 0.114077696. The reason why 'MDT' assets get the largest weight is largely due to the fact that it belongs to a medical company. It is providing long-term services to patients with chronic diseases, has a good market demand and has an excellent development history. As an energy company, VLO's development is limited by environmental protection and carbon emission control policies, which may be the reason for its minimum Weight.

However, after textual research, the beta value can't fully explain the expected return of assets. Therefore, two risk factors are added to the FF3F model, as shown in the regression results in Table 6.

Table 6. FF3F model regression data

	JLL	MDT	Foxconn	VLO	WMT
SMB RP	0.00193	0.00193	0.00193	0.00193	0.00193
HML RP	0.003527	0.003527	0.003527	0.003527	0.003527
FF3F Mkt Beta	1.099003	0.793912	0.366414	1.525735	0.621245
FF3F SMB Beta	0.70618	-0.34834	1.231564	0.596559	-0.73605
FF3F HML Beta	0.748826	-0.02492	0.279738	1.413311	-0.23354
FF3F E(R)	0.014106	0.007276	0.008507	0.019126	0.004624
FF3F alpha	0.003176	-0.00264	-0.00674	0.007454	0.004145
E(R) with alpha	0.017282	0.004637	0.00177	0.026579	0.008769

The average excess return of assets in FF3F model is divided into alpha contribution, scale factor, market factor contribution and value factor contribution. Alpha depends on the excess return of active investments (beta cannot be explained). As shown, 'MDT' and 'Foxconn' have negative alpha values.

Except that the market information beta has no negative value, the beta values of the other two factors all have negative values of asset ‘MDT’ and 'WMT.

Table 7. Maximum Sharpe ratio results for the FF3F model

	JLL	MDT	Foxconn	VLO	WMT
FF3F weight	0.355043176	0.162713427	0.236651733	0.245591664	0
Expected Return	0.012902469				
Variance	0.005246486				
Standard deviation	0.07243263				
Sharp ratio	0.141357506				

‘JLL’ has the largest weight, ‘MDT’ has a relatively small weight, and ‘WMT’ has a zero weight. The overall weight ratio is not very harmonious (See Table 7). Combined with the data in Table 6, ‘WMT’ does not perform very well in terms of market capitalization factor and size factor, while ‘JLL’ does the opposite. On the other hand, as a real estate asset, ‘JLL’ has a large and stable market demand and good historical data, so it has the largest weight. Compared with the 11% Sharpe ratio of the CAPM model in Table 5, the FF3F model has a better Sharpe ratio of 14%, which achieves the optimization of the portfolio.

5. Conclusion

Based on all the information summarizing above, portfolio building is usually based on a specific hot industry with good prospects and a stable history of development. This paper aims to help investors make rational investment choices in pharmaceutical, real estate, technology, energy and retail services industries. This paper mainly uses covariance to select assets, and uses CAPM model and FF3F model to find the best portfolio weight. In the latter two models, this paper uses the maximum Sharpe ratio to match the weights. Combined with the specific data of the capital asset pricing model and the Fama-French three-factor model, the reasons for the weight allocation of assets are further clarified. In the CAPM model, the weight of the pharmaceutical health and service retail industry is excellent. However, in the FF3F model, the real estate and technology industries have a larger proportion of the weight. And there is zero weight in the service retail industry. Despite these results, it is undeniable that these models have some shortcomings. Even the FF3F model based on the extension of CAPM model still has many unexplained parts. The risk factors in the real market are far more than the classic three in the model, so comprehensive consideration should be made.

In this paper, it only optimizes the portfolio from CAPM and FF3F model. For some extent, it is not perfect, because both models have some shortcomings. The selected data is only from five directions, of course, there are more areas worth exploring.

References

[1] Sharpe W F. Capital asset prices: A theory of market equilibrium under conditions of risk. The journal of finance, 1964, 19 (3): 425 - 442.

[2] Black F, Litterman R. Global portfolio optimization. Financial analysts journal, 1992, 48 (5): 28 - 43.

[3] Golmakani, H. R., & Fazel, M. Constrained portfolio selection using particle swarm optimization. Expert Systems with Applications, 2011, 38 (7), 8327 - 8335.

[4] Fisher J D, Sirmans C F. The role of commercial real estate in a multi-asset portfolio. Journal of property management, 1994, 59 (1): 54 - 9.

[5] Simonian J. Mixed Ag: A Regime-Based Analysis of Multi-Asset Agriculture Portfolios. The Journal of Portfolio Management, 2020, 46 (6): 135 - 146.

[6] Anghel, M. G., & Paschia, L. Using the CAPM model to estimate the profitability of a financial instrument portfolio. Annales Universitatis Apulensis: Series Oeconomica, 2013, 15 (2), 541.

[7] Olivier l., Michael W., Improved estimation of the covariance matrix of stock returns with an application to portfolio selection, Journal of Empirical Finance, 2003, 10 (5), 603 - 621.

[8] Balteș, N., Dragoe, A. G. M., & Ardelean, D. I. Study Regarding the Assets Evaluation on the Financial Market through the C. A. P. M. Model. Studia Universitatis Vasile Goldiș Arad, Seria Științe Economice, 2014, 24 (3), 78 - 87.

[9] Zhang P, Meng XH. The Market Application Analysis of CAPM Model. AMM 2013, 380 – 384.

[10] Jiao W, Lilti J J. Whether profitability and investment factors have additional explanatory power comparing with Fama-French Three-Factor Model: empirical evidence on Chinese A-share stock market. China Finance and Economic Review, 2017, 5 (1): 1 - 19.

[11] Suh D. Stock returns, risk factor loadings, and model predictions: A test of the CAPM and the Fama-French 3-factor model. West Virginia University, 2009.