Portfolio Optimization for Several Industries among the U.S. Stock Market

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Abstract. Optimizing portfolio has been a popular topic since it was proposed because it can reduce the investment risk. This study selected five active stocks from different industries, including Online E-Commerce, Commercial Banks, Motor Vehicles, Mobile Communication Production, and Telecommunications. Then they were allocated into five kinds of portfolios, which are tangency portfolios and minimum variance portfolios under Capital Asset Pricing Model and Fama-French three-factor Model, as well as the 1/N portfolio. The results found that 'BAC' and 'T' have the largest weight and the lowest weight respectively in the two models. The comparison of the five portfolios showed that the portfolio with the highest cumulative return is the tangency portfolio under FF3F Model, and the 1/N portfolio also performed well. Only these two portfolios outperformed the SPDR S&P 500EFT Trust. This research may help investors focused on the five sectors mentioned above to have a better idea of how to allocate their capital.

Key words: Capital market, factor model, portfolio optimization.

1. Introduction

The American economics Harry Markowitz first derived a pathbreaking mathematical model for portfolio selection in 1952, the mean-variance model, which is widely used to solve allocating risky assets' problems and impact profoundly on the global financial market [1]. In this model, the portfolio risk is quantified as variance, and the investors prefer to minimize it because their capital is limited, and they are afraid to lose the principal [2]. In general, the weaker the investors' risk tolerance is, the higher their loss aversion level is likely. Therefore, the optimizing portfolio method has been a hot topic for numerous scholars. Many studies indicate that diversification can reduce risk to a certain extent [3], and research on portfolio optimization have evolved considerably recently.

Previous studies often concentrated on the multivariate modeling of the market development trend and analysis, or some new theories from the macro perspective under certain economic environment. For example, Drake, Fabozzi, and Fabozzi proposed a portfolio theory and a capital market theory to be as a framework of security's risk and return measurement, and they also revealed the existing relationship between expected return and risk [4]. Feng Yongfu and other scholars investigated the volatility of the Chinese A-share market by mining data [5]. They also conducted a maximum likelihood estimation of the returns and used a theoretical-empirical model which can calibrate the volatility of the market well. Their model will contribute to simulating and forecasting, which can help local governments regulate and the individuals invest. Procacci and Aste pointed out three sources of error associated with the complex portfolio modeling system, including uncertainties resulting from parameters' sampling error, oversimplifying hypothesis, and intrinsic non-stationarity of these systems. The last two can be quantified after analyzing parameters of the training sets of various lengths. They also concluded the result that the optimal portfolio is related to its holding period. Mo and Chen used the topological structure of the financial network to select stocks to build the portfolio and then used the random matrix theory to improve the performance of the portfolio by reducing its risk [7]. However, few studies aim to concentrate on industries like Online E-Commerce, Commercial Banks, Motor Vehicles, Mobile Communication Production, and Telecommunications. It is obvious that Online E-Commerce has become increasingly important in a strategic perspective [8], as well as the other four industries, because they are an integral part of people's lives.

This paper focuses on five representative companies selected from each industry mentioned before. First, this paper collected the adjusted closing price of the five companies during the period between

December 2016 and December 2021, and the data also was checked so that they matched with each other in the time dimension. Second, different methods was used to analyze the portfolios' efficiency, which contributes to better decision-making for investors interested in certain fields. Arithmetic mean returns and expected returns based on Capital Asset Pricing Model (CAPM) and Fama-French Three-Factor Model (FF3F Model) are used to build up the tangency portfolios and the mean-variance portfolios. This paper compared the changes of each portfolio's weights and tried to explain the reasons. Additionally, naïve strategy (1/N portfolio) was concerned as well, because Demiguel, Garlappi, and Uppal indicate that the very simple portfolio performed better when comparing its Sharpe ratio with those of other optimizing models [9]. Finally, all portfolios were compared to find out which one has the highest cumulative return.

The remaining parts of this paper includes four sections. Section 2 introduces the chosen five stocks and their data used in the future study of this article. Section 3 describes the methods of getting the optimal portfolios. Section 4 and Section 5 present results with graphs and figures, as well as conclusion and discussion.

2. Data

The data used later in this paper are from the most active stocks section in Yahoofinance (https://finance.yahoo.com/most-active). This paper selected five representative companies, including AMZN, BAC, F, NOK, NVDA, and T. These stocks are considered chosen because they performed actively, and they belong to different industries in order to diversify. Their five years monthly adjusted closing prices from December 2016 to December 2021 were downloaded for the portfolio construction. The price data of SPDR S&P 500EFT Trust was also downloaded to represent market sentiment. In this paper, prices were transferred into differential returns, and the processed data were separated into a training set (2017.01-2020.12) and a test set (2020.01-2021.12). The first set is used for constructing the portfolio, and the other one is for the performance evaluation of the asset allocations. The stocks' descriptive statistics of the training set are illustrated in the Table 1 below.

Table 1. Descriptive statistics of the selected stocks (Training Set)

	AMZN	BAC	F	NOK	T
average return	3.44%	1.17%	0.19%	0.26%	-0.01%
standard deviation	8.37%	8.28%	9.05%	9.83%	5.54%
maximum return	26.89%	18.82%	19.02%	20.58%	9.92%
minimum return	-20.22%	-25.51%	-30.60%	-27.87%	-17.23%

It is illustrated in the graph that the 'AMZN' is the stock with the highest average return, while the lowest is -0.01% from the 'T'. Regarding standard deviation, the 'NOK' and the 'T' are the highest and the lowest respectively. Additionally, the 'T' has the lowest maximum return and the highest minimum return, while the highest maximum return is the 'AMZN,' and the lowest minimum return is the 'F'.

3. Method

3.1 Mean-Variance Analysis

A mathematical tool, the mean-variance analysis enables related investors to allocate assets according to defined objectives. It is a useful approach to balancing return and risk and making trade decisions rationally when constructing portfolios. In this framework, each asset is a random variable obeying a probability distribution. The arithmetic mean and the variance of one asset's return are

given the economic meanings, which are the expected return and the risk respectively [10]. The relevant formula are listed below:

$$\sum_{i=1}^{n} w_i = 1 \tag{1}$$

Where i is the serial number of $asset_i$ in the portfolio, and w_i is the weight of the $asset_i$. The total number of assets is n. In this paper, n is equal to five.

$$E(R_P) = \sum_{i=1}^{n} w_i E(r_i) \tag{2}$$

Where the 'E' and the 'P' is shorthand for 'expected' and 'portfolio' respectively. The R_P is the portfolio return and r_i is the $asset_i$ return.

$$\sigma_P^2 = \sum_{i}^{n} \sum_{j}^{n} w_i w_j Cov(r_i, r_j) \tag{3}$$

Where σ_P^2 is the variance and stands for the volatility of the portfolio, and $Cov(r_i, r_j)$ is the covariance which is used to measure the overall error between the $asset_i$ return and the $asset_j$ return.

$$Sharpe\ Ratio = \frac{E(R_P) - R_f}{\sigma_P} \tag{4}$$

The R_f means an interest rate without credit risk and market risk, and σ_P is the portfolio standard deviation.

3.2 Capital Asset Pricing Model

The CAPM, first developed by Sharpe et al. in 1967, is widely recognized as a significant return forecasting model in securities investment [3]. Although there are many new models after then, it is still classic, and its central status is undoubtable in financial economics despite of some critiques [11]. It assumes that when the capital market reaches the equilibrium, the risk and return of each asset will become balanced, which means the marginal price of risk will not change. In this case, the return of an asset consists of the risk-free rate and the risk premium rate. The higher the risk is, the correspondingly higher the value of the latter one will be. The well-known equation is showing below:

$$E(r_i) = R_f + \beta_{1,i}\big(E(R_m) - R_f\big) \tag{5}$$

In this paper, $\beta_{1,i}$ is a coefficient with the interval range of 0.5 to 2.0 that measures the systematic risk of $asset_i$, and $E(R_m)$ is the market expected return.

3.3 Fama-French Three-factor Model

A three-factor model was proposed by Fama and French, two renowned economists, for analyzing stock returns and argued that the added two factors can compensate for the risk factors not captured by market risk in CAPM. This model performs well in assessing portfolio when compared with market returns [12]. Its equation is shown below:

$$E(r_i - R_f) = \beta_{1,i}E(R_m - R_f) + \beta_{2,i}E(SMB) + \beta_{3,i}E(HML) \quad (6)$$

The $E(SMB)$ is the expected returns from low market capitalization stocks less returns from high market capitalization stocks, and the $E(HML)$ is the expected returns from high B/M stocks less returns from low B/M stocks. Respectively, $\beta_{2,i}$ as well as $\beta_{3,i}$ are the risk factor coefficients.

4. Results

The five companies' expected return can be calculated using the CAPM model and the FF3F model, and the results are shown in Table 2.

Table 2. Expected returns under CAPM and FF3F Model (Training set)

	AMZN	BAC	F	NOK	T
CAPM	1.010%	1.256%	1.077%	0.774%	0.732%
FF3F	0.792%	1.439%	1.321%	0.801%	0.790%

The expected returns under the two models are slightly different, but the highest one and the lowest one are same, i. e. the 'BAC' and the 'T'. The regression results of five companies' beta coefficients are shown in Table 3.

Table 3. Some details of five stocks under CAPM and FF3F Model (Training set)

	AMZN	BAC	F	NOK	T
$\beta_{1,i}$ (CAPM)	1.09	1.45	1.18	0.74	0.68
$\beta_{1,i}$ (FF3F)	1.40	1.31	0.84	0.71	0.60
$\beta_{2,i}$ (FF3F)	-0.65	-0.22	0.70	0.03	0.15
$\beta_{3,i}$ (FF3F)	-0.93	1.01	1.05	0.13	0.26

It is shown in Table 3 that 'T' has the lowest market beta no matter in which model, while 'BAC' and 'AMZN' wins the highest market beta respectively in FF3F model and in CAPM. This paper calculated the Sharpe ratios in two cases which is showing below in Table 4.

Table 4. Sharpe ratios under CAPM and FF3F Model (Training set)

	AMZN	BAC	F	NOK	T
CAPM	8.84%	11.90%	8.91%	5.12%	8.33%
FF3F	6.23%	14.11%	11.60%	5.39%	9.38%

The highest Sharpe ratio and the lowest one under two cases are same, i. e. BAC and NOK. Under these two models, the portfolio can be constructed with the objectives of maximum Sharpe Ratio, minimum variance, and equal weights. The results are shown in the following tables.

Table 5. Weights of tangency portfolios under CAPM and FF3F Model (Training set)

	AMZN	BAC	F	NOK	T
CAPM	27.03%	30.34%	11.77%	1.94%	28.93%
FF3F	3.13%	53.77%	19.32%	4.23%	19.56%

Table 5 illustrates the maximum Sharpe ratio portfolios, which is also called tangency portfolios. This kind of portfolios is the adoption of the mean-variance efficient frontier. After comparing 4 portfolios, Theron and Vuuren [14] concluded that the TG portfolio outperforms others at most of the

time, with positive cumulative return and relatively low long-term volatility. TG portfolios in this paper prefer to give 'BAC' a high weight.

Table 6. Minimum variance portfolios under CAPM and FF3F Model (Training set)

	AMZN	BAC	F	NOK	T
CAPM	26.10%	0.00%	4.50%	5.81%	63.59%
FF3F	26.10%	0.00%	4.50%	5.81%	63.59%

Table 6 presents the weights of minimum variance portfolios. This strategy is of great concern over the past several years as well, especially when a financial crisis happens, and risk management is needed urgently [15]. Bednarek and Patel [16] came out the result that the MVP portfolio tends to long low beta stocks and short high beta stocks, which did happen in this paper. Both models seem to choose BAC as the greatest weight stock, and it is possible because the Sharpe ratio of FF3F Model is higher than that of CAPM, the weights increase when change model from CAPM to FF3F Model. When it comes to minimum variance portfolio, the reliable stock is T for the two cases.

The data used to construct the portfolio above are from the training set. When the weights are taken into the test set, the returns are shown in Table 7.

Table 7. Yearly returns of the test set.

	return
tangency portfolio (CAPM)	27.95%
tangency portfolio (FF3F)	48.97%
minimum variance portfolio (CAPM)	4.92%
minimum variance portfolio (FF3F)	4.92%
equal weights portfolio	45.28%

The return of SPDR S&P 500EFT Trust in the same year is approximately 28.82%, which means the only two cases where the asset portfolio outperforms it are maximum Sharpe ratio (48.97%) and equal weights (45.28%). It can be observed from Figure 1 and Figure 2 as well.

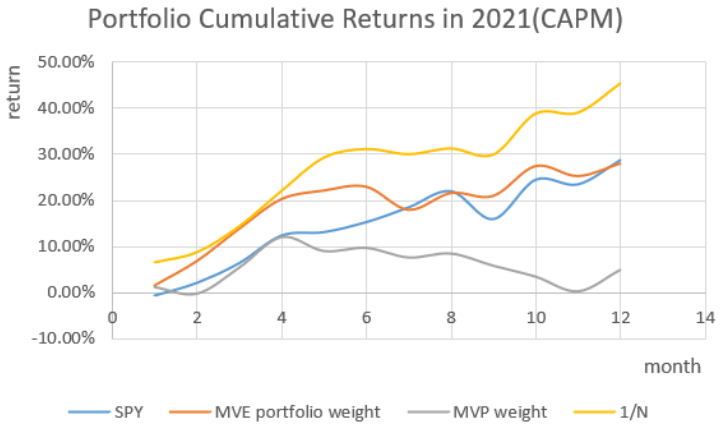


Figure 1. Comparison between 4 portfolios under CAPM model

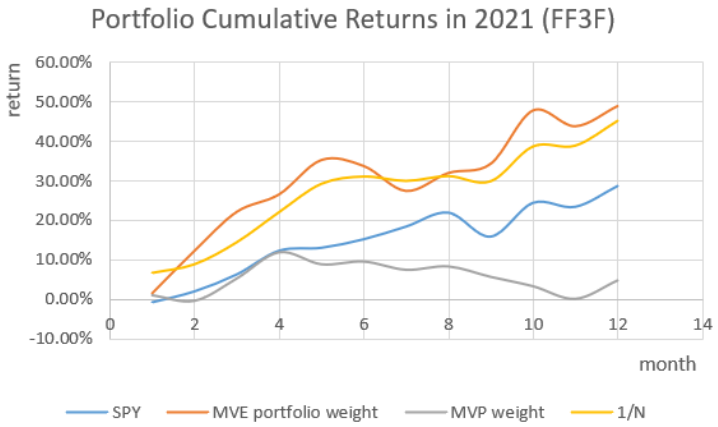


Figure 2. Comparison between 4 portfolios under the FF3F model

5. Conclusion

In summary, Existing asset portfolio studies tend to focus on the whole market or a specific popular sector, while this paper provides five portfolios (maximum Sharpe ratio and minimum variance for CAPM model, maximum Sharpe ratio and minimum variance for FF3F model, and equal weights) consisting of five stocks in different sectors for diversification purposes. In this paper, the expected return of each stock in a specific period and the covariance matrix are calculated based on the data from Yahoo Finance. After applying the mean-variance analysis to optimize the portfolio and construct the tangency portfolio and the minimum volatility portfolio, the cumulative portfolio returns for 2021 are available using the test set data.

The comparison with SPY shows that only the Sharpe ratio maximizing portfolio and the equally weighted portfolio under the FF3F model outperforms the stock index, with the former having the highest returns. 'T' has a share of over 60% in the minimum variance assets allocation, indicating that it is a worthwhile long position for risk-averse investors. In conclusion, the asset portfolio constructed by applying the FF3F model is superior, with a maximum Sharpe ratio portfolio providing the highest returns and a moderate risk. The minimum variance strategy, while reducing risk, also significantly reduces expected returns.

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