Comparison of the Pricing Model for Different Types of options

Xianglong Zeng *

Institute of mathematics, Southwestern University of Finance and Economics, Sichuan, China

*Corresponding author: 161007106@stu.cuz.edu.cn

Abstract. Contemporarily, option is one of the widely used underlying assets to hedge the risks and construct portfolio in finance field. As a matter fact, with different regulations and trading rules of various transaction center around the world, there are plenty of types of options. On account of the differences in the trading rules, the option pricing models vary a lot. On this basis, this study will select three common types of options (i.e., European option, American option, and Asian Option) in order to detailly demonstrate the differences in the pricing. To be specific, the formulae as well as the empirical analysis will be presented to vividly prove the statement. According to the analysis, the differences between the pricing procedure and models will be exhibited. In addition, the current drawbacks for the models will be illustrated and suggestions for further study will be clarified accordingly. Overall, these results shed light on guiding further exploration of option pricing from the theoretical as well as empirical sides.

Keywords: European Option; American Option; Asian Option.

1. Introduction

Option, with its unique charm in risk management and portfolio investment, provides a more effective and flexible investment tool for investors to arbitrage and avoid risk transactions. Options have been around since ancient times. The first record of options trading is a contractual agreement in the Book of Genesis. An option-like contract signed by Jacob around 1700 BC to marry Rachael, Laban's younger daughter, gave Jacob permission to marry Rachael in exchange for agreeing to work for Laban for seven years. According to the definition of the option, Jacob got the "right but not the obligation" to marry Rachael with the "royalty" of seven years' labor. In modern times, the main purpose of option trading is to manage the risk of price fluctuations. The earliest recorded use of options for risk management occurred in the Netherlands in the late 1630s. With the development of economy, there are more and more financial derivatives. Option is a kind of financial derivatives, which plays an important role in financial derivatives. Options are a product of Western economies, typically "the company pays, the market pays ", whereas China is more accustomed to incentivising employees with bonuses. However, with the development of China's capital market, options began to be active in China's market. The characteristics of option decide the development history of option in foreign countries. By collecting relevant data of the development of the domestic option market, this paper analyzes the development of the option market, studies the advantages and disadvantages of the option in the Chinese market, and forecasts the development prospects of the option in China [1].

As it known to all, Norbert Sehmitz proved that the actuarial formula of option pricing proposed by Mogens Bladt and Tina Hvid Rydberg was wrong with a counter example. On the basis of revising the actuarial formula proposed by Bladt and Rydberg, this paper quantitatively studies the composition of option value from the perspective of evaluating actual loss and corresponding probability distribution, obtains the option pricing model based on actuarial method, and further deduces the classic Black-Scholes option pricing formula. The actuarial method overcomes the shortcomings of B-S model based on risk-free arbitrage and replication, such as strict assumptions and complicated formula derivation, and points out the differences between actuarial pricing and B-S option pricing methods. Finally, an example is given to discuss the application of actuarial method in option pricing theory, which provides theoretical and practical reference value for the reasonable determination of option price in practice [2]. The behavior model of stock price is studied in the option pricing model of stock price subject to jump-diffusion process. Assuming that the jumping process of stock price is a special updating process, a behavior model of stock price obeying the

jumping diffusion process is established. Under the assumption of risk neutrality, the pricing formula of European option based on stock is derived [3]. Definitely, a compound real option model for start-up pricing assume that the research and development of a new product or business model will bring cash flow to the enterprise in the future, and the start-up will gain a small position in the product market due to patent protection. The successful research and development of a new product or new business model represents the potential growth value of the enterprise. In this paper, the value of the entrepreneurial enterprise is assumed to be its potential growth value. Its total market value GH can be understood as a physical growth option GX plus an exchange option for marketing capital input GM [4].

In order to give people who are just beginning to learn finance a clear idea, this paper discussess different types of options and their limitaions from the perspective of their basic models. Specfically, the European option, American option and Asian option are analyzed and evaluated. The rest part of the paper will be organized and separated as follows. The Sec. 2 will give a brief introduction of the option. The Sec. 3 will discuss the European option. The Sec. 4 will introduce the American option an then the Asian option. Eventually, a brief summary will be given in the Sec. 5.

2. Description

Derivatives represented by options are an important part of the entire capital market. Option trading has multiple functions such as price discovery, market efficiency improvement and risk hedging mechanism improvement. The core of option business is pricing [5]. Option is a contract that gives the people who have the option the privilege that before a particular time, people are free to decide whether they get the stuff at an unchangeable price or sell their property before a particular time. The definition of the option has something people have to consider:

- An option is a privilege. There are least a buyer and seller in an option contract. People can enjoy the rights and there are no obligations.
- Markers of options. what you choose to buy or sell is called the marker of an option. Stocks, currencies, stock indexes, government bonds, commodity futures and so on are involved.
- Maturity date. The date on which the mutually agreed option expires is called the "expiration date.
- Option execution. The act of buying or selling an underlying asset under an option contract is called an exercise.

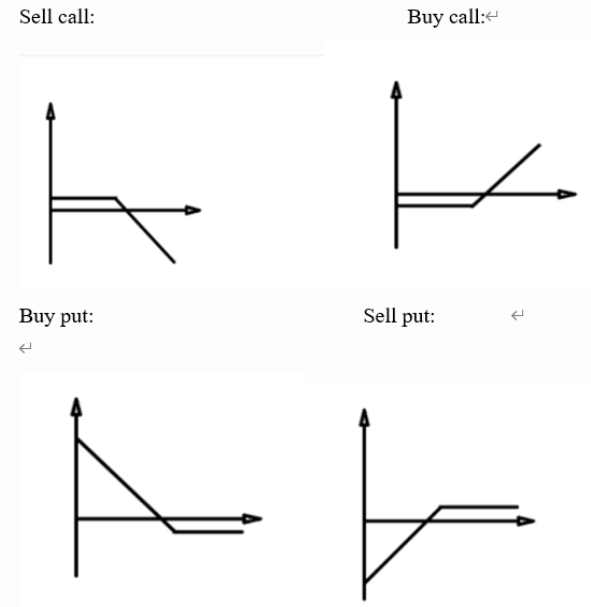


Fig. 1 The yield curve of the long and short side of the call option and the long and short side of the put option.

Call options and put options consists of the option. The call option means that a buyer of an option that can buy a specific commodity on the expiration date after paying the option price to the seller, but there is no obligation to buy it. However, the option seller has a put obligation, that is, the option seller has the obligation to sell the agreed commodity when the option buyer decides to take advantages. A put option means that the buyer of an option that can sell a specific commodity on the expiration date after paying the option premium to the seller, but there is no obligation to sell. However, the option seller has a call obligation, that is, the option buyer decides to take advantage of the option, the option seller is obligated to buy the agreed commodity. The yield curves for different situations of are shown in Figure 1.

3. Types of Options

3.1 European Option

European Options are options that buyers can only use the rights before the expiration date of the option. In this topic, here's a new financial innovation tool in the field of foreign exchange, the European foreign exchange option. It is separated into the call option and put option, which its value depends on the exchange rate.

The European option pricing formula is derived as follows. Let x be the delivery price of the option, and the option price f must satisfy the boundary conditions $f = max(s - x, 0)$ (call) or $f = max\{x - s, 0\}$ (put). Thus, the equations are simultaneous as

$$\frac{\partial f}{\partial t} + \frac{rs\partial f}{\partial s} + \frac{\sigma^2 s^2 \partial^2 f}{2} \partial s^2 = rf \tag{1}$$

From this solution, European call option prices $a = sN(p) - xe^{-r(T-t)}N(q)$ and the put option prices is $b = xe^{-r(T-t)}N(-q) - sN(-p)$. Here,

$$p = \frac{\ln\left(\frac{s}{x}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \tag{2}$$

$$q = \frac{\ln\left(\frac{s}{x}\right) + \left(r - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \tag{3}$$

$$N(x) = \int_{-\infty}^{x} e^{-t^2/2} dt \tag{5}$$

The inferences are as followings. According to the assumption of no arbitrage, simply change a real fundamental probability measure P to an equivalent P, under the assumption that r=0 (i.e., R =1, for all t), the price of an undetermined claim H is not E{H} but E*{H} (if R is not constant and does not equal 1, then, the discounted debt. The expectation of the right is E*{e^{-R_t}H}). In addition, the way to derive this price is to look for the probability P*. The price process in continuous time $S = (S_t)_{t \geq 0}$. A sufficient condition for constant expectation is that S is a martingale (i.e., $t \to E\{s_t\}$ is constant [6]).

3.2 American Option

American option is a type of option that can be exercuted on any day after the expiration. This means that the option holder can choose whether he exercises the option contract on trading time. American options allow the option holder to exercise the right to sell (in the case of a put option) the underlying asset or buy (in the case of a call option) on or before the expiration date.

As for American call option pricing with no dividend. The buyer of the option, which is in the active state, immediately exercises the option and sells the stock in the stock market to realize the return S-E. However, this does not mean that the option is eligible for early exercise. On the contrary, this practice is extremely unwise, because it makes the buyer only receive the implied worth of the option, but miss their time value. To be specific, because of the rising stock price, the opportunity to increase the value of the option (as exhibited in Fig. 2). Therefore, the buyer should continue to maintain the long position in the hope of obtaining higher returns. If the buyer expects the stock price to be overvalued and the option to move to a two-flat or imaginary state, the buyer should not exercise

the option but sell it. In order to obtain both the connotation value and the time value. To sum up, American call options are not subject to early exercise under any circumstances. As a result, the price is the same as a European call option, i.e., $C_A(s,t) = C_E(s,t)$.

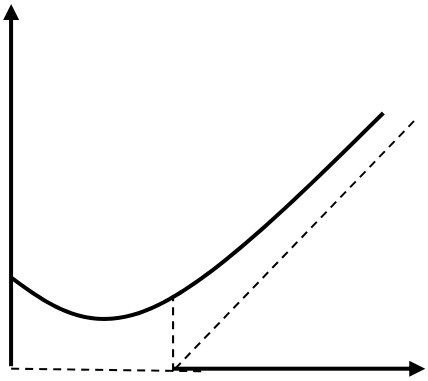


Fig. 2 Value of the option.

Regarding to American put option pricing with no dividend, because in the real world, stock prices can't go down indefinitely, that's S > 0. Therefore, when the put option is at the deep real value, the implied value has basically reached the maximum value, while the time value may have a negative growth, that is, s shows a trend of changing in the opposite direction (as presented in Fig. 3).To get away from the loss of time value, the s drops to a certain extent (denoted as $s_{f(t)}$).), the option should be executed to obtain the yield E-S, otherwise continue to maintain a position, and price is the equal to the European put option (as depicted in Fig. 4) [7].

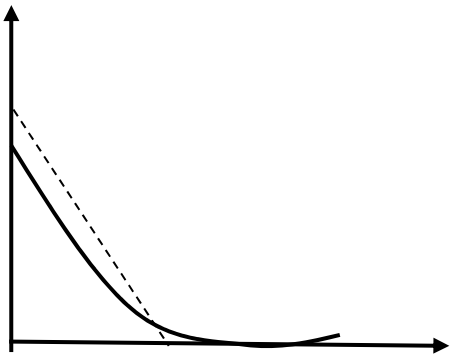


Fig. 3 Trend of changing

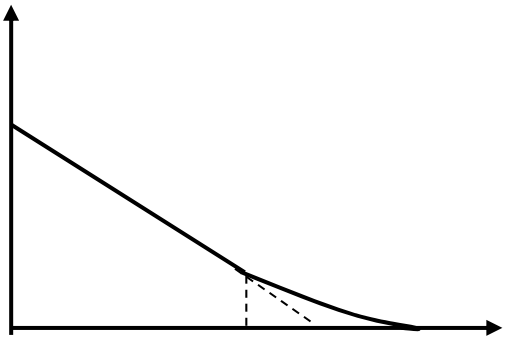


Fig. 4 Price

American options usually have a higher value because they have a larger margin than their European counterparts. Nevertheless, there are obvious exceotions. The foreign exchange options and some stock index options. Therefore, American-style option is more changeable than European option because it gives the buyer more opportunities, while the seller is always faced with the risk of performance of the contract. Therefore, American-style option's royalty is comparatively high than others.

3.3 Asian Option

As a matter of fact, the difference with the standard option is that the yield of the option at expiration is determined by the mean volume of the targets during a certain period rather than the current market price of the underlying asset. Followings are some functions:

- Avert artificially hyping stock prices.
- Lessen the unknowable deal and damaging the interests of the firm.

However, the sub-option which appeared in the 1990s is obviously different from the standard option, which is essentially an innovation of the European option. The common point between the Asian option and the European option is that they only allow the investor to exercise on the expiration date T. The difference is that European options decide whether to execute the option contract based on the level of the price S(T). The sub-option determines whether to execute the option contract in term of the level of the average stock price A(T) within the contract period [0,T]. The expression of A(T) is as follows:$A(t) = \frac{1}{t}\int_0^\tau S(\tau)\,\mathrm{d}\tau$. Due to the different calculation methods, sub-options can be divided into Average Rate Option and Average Strike Option. The use of A(T) in the two is different: the former uses A(T) to replace the stock price S(T) on the expiration date of European options to avoid the risk of price fluctuations. The latter replaces the strike price E with A(T). The difference between the three is shown in Table 1 [8].

Table. 1 The option yield expression for different types.

Class of options	Option yield expression
European Option	MAX$(S(T)-E,0)$
Average-priced sub-options	MAX$(\mathrm{A}(T)-E,0)$
Average strike price sub-option	MAX$(S(T)-A(T),0)$

4. Limitations&Prospects

Nevertheless, for about the European option. It is discussed in a financial theory, the American option may be superior to the European option after thinking about some particular effects (like cash dividends).For instance, when Company A suddenly declares the cash dividend of a higher amount than expected, the holder can immediately demand the performance of the agreement, convert the option into stock, and receive the cash dividend. Holders of the company's European options will have to sit back and miss out on early stock swaps and cash dividends. However, apart from this particular factor, they found that the American option and the European option were no better than the other. Intuitively, if their investment options are a privilege. People all know the idea that the right is changable means it can be valuable. European options are less flexible than American options. However, the truth is that after knowing the calculation of the value, people now accept that the value of the European option and the American option are the same, except for things like cash dividends.

When it comes to American option. In the pricing of financial derivative products, compared with the traditional approaches, Monte Carlo simulation method has four advantages:

- the standard error and convergence rate of the simulation estimate are strongly independent from the dimension of the question;
- more flexible;
- the state variables can be subject to the usual random process, without being restricted to the assumption of normal distribution;
- it can be used for pricing of path-dependent and derivative products with American execution characteristics.

The weighted least square pseudo-Monte Carlo simulation (WLSQM) method uses Faure sequence and variance reduction techniques to reduce the variance of the estimated results. The randomization of Faure sequence avoids the problem of determining periodicity of sequence. The fitting result of weighted least-squares regression is better than that of ordinary least-squares when

estimating the return expectation of option holders who continue to hold the option, thus no large and unreasonable result is produced [9].

Finally, Asian option also has its advantages. Taking arithmetic mean Asian call option pricing as an example, the advantages and disadvantages of two typical models (i.e., MC and Quasi-MC) in option pricing are compared and discussed. The specific step is to compare the difference between the two types of methods in low and high dimensions by increasing dimension successively. The result is that for both low and high dimensions, the quasi approach shows better performaces than the pure scenarios after a short period of being inferior to the MC method [10].

5. Conclusion

In summary, this paper investigates the pfricing models for differnent types of options. To be specific, European option, American option and Asian option are selected to analyze with corresponding empirical study and formaule. According to the analysis, they all have advantages in calculating with the Monte Carlo method. Nevertheless, there're some troubles when people actually using the model to price the option. In the future, one may gather the advantages from various options. Overall, these results offer a guideline for combination of various options and pricing model development for further research.

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