Option Pricing Based on Black-Scholes Model, Monte Carlo Method and Binomial Tree Model

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Abstract. Contemporarily, option is one of the widely implemented underlying assets to hedge the risk of portfolio and satisfy certain trading motivation for investors. To obtain extra return based on investment, it is crucial to utilize and apply an accuracy and suitable tool to price option. On this basis, this study focused on comparing and applying different model to achieve the goal of option pricing. To be specific, the Python code was used to help make calculations and list the results, with three functions to price options with three diverse methods. This paper chose Alphabet Inc., Amazon, Meta Platforms, Spotify, Sunrun, and Tesla, Inc. and priced the options using the historical market data from July 20, 2022, to October 20, 2022. Reasonable results for each option are achieved with differences between each method, offering price ranges for the options. These results offer a guideline for applying different methods to option pricing and compare the effectiveness of the three methods.

Keywords: Option pricing; Black-Scholes Model; Monte Carlo Method; Binomial Tree Model.

1. Introduction

An option is a contract in the financial markets that gives the holder the right to purchase certain quantity of a specific trading products or underlying assets on a specific time range period [1-3]. In particular, as a special type of underlying assets, it has different trading features compared to other traditional trading underlying assets (e.g., stocks and futures) [4, 5]. To put the trading tool of option in a certain portfolio, it is important to denote the utilizing method for investing options, hence the pricing process for option is an inevitable task.

As a matter of fact, the valuation of the option determines the holder's profit and loss [6-8]. In general, there are various approaches for pricing options. In research by Black and Scholes [9], they proposed a math expression for simply pricing the products with a known expiration date, and the method now is known as the Black-Scholes Model. Boyle initiated the use of the Monte Carlo method in option pricing, and it is a famous way of option pricing [10]. According to the research of Rendleman and Bartter [11], in their research, they used an iterative approach that utilized multiple periods to value American options, and now the method is called the Binomial tree model.

To compare the effectiveness of the option models on different underlying assets, this paper chooses Alphabet Inc., Amazon, Meta Platforms, Spotify, Sunrun, and Tesla, Inc. To examine these questions, we chose the three most used option pricing models. They are the Black-Scholes model, the Monte Carlo method, and the binomial tree model. In this project, these three models are implemented in Python. The result part of the paper will be separated as follows. The Sec. 2 will focus on processing data and the method adopted. The Sec. 3 will discuss the result and analyze comparison. The Sec. 4 will concentrate on the limitation and the prospects for future. Eventually, a brief summary will be given in Sec. 5.

2. Data & Method

2.1 Data

There are multiple types of underlying assets or financial instruments of the option, such as stocks, government bonds, currencies, etc. The option pricing of stocks is accomplished in this project. Regarding stock selection, several principles need to be followed. First, the stocks should be highly

liquid, which makes applying option pricing models to them meaningful since their liquidity affects the information provided by the stock price. The volume of stocks indicates investor demand for their option and as the volume increases, the option pricing will become more efficient. Therefore, the selected stocks need to have high volumes. What's more, to simplify the option pricing process, the companies of the selected stocks should be companies that do not pay dividends.



Fig. 1 The return of Google, Amazon, and Meta Platforms. The benchmark is set to Google.

According to the stock selection principles given above, the stocks of six companies are chosen: Alphabet Inc., which is known as Google, Amazon, Meta Platforms, Spotify, Sunrun, and Tesla, Inc. The price trends of the companies are given in Fig. 1. The three approaches of option pricing covered in this report work well only for European options, so European options of the stocks are selected. Historical market data from July 20, 2022 to October 20, 2022 of the selected stocks are downloaded from Yahoo Finance API, and adjusted close prices are used in the models. The close price on October 20, 2022 is used as the spot stock price as well as the strike price. London Interbank Offered Rate (LIBOR) on October 20, 2022 is used as the risk rate, which is 5.34%.



Fig. 2 The return of Spotify, Sunrun, and Tesla. The benchmark is set to Spotify.

2.2 Models

The important parameter volatility (denoted by the symbol Sigma) is required in all three option pricing models. It represents the magnitude of stock price fluctuations. Volatility is calculated by the following formula:

$$\sigma = \frac{s}{\sqrt{\tau}} \tag{1}$$

where

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(u_i - \bar{u})^2} \tag{2}$$

$$\mathrm{T} = \frac{Length\ of\ time\ interval}{252} \tag{3}$$

$$u_i = \ln(\frac{S_i}{S_{i-1}}) \ , i = 1, \dots, n \tag{4}$$

and n+1 in this project is the number of trading days from July 20, 2022 to October 20, 2022. Si is the adjusted closing price for each day, τ requires the length of time interval (1 day) divided by 252, because it is measured in years, assuming a year with 252 trading days.

2.2.1 B-S model

The B-S model is one of the most famous option pricing models, and the Black-Scholes formula derived from this model gives a theoretical estimate of the European option price. In this model, there are two probability coefficients d_1 and d_2. d_1 and d_2 are calculated as follows:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right)+T(r+\frac{\sigma^2}{2})}{\sigma\sqrt{T}} \tag{5}$$

and

$$d_2 = d_1 - \sigma\sqrt{T} \tag{6}$$

Below are the formulae used to calculate the call price (C) and put price (P) of the option:

$$C = N(d_1) - N(d_2)Ke^{-rT} \tag{7}$$

and

$$P = N(-d_2)Ke^{-rT} - N(-d_1)S \tag{8}$$

2.2.2 MC option model

The MC option model applies the Monte Carlo method to calculate the value of an option. Usually, the standard model for evolution is generated by the Wiener process as below. Repeating this equation allows multiple potential future asset paths to be generated, where ε is a random number sampled from a standard normal distribution.

$$S_T = S_t e^{\left(\mu-\frac{\sigma^2}{2}\right)(T-t)+\sigma\epsilon\sqrt{T-t}} \tag{9}$$

Here, ST is the average value of the asset price and K is the strike. The formulas below are used to calculate the call price (CT) and put price (PT).

$$P_T = PV[E(\max(0, K - S_T))] = e^{-r(T-t)}E(\max(0, K - S_T)) \tag{10}$$

$$C_T = PV[E(\max(0, S_T - K))] = e^{-r(T-t)}E(\max(0, S_T - K)) \tag{11}$$

2.2.3 Binomial option pricing model

The binomial option pricing model assumes that stock prices move in only two directions, upward and downward. The basic assumption of the binomial option pricing model is that the price of a security moves in two possible directions over a given time interval: up or down. The main function of this combination is to give a way to price call options. Unlike futures, where the hedging transaction, once established, does not need to be changed, options hedging is constantly adjusted until the option expires:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{12}$$

$$d = \frac{1}{u} \tag{13}$$

$$p = \frac{e^{r\Delta t}-d}{u-d} \tag{14}$$

Here, u is the magnitude of the stock's rise and d is the magnitude of the stock's fall. p is the probability of the stock's rise.

2.3 Processing

Firstly, data from 6 companies were downloaded from Yahoo by yfinance in python. Then, some parameters such as T, r, K were initialized, while sigma would be computed as well by definition. After that, option pricing using each model was applied by operating each function to compute the formula. Ultimately, the results are sorted in an entire sheet.

3. Results & Discussion

3.1 B-S Model

The results of the Black-Scholes model for the six companies are presented in Table. 1. Seen from the results, the call and put the price of Sunrun is 7.123455 and 6.101149. The call and put the price of Amazon are 23.21716 and 17.22424. The call and put prices of Spotify are 19.9842 and 15.427. The call and put prices of Sunrun are 16.24203 and 11.04366. The call and put the price of Meta are 27.83786 and 20.99839. The call and put the price of Tesla is 50.3698 and 39.59139.

Table 1. Results of the Black-Scholes model

	Sunrun	Amazon	Spotify	Alphabet	Meta	Tesla
Black-Scholes Call Price	7.123455	23.21716	19.98942	16.24203	27.83786	50.3698
Black-Scholes Put Price	6.101149	17.22424	15.427	11.04366	20.99839	39.59139

3.2 Monte Carlo Option Model

The results of the MC method for the six companies are presented in Table. 2. Seen from the results, the call and put the price of Sunrun is 7.120396 and 6.104443. The call and put the price of Amazon are 23.20636 and 17.23427. The call and put the price of Spotify are 19.98029 and 15.43594. The call and put the price of Sunrun are 16.23423 and 11.05005. The call and put the price of Meta are 27.82499 and 21.01058. The call and put the price of Tesla is 50.34704 and 39.61424.

Table 2. Results of the Monte Carlo option model

	Sunrun	Amazon	Spotify	Alphabet	Meta	Tesla
Monte Carlo Call Price	7.120396	23.20636	19.98029	16.23423	27.82499	50.34704
Monte Carlo Put Price	6.104443	17.23427	15.43594	11.05005	21.01058	39.61424

3.3 Binomial Tree Option Pricing Model

The results of the model for the six companies are presented in Table. 3. Seen from the results, the call and put the price of Sunrun is 7.123345 and 6.101039. The call and put the price of Amazon are 23.21682 and 17.22391. The call and put the price of Spotify are 19.98912 and 15.42671. The call and put the price of Sunrun are 16.2418 and 11.04344. The call and put the price of Meta are 27.83745 and 20.99799. The call and put the price of Tesla is 50.36905 and 39.59064.

Table 3. Results of the binomial option pricing model.

	Sunrun	Amazon	Spotify	Alphabet	Meta	Tesla
Binomial Tree Call Price	7.123345	23.21682	19.98912	16.2418	27.83745	50.36905
Binomial Tree Put Price	6.101039	17.22391	15.42671	11.04344	20.99799	39.59064

3.4 Comparison & Suggestion

The call price of Sunrun is predicted to be between 7.120396 and 7.123455, while the put price seems to be between 6.101039 and 6.104443. The call price of Amazon is predicted to be between 23.20636 and 23.21716, and the put price seems to be between 17.22391 and 17.23427. The call price

of Spotify is predicted to be between 19.98029 and 19.98942, while the put price seems to be between 15.42671 and 15.43594. The call price of Alphabet is predicted to be between 16.23423 and 16.24203, and the put price seems to be between 11.04344 and 11.05005. The call price of Meta is predicted to be between 27.82499 and 27.83786, while the put price seems to be between 20.99799 and 21.01058. The call price of Tesla is predicted to be between 50.34704 and 50.3698, and the put price seems to be between 39.59064 and 39.61424.

To conclude, for the call price, Black-Scholes model always got the maximal result, and Monte Carlo model got the minimal result. For the put price, Monte Carlo model always got the maximal result, and the Binomial tree model got the minimal result. The prices calculated by all the methods are mainly similar, whereas there's an obvious difference between the Monte Carlo method and the other two models. The call price seems lower than others while the put price seems higher. This phenomenon happens might because the parameter selection in the binomial tree method and the MC method are not appropriate. For instance, the step size is insufficient, and the time interval is not short fairly.

4. Limitation & Prospects

Nevertheless, regarding the B-S model can only be used to price European-style options, no model is applied to American options in this project, which require combination of the multiple methods with stochastic process. Additionally, only companies not paying dividends are chosen for this project. For further research, one ought to calculate the valuation of the American option, making it more accurate firstly. Subsequently, it is also worthwhile to price the varied options, for example, the price of Asian options, the option of options. Moreover, option pricing on assets such as stocks where the companies pay dividends is what further study ought to focus on. As a matter of fact, some of the machine learning scenarios can also be adopted to help predict certain components (e.g., volatility) for the sake of improving the accuracy of the prediction. Meanwhile, the deep learning schemes can also be constructed for directly price different types of option.

5. Conclusion

In summary, this study focuses on the option pricing for six companies based on three different models. Successfully, all the put and call prices were calculated with 3 methods, and the data are availed to compare the effectiveness of different methods. A rough similarity of all the results revealed availability of the option pricing. Additionally, the difference seen between the methods of different precision conducted the essentiality of choosing some parameters. To simplify the process, only European options of companies that do not pay dividends are considered, so that what might be done in the future is to apply option pricing to also American options, as well as the methods considering the dividends. Overall, these results offer a guideline for applying different methods to option pricing and compare the effectiveness of the three methods.

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