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Spillovers in High-order Moments of Stocks, Foreign Currency Exchange and Bitcoin

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Abstract

This study investigates the higher-order spillover effects between stocks, exchange rates, and Bitcoin in the U.S. By using weekly data, the analysis focuses on realized volatility, realized skewness, and realized kurtosis innovatively introduces the jump statistic. A Vector Autoregressive model is constructed for Granger causality tests, and the Generalized Impulse Response Function is applied to analyze the dynamic interactions between different markets. The results show that there are varying spillover effects between Bitcoin and traditional financial assets across different higher-order estimators, underscoring strong, non-negligible interactions among third and fourth order moments. Policymakers should closely monitor the potential instability that Bitcoin poses to the global financial system and implement differentiated regulatory strategies for different markets.

Keywords

Higher-order Moments; Spillover Effects; Bitcoin; Financial Instability.

1. Introduction

Since its launch in 2009, Bitcoin has evolved from a niche digital asset to a major player in the global financial market, currently priced around $30,000 with a market capitalization exceeding $500 billion. Its price distribution exhibits significant skewness and kurtosis, driven by speculation, regulatory uncertainties, and market sentiment, resulting in high volatility and vulnerability to external shocks.

Traditional financial markets, such as stocks and exchange rates, also display distinct patterns of skewness and kurtosis. Stock markets typically show high kurtosis and lower skewness, indicating a “fat-tailed” distribution and an elevated risk of extreme price fluctuations. In contrast, exchange rate markets often feature bimodal distributions(Baele, L., & Inghelbrecht, K., 2022), with their moments reacting to economic events and policy changes. Analyzing spillover effects between Bitcoin and these assets, particularly regarding higher-order moments, helps clarify Bitcoin's role in the financial system and assists in managing volatility and risk.

This study seeks to uncover spillover effects among higher-order moments between Bitcoin, stocks, and exchange rates, focusing on risk transmission pathways amid increasing global economic uncertainties. Understanding these interdependencies is crucial for effective investor risk management and policy interventions during extreme events that could threaten financial stability.

We introduce the jump statistic to identify sudden price movements in Bitcoin and other assets, which often arise from news, policy shifts, or liquidity shocks. Analyzing jump spillovers enhances our understanding of Bitcoin's market influence.

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2. Related Studies

In financial market research, spillover effects are crucial for understanding how assets influence each other and transmit risk. Chen et al. (2019) identified significant spillover effects between commodity and stock markets during economic crises using the DCC-GARCH model. Recent studies, such as those by Wang et al. (2020), explore higher-order moment risk spillovers, showing how skewness and kurtosis impact market dynamics.

The foreign exchange market also exhibits complex spillover effects influenced by economic conditions. Duan et al. (2021) found that the COVID-19 pandemic intensified exchange rate volatility's effects on macroeconomic variables, while Khan et al. (2023) examined the impact of oil price fluctuations on emerging market exchange rates. Mihailov et al. (2020) focused on spillovers among currency pairs, highlighting their influence on exchange rate volatility.

Significant spillover effects exist between foreign exchange and stock markets, with varying characteristics. Wang and Li (2018) provided evidence of how exchange rate changes affect stock market performance in BRICS countries. Ariel and Liu (2019) explored the dynamic relationship between stock prices and exchange rates in emerging markets using nonlinear models to analyze spillover effects, emphasizing the complexity of market volatility.

Bitcoin's volatility impacts traditional asset markets significantly. Shahzad et al. (2020) noted Bitcoin's importance for diversified investment portfolios in G7 markets, while Bouri et al. (2020) highlighted its interaction with traditional financial markets. Gkillas et al. (2020) examined Bitcoin's role in risk transmission among oil, gold, and Bitcoin.

Despite extensive research, there is a gap in integrating Bitcoin into spillover models between stock and foreign exchange markets. This study aims to address that gap by analyzing how Bitcoin influences risk transmission across these markets.

Market jumps, characterized by sudden price fluctuations, are vital for risk management. Merton (1976) emphasized including jump risk in asset pricing, while Klein and Bredin (2018) demonstrated jump risk spillovers between stock and foreign exchange markets.

This study will use a multivariate VAR model to analyze the interrelationships among Bitcoin, stock markets, and foreign exchange markets in Japan, the U.S., and China. The paper is organized into six sections: the first introduces the research background and motivation; the second reviews literature; the third details data and methodology; the fourth presents results; the fifth offers conclusions; and the sixth lists references.

3. Methodology

This research investigates the spillover effects on realized distribution moments—realized volatility, jumps, realized skewness, and realized kurtosis—across Bitcoin, stock, and exchange rate markets. We discuss intraweek data adjustments and the calculation of daily realized moment estimators. Weekly returns were constructed from daily data for four assets, covering January 1, 2017, to August 31, 2024, across 400 calendar weeks. Bitcoin data is sourced from Bitcoincharts, while stock market and exchange rate data come from the Wind database, utilizing the S&P 500, CSI 300, and Nikkei 225 indices to represent the U.S., China, and Japan, respectively, along with USDCNY, USDJPY, and JPYCNY exchange rates.

Daily returns for each price series are calculated using the logarithmic difference between consecutive prices. Specifically, the daily return for the $t-th$ observation on the t-th day is given by:

$$r_{t,i} = \log(P_{t,i}) - \log(P_{t,i-1}) \tag{1}$$

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where $r_{t,i}$ denotes the daily returns, and $P_{t,i}$ is the price for the $i-th$ observation on day t, with i ranging from 1 to T.

For each day t, we calculate the daily realized volatility RV_t. using all intraday returns from the dataset. This RV_t serves as an estimator of the second realized moment, reflecting the dispersion risk associated with the price process and measuring the average deviation of observed returns from the mean return. The calculation method for RV_t for each day t is described as follows:

$$RV_t = \sum_{i=1}^{T} r_{t,i}^2 \tag{2}$$

We subsequently detect jumps by analyzing the realized volatility through the method suggested by Duong and Swanson (2015). This detection process relies on choosing a jump-robust realized volatility estimator. Here, we utilize the threshold bi-power variation ($TBPV_t$) estimator, following the approach of Corsi, Pirino, and Renò (2010), to maintain robustness in the presence of jumps. The jump statistic ($ZJ_t^{(TBPV)}$) is formulated as follows:

$$ZJ_t^{(TBPV)} = \sqrt{T}\frac{(RV_t - TBPV_t)RV_t^{-1}}{[(\xi_1^{-4} + 2\xi_1^{-2} - 5)max\{1, TQ_t TBPV_t^{-2}\}]^{1/2}} \tag{3}$$

In this context, TQ_t denotes the realized tri-power quarticity, which is computed using the following formula:

$$TQ_t = T\xi_{4/3}^{-3}\sum\nolimits_{i=1}^{T}\left|r_{t,i}\right|^{4/3}\left|r_{t,i+1}\right|^{4/3}\left|r_{t,i+2}\right|^{4/3} \tag{4}$$

which converges in probability to the integrated quarticity. To estimate the jump-free volatility, the threshold bi-power variation ($TBPV_t$) is employed, as defined by the following formula:

$$TBPV_t = \sum_{i=2}^{T}\left|r_{t,i-1}\right|\left|r_{t,i}\right| I_{\left\{\left|r_{t,i-1}\right|^2 \leq \theta_{i-1}\right\}} I_{\left\{\left|r_{t,i}\right|^2 \leq \theta_i\right\}} \tag{5}$$

In this context, $I_{\{\cdot\}}$ denotes the indicator function, with $r_{t,i}$ representing the daily return series and t indicating time at a daily frequency. A jump is deemed statistically significant when $ZJ_t^{(TBPV)}$ exceeds the critical value from the standard Gaussian distribution. Therefore, the jump component of daily realized volatility is defined accordingly. Here, $I_{\{\cdot\}}$ functions as an indicator to determine whether $ZJ_t^{(TBPV)}$ exceeds a specified critical threshold ϕ_α from the Gaussian distribution at a chosen significance level.

$$J_t = |RV_t - TBPV_t| I_{\left\{ZJ_t^{(TBPV)} > \phi_\alpha\right\}} \tag{6}$$

Realized skewness (RS_t) measures asymmetry risk and indicates potential crash risk by assessing the conditional skewness of daily returns. The daily realized skewness is calculated as follows and normalized by dividing by $RV_t^{3/2}$

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$$RS_t = \frac{\sqrt{T}\sum_{i=1}^{T} r_{t,i}^3}{RV_t^{3/2}} \tag{7}$$

The calculation of intraday realized kurtosis RK_t is described in Equation 8. This metric measures kurtosis risk in a univariate price process, indicating the thickness of the tails around the mean. To normalize the measurement, it is divided by RV_t^2.

$$RK_t = \frac{T\sum_{i=1}^{T} r_{t,i}^4}{RV_t^2} \tag{8}$$

Next, we perform Granger causality tests within a four-variable vector autoregressive (VAR) framework to assess the directional relationships between the four markets being analyzed. A k-dimensional VAR model can be generally expressed as follows:

$$Y_t = \nu + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + \varepsilon_t \tag{9}$$

where Y_t represents a $K \times 1$ vector of variables, ν denotes the $K \times 1$ intercept vector, A is the $K \times K$ coefficient matrix, and ε_t refers to the $K \times 1$ error term vector.

After verifying stationarity and cointegration using the ADF unit root test and Johansen test, we conducted the Granger causality analysis and generated Generalized Impulse Response Function (GIRF) plots. The model's lag order was selected based on AIC and BIC. However, a very low lag order (e.g., lag of 1) can cause a rapid decline in GIRF responses, limiting the capture of dynamic interactions. In such cases, we prefer the lag order determined by the LR test..

The analysis of the Generalized Impulse Response Function (GIRF) provides insights into the causal relationships among Bitcoin, stocks, and exchange rates. Specifically, the GIRF measures the system's response to a one-standard-deviation shock in the $j-th$ variable at time t, as observed at time $t+h$,. This response is calculated using the formula:

$$\hat{\psi}_j(h) = \sigma_{jj}^{-1/2} \prod\nolimits_h \sum_{\varepsilon} e_j\, , h = 0,1,2,\cdots \tag{10}$$

Here, $\sum_\varepsilon = \{\sigma_{ij}\}$ represents the $K \times K$ variance-covariance matrix related to the error term $\boldsymbol{\varepsilon_t}$, where e_j is a $K \times 1$ vector with the $j-th$ element set to 1 and all other elements set to zero, applicable for $i, j = 1,2, \cdots K$. The term $\prod_i$ refers to a $K \times K$ coefficient matrix, obtained from the infinite moving average form of the previous equation. Additionally, the matrix $\prod_i$ can be derived recursively using the formula for $\prod_0$, which is equivalent to I_K, denoting a K-dimensional identity matrix.

$$\prod\nolimits_i = \begin{cases} \sum_{j=1}^{i} \prod_{i-j} A_j\, , \quad i = 1,2,\cdots p \\ \sum_{j=1}^{p} \prod_i - jA_j\, , i > p \end{cases} \tag{11}$$

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4. Empirical Results

Using the methodology outlined in Section 3, we conducted Granger causality tests and GIRF analyses for Bitcoin, stocks, and exchange rates in the US, Japan, and China. The results of the Granger causality tests for the realized moment estimators of the three assets in the three-variable VAR system are displayed in the tables. The p-values of the tests are given in parentheses. Panels A, B, C, and D respectively cover the realized second moment (RV), jumps (ZJ), the realized third moment (RS), and the realized fourth moment (RK). In each panel, the vertical axis represents the dependent variables in the system, while the horizontal axis represents the explanatory variables. The graphs illustrate the response values of the four assets over 10 lags after receiving a shock.

Table 1 presents the spillover relationships between Bitcoin and the S&P 500 index, as well as the USD/JPY exchange rate in the U.S. market. Panel A demonstrates the Granger causality relationships among the realized volatilities (RV) of the variables over the week. We observe a significant bidirectional Granger causality between Bitcoin and the RV of the S&P 500. Additionally, the RV of USD/JPY has a unidirectional Granger causality on both the S&P 500 index and Bitcoin's RV.

Panel B of Table 1 presents the results of jump analyses based on discontinuous estimators. We also observe a bidirectional Granger causality between Bitcoin and the S&P 500 index. Furthermore, USD/JPY not only continues to Granger cause Bitcoin's jumps, but also exhibits a bidirectional Granger causality on the jumps of the S&P 500 index.

In the analysis results of realized skewness presented in Panel C, Bitcoin's Granger causal effect on stocks disappears, but it re-emerges as a Granger cause of USD/JPY, with a bidirectional causal relationship between the two. The unidirectional Granger causality from the S&P 500 index to Bitcoin remains. In terms of realized kurtosis, as shown in Panel D, Bitcoin's impact vanishes, leaving only the bidirectional causality between the S&P 500 index and USD/JPY.

The GIRF results in Panel A of Figure 1 indicate that the realized volatilities (RV) of the two variables show a negative response to Bitcoin shocks in the early periods, although this negative response gradually approaches zero around the fifth period and remains within the negative range throughout the observation period. For shocks to the S&P 500 index, Bitcoin quickly exhibits a significant positive feedback, which then rapidly declines and eventually converges negatively toward zero. Bitcoin's RV mostly maintains a negative response to shocks from USD/JPY, except for a positive response in the third period. The stock market generally responds negatively to shocks from USD/JPY.

The GIRF results for jumps in Panel B show that Bitcoin shocks generally have a strong positive initial impact on U.S. stocks and USD/JPY, which later turns negative. The positive response of the S&P 500 index to shocks from other variables typically ends within the first three periods, after which it fluctuates around zero. The responses of Bitcoin and the S&P 500 index to shocks from the exchange rate market demonstrate significant volatility but are generally positive.

The GIRF results for realized skewness show significant differences compared to RV and jumps, with more pronounced volatility. After experiencing shocks from Bitcoin, the S&P 500 index and USD/JPY exhibit strong fluctuations, initially positive and then negative, with this trend persisting until the tenth period. The spillover effects of shocks in the stock market and exchange rate market also fluctuate noticeably around zero.

The realized kurtosis within the week emphasizes the short-term effects of shocks on the variables. Overall, when subjected to shocks from other variables, Bitcoin shows fluctuations around the X-axis in the early stages, gradually trending toward zero in the later stages. The response of the S&P 500 index's RK to shocks from Bitcoin exhibits a pattern of first positive,

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then negative, and back to positive fluctuations. In contrast, the response of the USD/JPY exchange rate's RK is generally negative.

Table 1. VAR Granger causality tests among realized estimators of Bitcoin and US markets

Variables	Bitcoin	S&P500	USDJPY	All
Panel A: realized volatility				
Bitcoin	-	16.422**	5.4929	23.613**
	-	[0.012]	[0.482]	[0.023]
S&P500	14.323**	-	2.1824	16.445
	[0.026]	-	[0.902]	[0.172]
USDJPY	16.877***	32.795***	-	50.156***
	[0.010]	[0.000]	-	[0.000]
Panel B: jumps				
Bitcoin	-	31.743***	5.2186	36.436***
	-	[0.000]	[0.516]	[0.000]
S&P500	92.505***	-	12.322*	2000.81***
	[0.000]	-	[0.055]	[0.000]
USDJPY	23.555***	30.479***	-	50.851***
	[0.001]	[0.000]	-	[0.000]
Panel C: realized skewness				
Bitcoin	-	6.7736	22.812***	30.659**
	-	[0.661]	[0.007]	[0.032]
S&P500	15.18*	-	9.7938	25.249
	[0.086]	-	[0.367]	[0.118]
USDJPY	14.973*	10.731	-	23.089
	[0.092]	[0.295]	-	[0.187]
Panel D: realized kurtosis				
Bitcoin	-	1.1808	1.4892	2.6729
	-	[0.758]	[0.685]	[0.849]
S&P500	1.4818	-	10.658**	11.551*
	[0.686]	-	[0.014]	[0.073]
USDJPY	8.0303**	1.639	-	9.5279
	[0.045]	[0.651]	-	[0.146]

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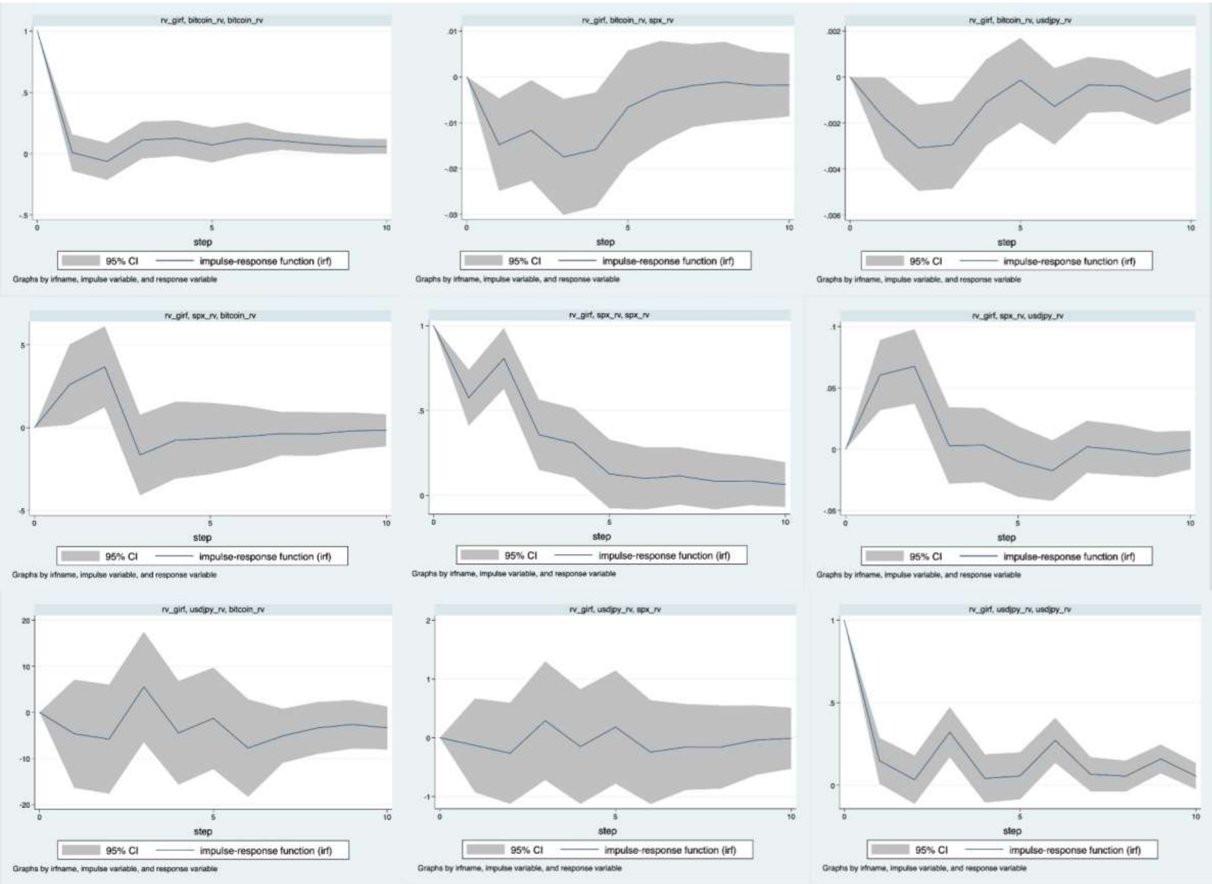
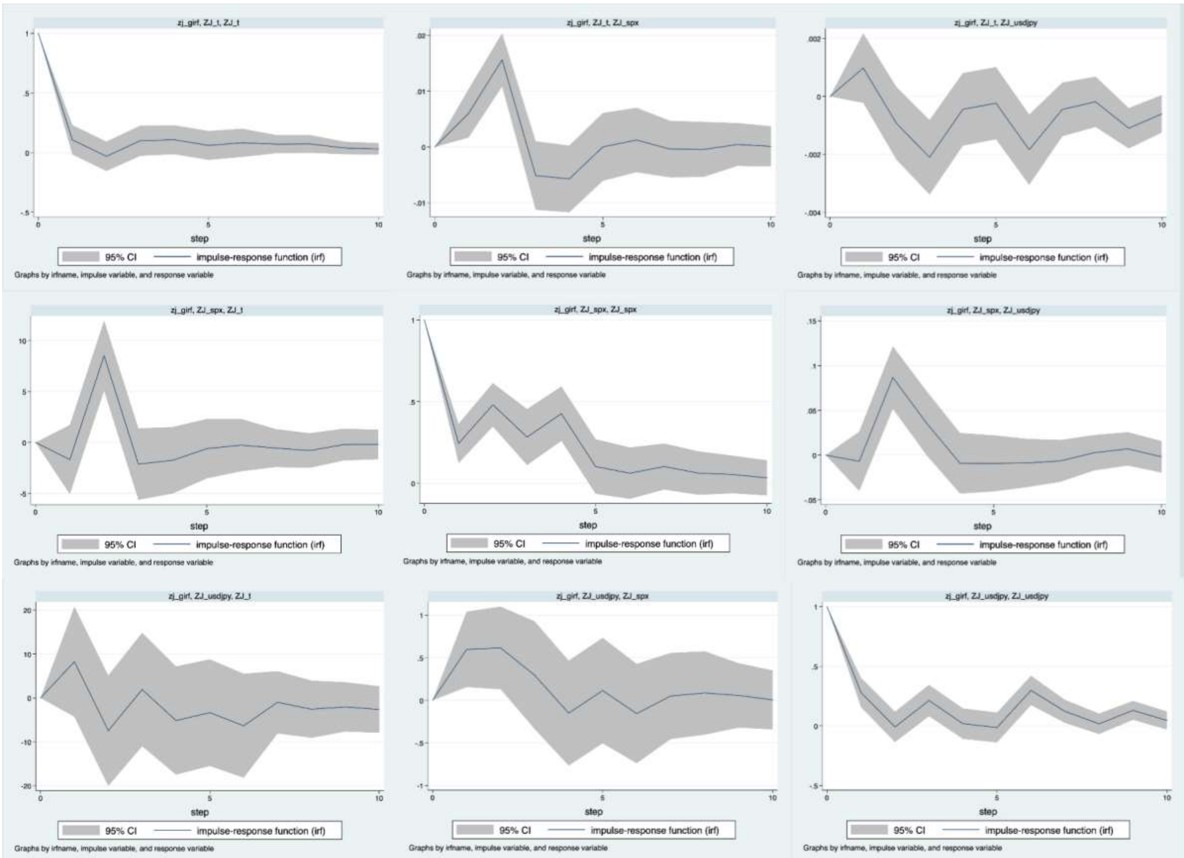


Panel A: realized volatility



Panel B: jump

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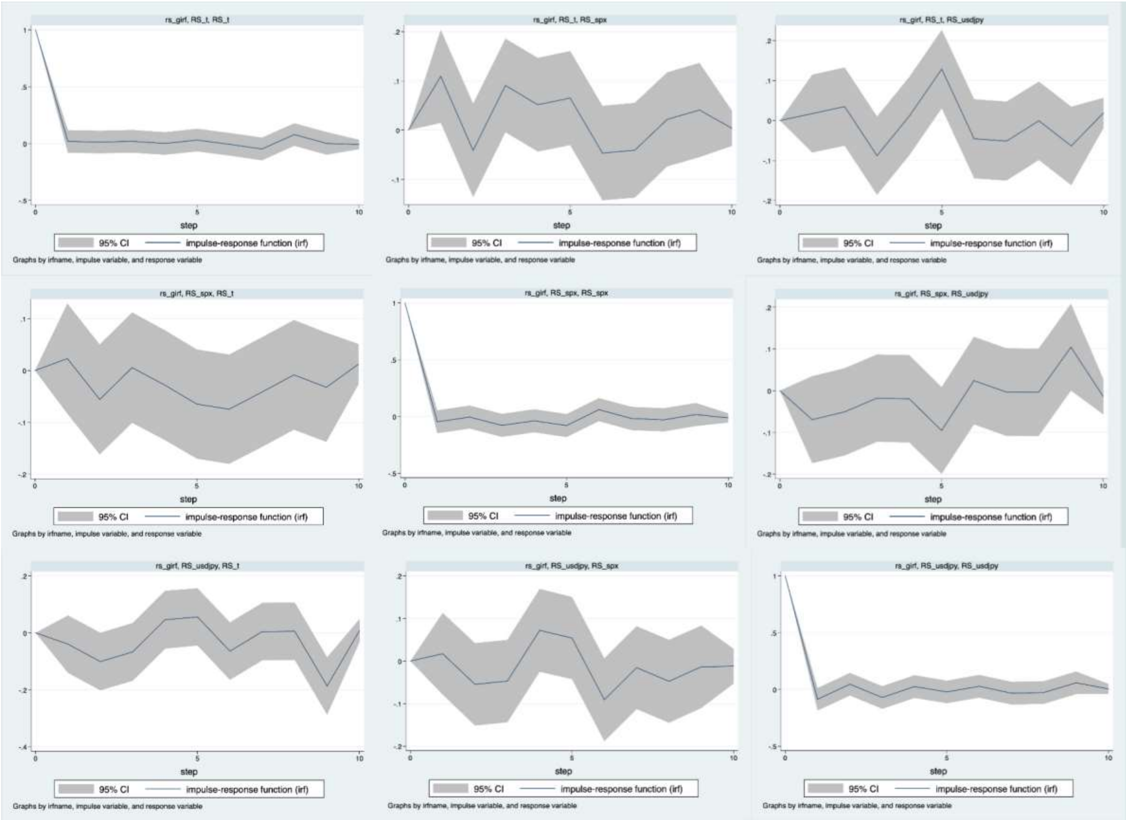
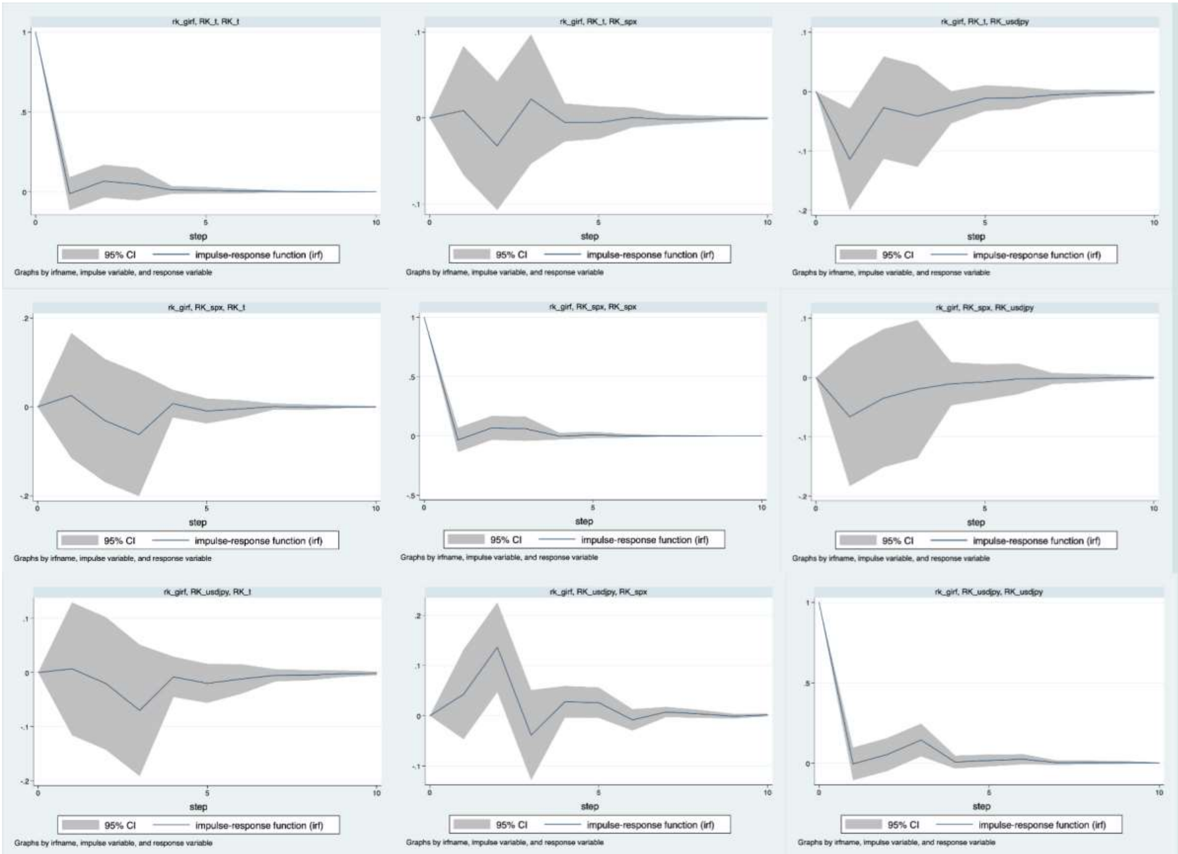


Panel C: realized skewness



Panel D: realized kurtosis

Figure 1. GIRF for a shock to Bitcoin, US stock and USDJPY

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5. Conclusion

This paper analyzes the higher-order spillover effects among stocks, exchange rates, and Bitcoin ofthe U.S. based on weekly data. It investigates realized volatility (RV), jump statistics, realized skewness (RS), and realized kurtosis (RK) to explore these relationships. By constructing a VAR model, the study conducts Granger causality tests and utilizes GIRF analysis to examine the dynamic interactions among these markets.

The results reveal significant bidirectional spillover effects between Bitcoin and traditional financial assets in the U.S. market, notably with the S&P 500 index and USD/JPY exchange rate. Specifically, Bitcoin's realized volatility exhibits a strong bidirectional Granger causality with the S&P 500. Jump statistics further indicate a substantial mutual influence between Bitcoin and these assets, particularly in response to shocks. The closer relationship between the U.S. market and Bitcoin, characterized by more bidirectional Granger causality, is attributed to their deep connections within the global financial system. This aligns with findings from studies like Bouri et al. (2017), highlighting Bitcoin's significant impact on traditional assets.

This research provides important policy implications. Bitcoin significantly affects not only market volatility but also higher-order risks like skewness and kurtosis, which are essential for accurate risk assessments and asset pricing. Policymakers must recognize Bitcoin's role in transmitting volatility to safeguard market stability amid rising global volatility.

Finally, as Bitcoin's market grows, it may increase global market instability. Ongoing monitoring of Bitcoin-related risks and developing early-warning systems will be crucial for managing future financial risks. This paper integrates Bitcoin into spillover effect models between stock and exchange rate markets, challenging the traditional view of exchange rates as primary risk transmitters.

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