

Private Pension Institutions in China: Public Economic Analysis

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Abstract

In recent years, China 's private elderly care industry has developed rapidly, which has aroused widespread concern in academia and society. In this paper, the research team answered the questions of why the private elderly care industry has more potential in China and why public policy formulation is important for the private elderly care industry. They established a sample mathematical model of China 's elderly care system from the perspective of public economics and microeconomics, conduct empirical analysis and find the exact function form. This study is systematic and comprehensive, has some implications for understanding the development of China 's private elderly care.

Keywords

Microeconomics; Public Policy; Public Economics; Aging; Elderly Care.

1. A Primary Modeling of the Elderly Care System

In the early days of the founding of the People 's Republic of China, the pension system was still dominated by family pension. However, with the gradual establishment of the social welfare system, some of the responsibilities have shifted from the family to the state and units. In the city, the unit arranges the welfare of employees and their families ; in rural areas, the " five guarantees " system is implemented to provide basic living security for the elderly who lack labor ability or have no basis for life.

After the reform and opening up, with the development of social economy and the intensification of population aging, the traditional family pension model is gradually difficult to adapt to the actual needs. In this context, China began to gradually promote the reform of the socialization of pension services. On the one hand, pension institutions gradually transformed into social welfare departments, open to the whole society ; on the other hand, the reform of community service system and grass-roots social governance system also provides strong support for the new pension.

At present, the supply of China 's elderly care industry market does not match the strong and clear demand for pension services for the elderly, and the contradiction between supply and demand is prominent. Taking the elderly care service as an example, the elderly who choose the three old-age care modes of home, community and institution have different degrees of demand for old-age care services. These elderly people 's demand for institutional care services is a rigid demand. Due to the dual-track system of China 's elderly care institutions, there are obvious differences in location, pension facilities, cultural atmosphere and medical care between public and private nursing homes, resulting in high occupancy rate of public nursing

homes and low occupancy rate of private nursing homes (Wang Mei, 2019). Pension institutions have the characteristics of large investment, high risk, slow effect and low profit. Many private pension institutions are supported by government subsidies. In the following, we will build models to demonstrate why this subsidy support needs to be sustained (why private elderly care institutions have more potential) and why these policies are so important in the development of private aged care institutions.

First, let's focus on the fact that public elder care system services can be considered a special public good due to their significant non-competitive and non-exclusive properties. As we all know, public goods are usually defined as a product that anyone can consume, regardless of whether a person is able to pay for it or not. Once created, it is not subject to the exclusion principle. Public goods also assume that the additional cost of providing another unit is at least negligible. In China, elderly care in the public sector is almost public welfare, although it has certain fees. The elderly care service industry is public welfare and non-profit. Although the industry is profitable, at present, the operation of the vast majority of pension service institutions is still in the stage of meager profit or even loss, and the vast majority of public institutions are in a state of extreme dependence on financial subsidies. The Chinese government emphasizes that public pension service institutions should give full play to the role of the final bottom of old-age security. All expenses of the elderly without the ability to pay in public nursing institutions are free according to the provisions of the Law of the People's Republic of China on the Protection of the Rights and Interests of the Elderly, which means that public pension services can be considered as a public good. Then, in the case that we only consider the supply of public sector elderly care industry, assuming that the total amount of public elderly care industry services in a region is D , n rational households, and the demand for D by household i ($i = 1, 2, \dots, n$) is d_i , then the total demand of the institute D is:

$$D = \sum_{i=1}^n d_i, d_i = d_{i+1}, i = 1, 2, \dots, n$$

and the amount of the whole region is:

$$\text{Total } D \text{ in region} = \sum D$$

The profit π of the PEC consumed by the household i is:

$$\pi = d_i R(D) - \frac{C(D)}{n}$$

Where R denotes the revenue of public goods every household receives per unit of public goods. $C(D)$ denotes the cost per unit. Assuming that each household consumes public resources PEC with the goal of maximizing individual interests, the complete information static game equilibrium of n rational households is satisfied:

$$\begin{aligned} FOC: \max \pi &= d_i R(D) - \frac{C(D)}{n} \\ s. t. D &= \sum_{i=1}^n d_i, i = 1, 2, \dots, n. \end{aligned}$$

The optimal consumption d of household i satisfies the following first-order conditions:

$$\partial d_i R(D) - \frac{C(D)}{n} \partial d_i = 0$$

$$d_i \left(\lim_{\Delta d_i \rightarrow 0} \frac{\Delta R(D)}{\Delta d_i} \right) \times \left(\lim_{\Delta d_i \rightarrow 0} \frac{\Delta D}{\Delta d_i} \right) + R(D) - \frac{\left(\lim_{\Delta d_i \rightarrow 0} \frac{\Delta C(D)}{\Delta d_i} \right) \times \left(\lim_{\Delta d_i \rightarrow 0} \frac{\Delta D}{\Delta d_i} \right)}{n} = 0$$

In order to ensure the universality of the format and easy to understand, all the derivation processes in the following are written in the limit form. Then, both sides are divided by $\lim_{\Delta d_i \rightarrow 0} \frac{\Delta D}{\Delta d_i}$:

$$d_i \left(\lim_{\Delta d_i \rightarrow 0} \frac{\Delta R(D)}{\Delta d_i} \right) + R(D) - \frac{\lim_{\Delta d_i \rightarrow 0} \frac{\Delta C(D)}{\Delta d_i}}{n} = 0$$

Because we assume that $d_i = d_{i+1}, i = 1, 2, \dots, n$, we can use d represents every d_i . Using D^* represents D that satisfies the above conditions, if n first-order conditions are added, we get:

$$\sum \left(d \lim_{\Delta d \rightarrow 0} \frac{\Delta R(D)}{\Delta d} + R(D) - \frac{\lim_{\Delta d \rightarrow 0} \frac{\Delta C(D)}{\Delta d}}{n} \right) = 0$$

$$\lim_{\Delta d \rightarrow 0} \frac{\Delta R(D)}{\Delta d} \sum d + nR(D) = \lim_{\Delta d \rightarrow 0} \frac{\Delta C(D)}{\Delta d}$$

$$D \lim_{\Delta d \rightarrow 0} \frac{\Delta R(D)}{\Delta d} + nR(D) = \lim_{\Delta d \rightarrow 0} \frac{\Delta C(D)}{\Delta d}$$

Finally:

$$D^* R^{(1)}(D^*) + nR(D^*) = C^{(1)}(D^*)$$

Here (1) is the meaning of the first derivative. Then we finally find the amount of demand D^* when: (1) public goods; (2) maximizing individual interests. Now, let us turn to another situation: only consider the supply of private sector elderly care industry, assuming that the amount of elderly care industry services is q and p the price per unit every household faces, we have:

$$p(\text{demand}) = p(q)$$

where demand denotes the willing of household to pay money for the elderly care. Then the complete equilibrium of one household is satisfied the above.

Through the same method we used above, we can find q^* satisfies the above conditions, and a corresponding r (which we will use in the following part, represents the revenue the elderly receives, just like R). Now we have the equilibrium price and profit in both situation when there is only public sector and when there is only private sector. In the following part, we will use an empirical method to find the details of them, and make a comparison between these two situations and then, finally combined them into one for further research.

2. A Empirical Measure of the R(D), C(D), p(r) and p(q)

First, We use the following indicators and corresponding weights to measure the value of R&r, and then find the relationship between this value and D, p.

Previous studies have often used questionnaires to measure the revenue of services for the elderly (Zhang Lin, 2024). However, we believe that this evaluation method is not rigorous enough. On the one hand, the elderly, as a group of people with weak physical and mental strength, are difficult to understand the content of the questionnaire ; on the other hand, even if they understand the content of the questionnaire, they often cannot give accurate numerical answers to quantitative questions. In summary, we decided to use a simple mathematical equation to establish the relationship between R&r and several classic evaluation factors(see table 1).

Table 1. Variable Description

Variable	Description
RH	(1)recipe health. We measured the health of the diet by the Chinese Healthy Eating Index, CHEI.
NW	(2)The number of nursing workers per capita.
PA	(3) Per capita construction area, per capita floor area.
PI	(4) Per capita project investment.
ID	(5)The added value of informatization degree. The measurement of remote inquiry, remote family visit, and Internet popularity is divided into three levels : 1, 2, and 3.
CV	(6) Cultural added value. The cultural infrastructure and cultural atmosphere are measured at three levels : 1, 2 and 3.

Attention: The above data have been standardized by range.

Based on the data of some public and private elderly care institutions in China(will be listed following), with the results of the questionnaire survey and telephone interview as a reference, we obtain the following regression equation:

$$2RH + 2NW + PA - TC + ID + CV = R\&r$$

So far, R and r have been found. Although it may not be rigorous, the research topic of this paper is not to accurately measure the quality of service. The above equation is enough to study the theme of this article from a qualitative point of view[15]. Now based on the data above, we try to use slightly complex mathematical tools to explore the relationship between R and D, r and q.

In the process of looking for the data of various public and private elderly care institutions, we use the methods of actual investigation and network search, not only through field visits to institutions in Shanghai, Beijing and other places to understand the specific facilities and activities to obtain data, but also through telephone visits, website search and other ways to further obtain data.

After a long data search process, we finally selected the data of 24 public institutions and 24 private institutions with more comprehensive data content, relatively representative and belonging to different administrative divisions(The principle we choose is that in the smallest administrative area where the nursing home is located, there is only one nursing home or basically only one nursing home. This means that we can aggregate the elderly in these districts we selected [about 390,000 exactly] for subsequent analysis) from the large amount of data obtained (the following table).

Then we used the method of range standardization to process a number of data, and transformed the data of the organization into data between 0 and 1, which was convenient for observation and subsequent processing.

Table 2. Selected data on elderly care institutions in various regions

Public Institute	Area	population	DAYS[16]	D	RH	NW	PA/m2	TC/km	ID	CV	R	Total Income of the Institute(C)
Pub1	Beijing	60	30	1800	70	0.20	25	41	2	2	3.05	420000
Pub2	Beijing	226	30	6780	78	0.12	22	40	3	3	3.69	1310800
Pub3	Beijing	1000	30	30000	77	0.18	100	46	3	3	5.48	15000000
...	...											
			Total D:	187738								
Private Institute	Area	population	DAYS	q	RH	NW	PA/m2	TC/km	ID	CV	r	p
Pri1	Shanghai	852	30	30	72	0.14	29	20	2	2	2.68	5750
Pri2	Guangzhou	180	30	30	66	0.18	22	30	1	2	1.94	4500
Pri3	Beijing	343	30	30	80	0.18	54	30	3	3	5.50	16474
...	...											
			Total D:	314087								

First of all, we conduct a descriptive analysis of the first three important indicators RH, NW, and PA that measure R&r. According to the surface figure, we find that compared with the private sector, the public sector provides fewer extreme high-value services (VAR.P=1.65), closer to the lower part of the overall space(Average=2.84). The private sector has more extreme high values (VAR.P=1.68) and higher average (3.43).This suggests that the private sector may have a larger customer base and a higher service quality ceiling. This will help us understand the conclusions below about the more potential of the private sector.

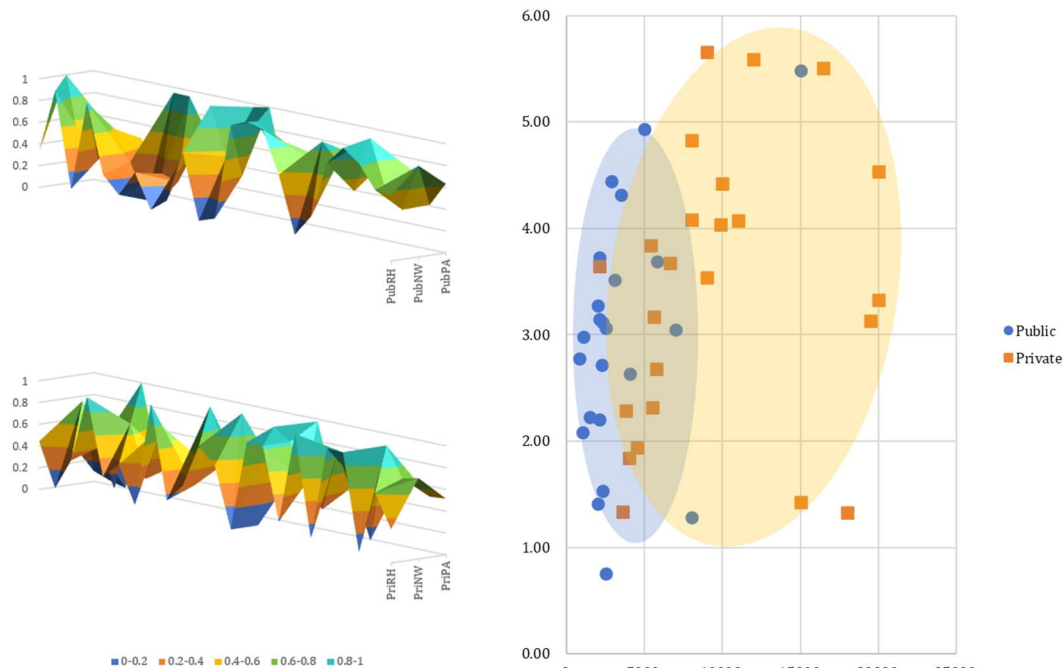


Figure 1. private sector may have a larger customer base and a higher service quality ceiling

After obtaining the above data, we split the R, D, C of the public institutions, and reorder them in a certain order to obtain the following table:

Table 3. R, C & D

R	C	D	R	C	D	R	C	D
5.48	15000000	30000	2.71	624340	8325	4.45	510400	5280
3.51	2480000	24000	3.15	502056	7172	2.63	445500	3300
-0.18	945000	13500	3.69	1310800	6780	2.23	150000	3000
2.08	360694	10821	0.76	500000	6000	3.27	160000	2400
1.28	369200	10069	3.72	420000	6000	1.41	140000	2100
1.53	690000	9000	2.98	220000	6000	3.05	420000	1800
2.20	630000	9000	4.93	985859	5915	4.32	175000	1500
3.11	690000	9000	2.77	148685	5576	3.07	100000	1200

The box diagram drawn from the above three columns of data is as follows. Using the box plot, we eliminate several data that are too extreme. After that, we will use the remaining data to estimate R(D) and C(D).

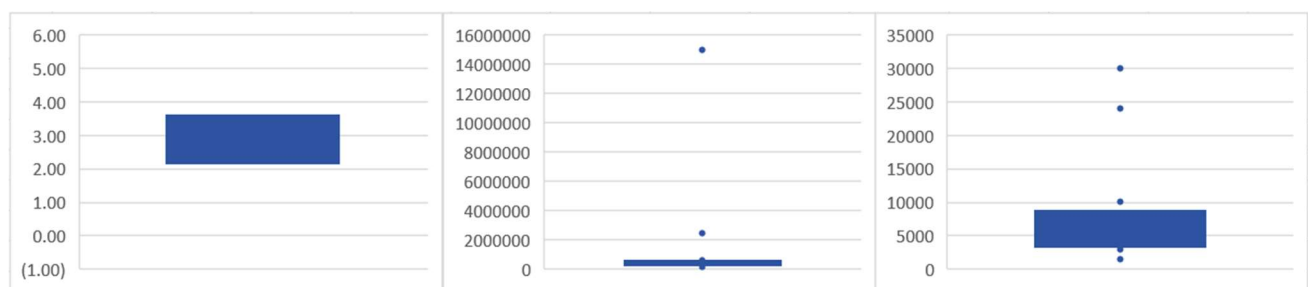


Figure 2. Delete extreme values through box plots

For the regression analysis of the new data set that excludes some extreme data, we find the specific form of the relationship between R(Vertical) and D.

$$R = -0.729 \ln D + 8.9081$$

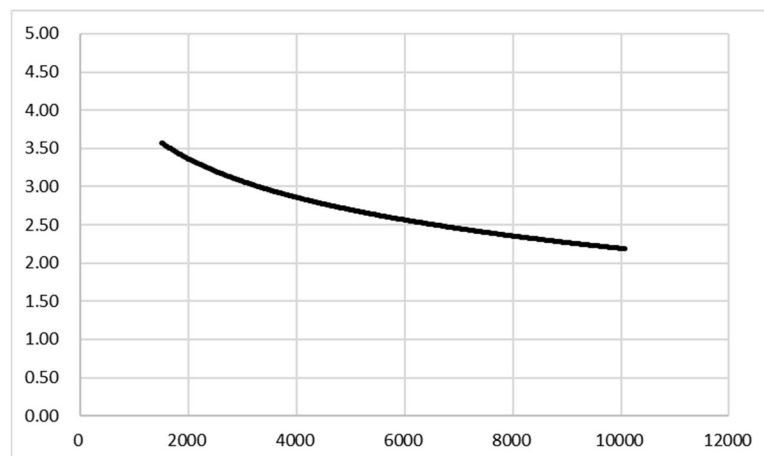


Figure 3. R(D)

Using the same method, we find the specific form of the connection between C(Vertical) and D.

$$C = 46.645D + 128705$$

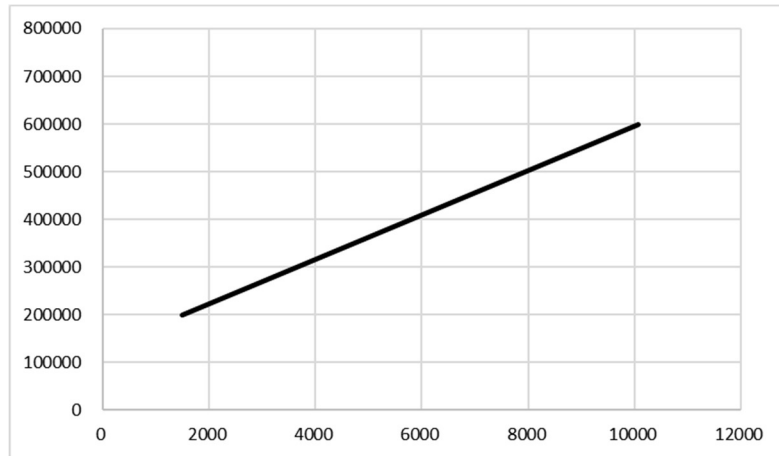


Figure 4. C(D)

Further more, we find R(P)(where P represents the price per person in public institute):

$$P = 1140.7e^{0.275R}$$

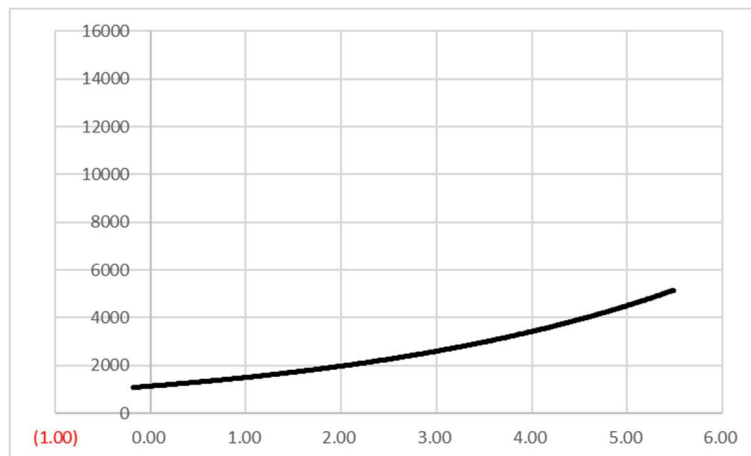


Figure 5. P(R)

So the P, R, C, D can summary by the following form:

$$\begin{pmatrix} P \\ C \\ e^{\frac{8.9081}{0.729}} \end{pmatrix} = \begin{pmatrix} 1140.7 & 0 & 0 \\ 0 & 46.645 & 128705 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} e^{0.275R} \\ D \\ 1 \end{pmatrix} = A \begin{pmatrix} e^{0.275R} \\ D \\ 1 \end{pmatrix}$$

Here A represents a constant matrix of rank 3. There are three variables P, C and R on the left side of the equation, and two variables D and R on the right side of the equation. In linear algebra, this means that P, C, and R can be uniquely represented by D[17]. That is to say, in the public sector, D is a variable that can significantly affect other variables. This tells us that for public aged care institutions, their service quality, total income, and the price faced by each elderly family are all affected by D (the total amount of consumption in the public aged care sector).

In these 3 cases above, the R square is 0.4011, 0.4119 and 0.3037 respectively. Although its value is not very high, it can already represent considerable significance in the analysis of microeconomics. These two curves tell us that for the public sector, at least in the 48 sample data we selected, D is negatively correlated with R and positively correlated with C. Considering the actual situation, this is easy to understand. For public elderly care institutions, if the number of elderly people increases, it will inevitably lead to a decline in service quality, which is in line with China 's social reality. If the care needs of the elderly in an area increase, the public sector will spend more on this, and they will have a corresponding increase in income.

Next, we estimate the private sector in a way which is little similar but not same at all. Since elderly care services in the private sector are often not public goods, we use microeconomic analysis of general merchandise. First, we calculate how Supply curves should be. We refer to the empirical method of Ding Suxia (2023) to construct the following mathematical model: By accumulating the different price groups($q\%$ [18]) on the supply side in proportion from small to large, the following curve can be obtained:

$$p = \int_1^{q\%} \text{the price } q\% \text{ Institutes want to receive for private care}$$

$$p = 2419.9e^{2.230\%}$$

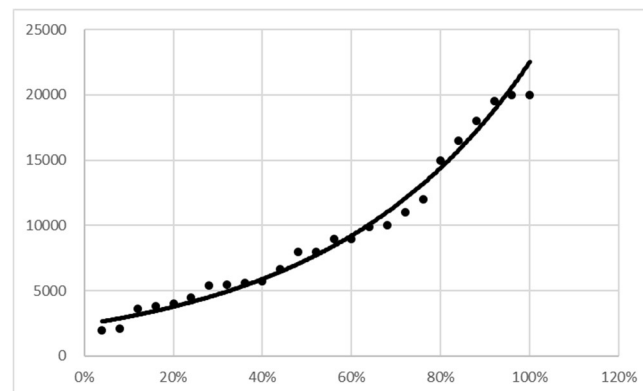


Figure 6. Supply Curve of the private

After the supply curve, we try to find the relationship between p and r by regression:

$$p = 5058.5e^{0.1398r}$$

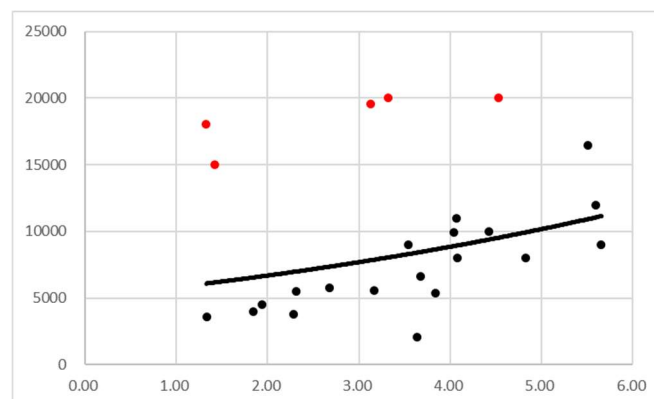


Figure 7. $p(r)$

According to the scatter plot, we can easily find that most of the points are around our function except for a small number of isolated points.

So far, we have found the specific forms of $R(D)$, $C(D)$, $p(q\%)$, $p(r)$. These functions are based on the huge amount of historical data and should have a good performance in the Chinese market in the past five years at least. In the following third part, we will use the method in the first section to find the equilibrium D^* & q^{**} under the equilibrium state, all public and all private conditions, respectively.

3. Why the Private Elderly Care System Should Be more Potential in China

Using the equation we used in section 1, Look the public sector elderly care as a whole:

$$FOC: \max \pi = d_i R(D) - \frac{C(D)}{n}, n = \sum \text{households} = 6258$$

$$D^* R^{(1)}(D^*) + n R(D^*) = C^{(1)}(D^*)$$

Where:

$$\begin{cases} R = -0.729 \ln D + 8.9081 \\ C = 46.645D + 128705 \end{cases}$$

$$\begin{cases} R^{(1)} = \frac{-0.729}{D} \\ C^{(1)} = 46.645 \end{cases}$$

$$46.645 + 0.729 = 6258 \times (-0.729 \ln D + 8.9081)$$

Using GeoGebra[19]:

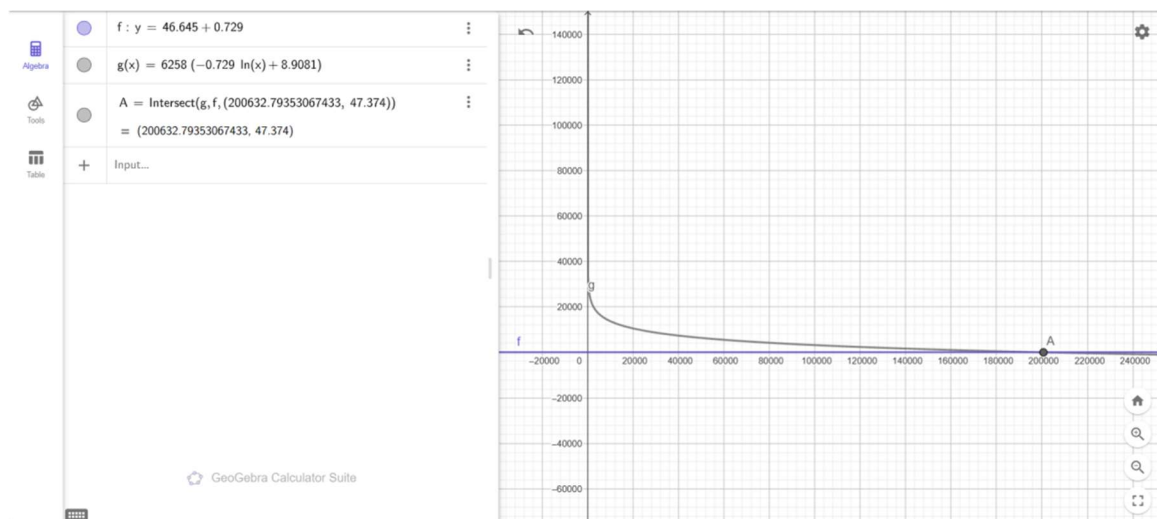


Figure 8. GeoGebra

With the intersect(g,f), we find the equilibrium Total $D=200632$. There are about 390,000 elderly people in the area involved in our sample. Assuming that each elderly person lives in an aged care facility, the total D per month is about 12.36 million. According to the above data table, the D value of one month is 187738, that is to say, only about 1.52 % of the elderly have beds in the public elderly service institutions. Then, under the current supply and demand conditions, if the utility maximization balance in our model can be achieved, the value of D is 200632. That

is, unless there are major changes in technology or income etc., the D value of the public sector can increase by up to 6.86 %.

Then, for private sector:

$$p(demand) = p(q)$$

We use the traditional supply and demand analysis to calculate the value at the equilibrium point. In order to ensure the validity of the comparison, we still use the above 390,000 elderly people in the region, and based on this data to draw the actual supply and demand curve. The supply curve is very easy to find in the above article:

$$p = 2419.9e^{2.2307q\%}$$

However, according to our data set to measure the different percentage of supply and the corresponding price, how to determine the total supply ? Without the total supply, it is impossible to determine the supply and demand analysis image in the following. Here, we try to use the equilibrium demand of the public sector to estimate the total possible supply.

According to the same method above, we calculate the supply curve[20] of proportional public elderly care institutions:

$$P = \int_1^{Q\%} \text{the price } Q\% \text{ household want to pay for public care}$$

$$P = 849.75e^{2.1511Q\%}$$

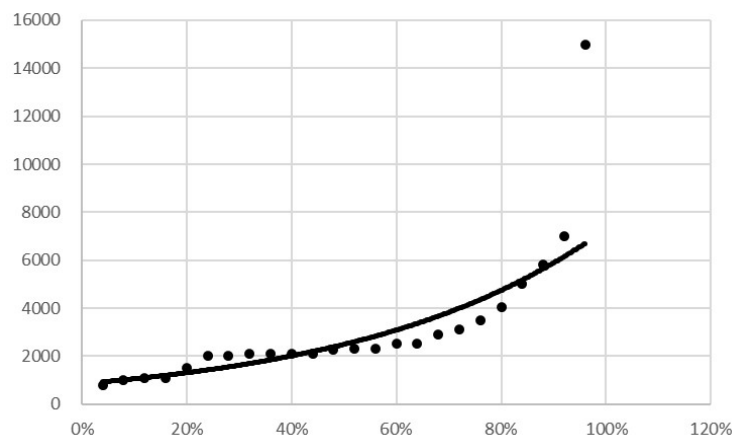


Figure 9. Supply Curve of the public

We have calculated that the D of the public sector in equilibrium is 200632, and its corresponding C is 9487184.64. The corresponding per capita consumption(shadow price) of this C is 1516.09 CNY, and its corresponding quantile in the proportional supply curve is 27.4 %. That's to say, based on the view of using samples to estimate the population, the elderly who really have the need for old-age care, the elderly care service unit they need is about 3.64 times that of 200632, that is, about 739,879. Therefore, the total possible max supply and demand is 739879.[21] So we can use the real q to replace q%.

The demand curve for this market is a very difficult problem. We cannot know everyone's inner willingness to pay by means of directly data collection. Fortunately, considering that supply is

roughly equal to demand from the perspective of the whole society, for each group, the price they pay is the price they are willing to pay. By summing up the above price data of public and private institutions and the corresponding number of people willing to buy, the demand curve of people in the whole society can be obtained.[22]

$$p = \int_1^{\text{demand}\%} \text{the price demand}\% \text{ households want to pay for private care}$$

$$p = 21675e^{-2.487\text{demand}\%}$$

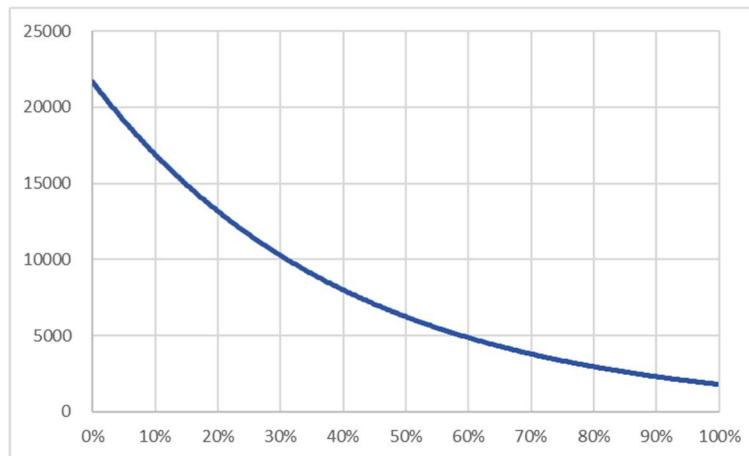


Figure 10. Demand Curve of the private

Then, using the real demand to replace demand% and putting them together, we have:

$$p = 21675e^{-3 \cdot 10^{-6} \text{demand}}$$

$$p = 2645.8e^{3 \cdot 10^{-6} q}$$

$$21675e^{-3 \cdot 10^{-6} \text{demand}} = 2645.8e^{3 \cdot 10^{-6} q}$$

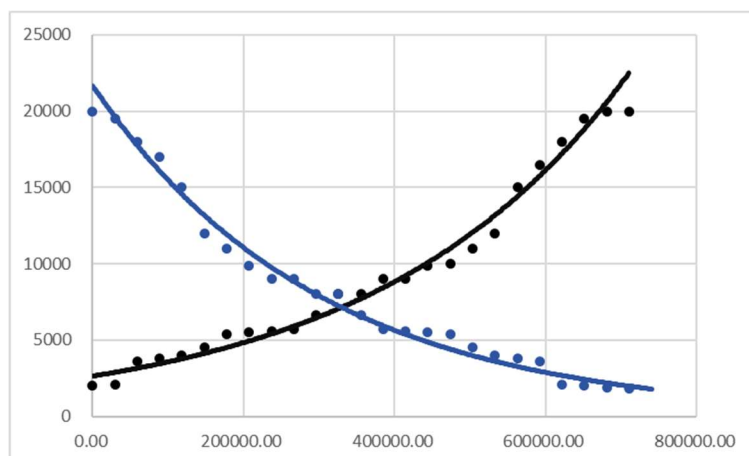


Figure 11. Equilibrium

Now, we have got the supply curve of the private part of the elderly care services, the demand curve and the balance point of the whole society. After automatic calculation by computer, we know that the D at this equilibrium point is about 350,531.01, 1.116 times of the total D of present private part, which is 314087 in total.

Then we find that according to our data set (which reflects the current economic situation), in the case of considering the entire market, if everyone 's optimal choice and market balance are achieved, then the potential growth rate of public institutions is only 6.86 %, while the growth rate of private institutions is as high as 11.60 % ! This theoretically proves why private elderly care institutions have more growth potential in China.

Table 4. Summary

	Total D of public sector	Total D of Private Sector
Present	187738	314087
Potential level	200632	350531
Growth rate	6.86%	11.60%

Now, we have shown that, on average in china's market, the potential growth rate of the public aged care sector (6.86%) is significantly lower than that of the private sector (11.60%) without external interference if markets are free-flowing, information is complete, and people are rational choice. In terms of empirical analysis, from 2012 to 2023, the number of pension beds in the country increased from 4.165 million to 8.23 million (China's State Council data), with a compound annual growth rate (CAGR) of 7.06 %, a total increase of about 97 %. During this period, the CAGR of the elderly population over 65 years old was 5.87 %, a total increase of 76 %. This means that about 27 % of the increase in the industry itself is not caused by the rise of the elderly population. Why is this ? According to our above calculations, we find that under the existing number of elderly people, the industry still has :

$$100\% \times \left(\frac{\text{Efficient Total D of Private and Public sector}}{\text{Total D of Present Private and Public sector}} - 1 \right)$$

9.832 % growth potential. If the elderly population maintains an annual growth rate of about 4.5 %, and the number of elderly people with needs for elderly care services increases simultaneously, then by 2027, the public and private elderly care industries will be expanded to :

$$\begin{aligned} & (1 + 0.045)^3 \times (1 + \text{growth potential of public}) \\ & (1 + 0.045)^3 \times (1 + \text{growth potential of private}) \end{aligned}$$

1.22 and 1.27 times. We find that these two figures are very close to the above 27 % increase. But the number 1.22 & 1.27 are only for 5 years, while the 27% above is for total 11 years of 2012-2023. Why has the output of private and public pension industries moving towards market equilibrium in the past few years been slower than what is theoretically possible? We believe that this is due to the inefficiency of market matching caused by factors such as policy, culture, and information asymmetry. In the following, we will discuss this in detail.

Futher more, in practice, the growth rate of the private sector may be higher in some areas and lower in others. Why is this?Just read forward.

4. The Private Sector will Play an Important Role in China 's Silver-haired Economy. Why Policies is Important During this Process?

In some economically backward areas, people 's ideas are relatively backward. For a long time, public elderly care institutions have been the mainstream of society; and the elderly have a

natural trust in the government. These two factors lead to a lot of backward ideas of the elderly are not willing to accept the emerging private elderly care institutions. If this type of elderly people are many in a certain area, the demand curve of the whole society faced by the private elderly care industry will move to the left. This will lead to a decline in the equilibrium output of the local elderly service industry and a decline in potential increment. In the end, the growth rate of private pension service industry in these areas is lower than that in other regions. When discussing the phenomenon that the number of demand-sides in private nursing homes is smaller than the theoretical number, the factor of 'relative deprivation' of the elderly and their families enlightens us. The so-called "relative deprivation" refers to the feeling of deprivation when people compare their situation with a certain standard or a certain reference and find themselves at a disadvantage. This feeling can produce negative emotions, which can be expressed as anger, resentment or dissatisfaction. In other words, relative deprivation is a feeling of 'we are entitled to but do not have', which will lead to changes in people's psychological expectations and comparison mechanisms. And this factor not only reflects the complex considerations of people in choosing nursing homes, but also reveals the potential impact of social psychology on the development of private nursing homes. The psychological burden of this relative deprivation will reduce their acceptance and satisfaction with private nursing homes, thereby reducing the well-being of the elderly and their families, and leading them to choose public nursing homes in the decision-making process. This will cause the demand side to shrink:

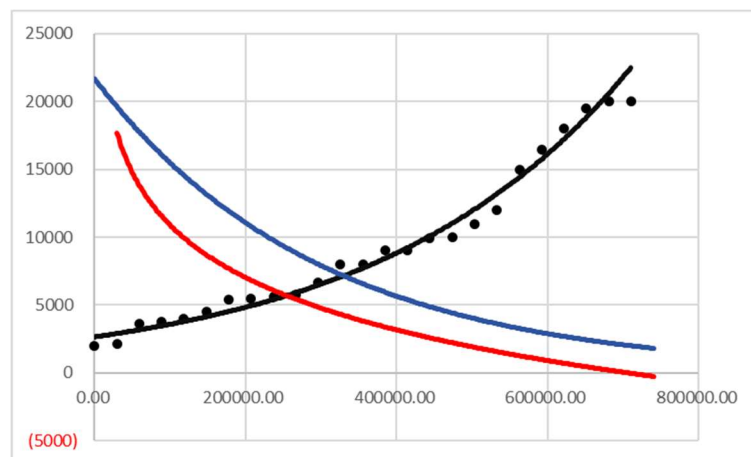


Figure 12. Culture and relative deprivation

It can be seen that due to the left shift of the demand curve, the total output has declined. If the government does not carry out any policy guidance, the impact of this relative deprivation on the growth rate of private nursing homes will become a vicious cycle process. If the elderly and their families choose private nursing homes less, the improvement rate of environmental facilities and cultural services in private nursing homes is likely to be lower than that in public nursing homes, or even bankruptcy.

In addition, the traditional Chinese filial piety culture will also lead to the left shift of the demand curve. In China, many people believe that it is irresponsible for their children to send the elderly to nursing homes. They feel that they have become a burden on their parents and are unwilling to take care of the elderly or spend time with the elderly. If the reason behind sending parents to nursing homes is really this, then this behavior is indeed a kind of 'unfilial' and forgotten performance. In fact, most of the children's intention is that the nursing home can provide more detailed services and more care, and the elderly in the nursing home can know their friends of the same age and alleviate loneliness. However, the misunderstanding of

the former leads to the fact that even if parents themselves agree to live better in nursing homes, they will also hinder their children's face from going to nursing homes to choose home care. Over time, private nursing homes lack funding sources, and the level of services and facilities will decline. Under this vicious circle, nursing homes will be difficult to operate. If there are not many policies to support them, the result will be:

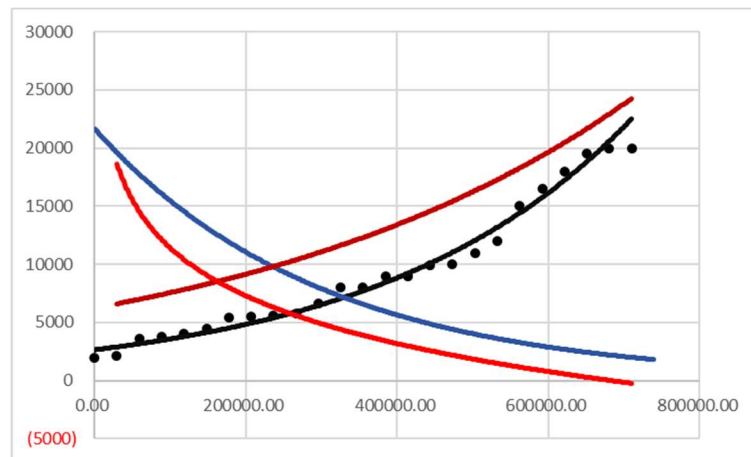


Figure 13. If the government does nothing

This leads to a larger reduction in output. That is to say, in the context of Chinese culture, only through the government's policy support and cultural guidance, can the private elderly care institutions develop healthily.

5. Summary: The Policy Recommendations for Supporting Private Sector

All in all, we first use the analysis methods of public economics and microeconomics in this paper to prove that the private elderly care industry generally has more development potential than the public elderly care industry within the acceptable error range. This paper shows that if there are appropriate policies, such as optimizing the business environment, reasonable cultural guidance and correct system design, the private pension industry will play an irreplaceable role as an important part of China's silver economy. It will bring greater well-being to the elderly in China.

Acknowledgments

Yan Xianzhen and Meng Deyang from Zoucheng No.1 Middle School provided very important assistance in the theoretical construction and quantitative calculation of this article during their high school years, which is crucial for us to achieve the analysis of the entire article. Although this article is only considered for publication in 2024, the contributions made by Yan and Meng in early 2023 were significant.

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- [13]Luo Xiaoman.2024”Quality Evaluation of Embedded Elderly Care Service: A Case Study of the Central Area in Shanghai”.Scientific Research on Ageing 12(5):120-125.
- [14]In China, ' elderly care industry ' and ' pension industry ' do not have much semantic difference. In this paper, the two are mixed without distinction.
- [15]If you have doubts about this, you can observe the mathematical process below. You will find that even if R and C have such a huge gap in the order of magnitude, as long as we do not change its dimension in the overall analysis process, it has little bad influence on the final result. As long as it can achieve the effect of sorting the quality of elderly care services, its mission is completed.
- [16]DAYS means the payment cycle of a single care service. In China, nursing homes are generally paid monthly, so we take 30 days here. Although deleting this data has almost no effect on our analysis, we decided to retain it in order to emphasize the time attribute of the number of service requirements D. On the other hand, it is very appropriate to use the number of hours to measure the labor factor of elderly care institutions. In most economics textbooks, factor L is calculated by time.
- [17]of course, we can also say that C can be uniquely represented by P, R, and D, but this correlation has no economic significance, so we do not discuss.
- [18]Note the distinction between q and q%. One represents the percentile, and one represents the true number.
- [19]A public mathematical drawing service website that can be used without payment or authorization.
- [20]Our image means that for the lowest part of the price, there may be only a minimum of 1 % of aged care institutions willing to sell their services. For slightly higher prices, there are more businesses willing to provide services. For the highest price, 100 % of merchants are willing to provide services. This is completely consistent with the definition of supply curve in microeconomics textbooks.
- [21]This is a more reasonable and realistic inference. We only need to follow the traditional supply and demand analysis, the public elderly care services as a big market for the whole society. In this case, we can estimate the equilibrium point of supply and demand, so as to estimate the maximum supply and demand of the whole society.

[22] This image means that we superimpose the demand curve of all possible consumers in the whole society, depicting the price each of them is willing to pay and summing up. This is consistent with the traditional definition of the demand curve.