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Study on the Pricing of Mortgage Default Insurance based on Options

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Abstract

In the process of improving the mortgage default insurance system, the conclusion of an insurance contract and the setting of premiums are undoubtedly crucial issues. Currently, the close cooperation between banks and insurance companies in China has provided favorable external conditions for research on mortgage insurance pricing. However, most of the existing research on mortgage default insurance pricing in China has not fully considered the impact of fluctuations in the value of mortgaged properties on insurance pricing. This paper conducts a detailed study of the pricing methods and process of mortgage default insurance, offering an option-based pricing solution and providing a theoretical basis for relevant policymakers.

Keywords

Mortgage Default Insurance; Option Pricing; Default Loss; Systemic Risk; Asset Portfolio.

1. Introduction

Mortgage insurance is a supplementary system corresponding to mortgage loans, designed to ensure the smooth realization of the mortgage right obtained by lending institutions when providing loans to home buyers. Depending on the type of risk covered, mortgage default insurance can be divided into life insurance for borrowers, property loss insurance for homes, and mortgage default insurance. Mortgage default insurance specifically addresses the default risk or credit risk of the borrower, and can be categorized into passive default risk and active default risk based on motivation.

Research on the pricing of mortgage default insurance, both domestically and internationally, mainly focuses on the following areas: Brueckner (1985) studied estimation methods for the operational costs of mortgage default insurance, focusing on how the uncertainty of future asset values affects the premiums of mortgage default insurance. Clauretie (1988) used empirical analysis to demonstrate the significant impact of regional diversification on the loan-to-value ratio of residential mortgage loans. Frye (2000), Hull (2005), and Fan (2012) examined how to use variable decomposition methods to address the numerous default issues encountered in the pricing of financial products. Chang, Wang, and Yang (2012) analyzed the impact of various macroeconomic factors on the pricing of mortgage loan insurance and proved this viewpoint through empirical analysis. Ming Pu, Gangzhi Fan, and Yongheng Deng (2013) developed an option-based model for the loan caps in reverse mortgages.

2. Basic Mortgage Default Insurance Pricing Model

Merton (1977) was the first to propose the protective function of a put option and to introduce option theory into deposit insurance. In this paper, if the value of the mortgaged property falls below the mortgage loan amount, the borrower can no longer repay the loan solely based on the market value of the property. We can similarly treat mortgage default insurance as a European put option, where the buyer of the option is the borrower, and the seller is the

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insurance company. The term of the option corresponds to the duration of the loan insurance, the underlying asset is the market value of the mortgaged property, and the strike price is the amount of the mortgage loan obtained by the borrower. During the life of the option, if the market value of the property falls below the mortgage loan amount, the insurer will pay the difference between the two values to the creditor, thus ensuring the value of the mortgage loan. The premium paid by the borrower is equivalent to the price of the put option, which completes the key step in the pricing of mortgage insurance. The main variables of the model include: the probability of an insurance event occurring, the extent of the loss of the insured asset, and the variance of the default loss.
Assume that the value of the mortgaged property follows a geometric Brownian motion:

$$dP_t = \mu P_t dt + \sigma P_t dW_t \tag{1}$$
$$P_0 = P > 0$$

where μ is the expected percentage growth in housing price, σ is the standard deviation in housing price, and W_t is a standard Brownian motion. α represents the remaining proportion of the home sale price which the individual has to repay, the duration of the mortgage loan is T, the loan principal and interest which the borrower needs to repay at time T will be the amount of αPe^{iT} , where i stands for the interest rate charged by the lender on this loan.For the default probability, since the solution to equation (1) is given by:

$$P_T = P\exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)T + \sigma W_T\right),$$

A straightforward calculation according to the optimal default condition gives the following probability of mortgage default:

$$Q = \Pr\left[P_T < \alpha Pe^{iT}\right] = \Pr\left[W_T < \frac{\ln\alpha + (i - \mu + \frac{1}{2}\sigma^2)T}{\sigma}\right] = \Phi\left(\frac{\ln\alpha + (i - \mu + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) \tag{2}$$

where Φ is the distribution function of a standard normal random variable. By (2), we can investigate how the effects of changes in the parameters of α, i, μ, σ and T upon the default probability Q. We obtain the following partial derivatives with respect to these parameters.

$$\frac{\partial Q}{\partial \alpha} > 0,\ \frac{\partial Q}{\partial i} > 0,\ \frac{\partial Q}{\partial \mu} < 0,\ \frac{\partial Q}{\partial \sigma} \gtrless 0,\ \frac{\partial Q}{\partial T} \gtrless 0 \tag{3}$$

Concerning the default loss of this loan at time T, it can be defined as:

$$L_T = \begin{cases} \alpha Pe^{iT} - P_T & if \quad P_T < \alpha Pe^{iT} \\ 0 & if \quad P_T \geq \alpha Pe^{iT}. \end{cases}$$

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Thus, the expected default loss can be written as:

$$l = E\left[e^{-rT}L_T\right]$$

We can treat L_T as the payoff of a put option with a strike price of αPe^{iT} , we then promote the following assumption: if the price of the mortgaged property follows a geometric Brownian motion, the expected loss (i.e., the fair premium) caused by the default of the mortgage to the lender can be expressed as follows:

$$l = \alpha Pe^{(i-r)T}\Phi(-d_-) - P\exp\left((\mu-r)T\right)\Phi(-d_+) \quad (4)$$

Where:

$$d_- = d_+ - \sigma\sqrt{T} = \frac{(\mu - \frac{1}{2}\sigma^2)T - \ln\alpha - iT}{\sigma\sqrt{T}}.$$

3. The Extension of Basic Model-Multiple Borrowers

In the insurance industry, the Law of Large Numbers and the Central Limit Theorem are among the most widely applied theories, and this is also true in the field of mortgage loan insurance. If the number of loans in the loan pool is sufficiently large, the risks caused by fluctuations in housing prices, especially unsystematic risks, can largely be diversified. This section considers the case of multiple borrowers and investigates the impact of loan diversification on mortgage insurance pricing.

Assume that each borrower is only allowed to purchase one property, the initial price of the house is fixed and equal, and all market participants have consistent expectations regarding any changes in housing prices. Therefore, the prices of N houses can be represented as an N-dimensional random variable that follows the geometric Brownian motion model, as described below:

$$dP_t^{(k)} = \mu P_t^{(k)}dt + \sigma P_t^{(k)}dW_t^{(k)}, \quad P_0^{(k)} = P \text{ for } \quad k = 1,2,3\ldots N \quad (5)$$

where $W_t^{(k)}$ represents the standard Brownian motion corresponding to the price change of the k-th property, and N is the number of mortgage properties and mortgage loans.

Since the factors affecting the overall change in housing prices can be decomposed into systematic and unsystematic factors, $W_t^{(k)}$ can be expressed as follows:

$$dW_t^{(k)} = \rho dZ_t + \sqrt{1-\rho^2}dB_t^{(k)} \; k = 1,2,3\ldots N \quad (6)$$

Equation (6) illustrates that the future cash flows of a property may be affected by both systematic and unsystematic fluctuations. In fact, similar to the CAPM model, the relationship between the future cash flows of a property and systematic fluctuations is linear. A large body of research has already provided empirical evidence on this.

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4. Conclusion

This paper extends the option-based pricing model by decomposing the price fluctuations of mortgaged properties into systematic and unsystematic factors. It explores the impact of related parameters on mortgage insurance pricing, and the model results provide a reference method for pricing mortgage default insurance.

Since factors such as loan-to-value ratio, choice of insurance term, and interest rates have significant effects on insurance pricing, insurance companies should assess risks based on factors such as the characteristics of the property, the borrower's creditworthiness, property value, and down payment ratio when implementing mortgage loans. They should then set corresponding rates. The choice of an appropriate insurance term should distinguish between existing homes and pre-sale homes. The premium payment method can be flexible, either annually or monthly, and can be adjusted based on the borrower's actual repayment situation.

Finally, for the comprehensive development and improvement of the mortgage default insurance system in China, in addition to standardizing the insurance products themselves and using reasonable pricing techniques, external support is also needed. Firstly, a well-established primary market is the cornerstone of the healthy development of mortgage insurance. Indicators such as loan quality, loan size, and transparency in loan approval are important measures of market completeness. At the same time, efforts should be made to improve the personal credit system and to formulate and improve relevant laws and regulations. This includes gradually establishing a credit system based on national credit management with support from private credit agencies, and improving the legal framework covering housing ownership, housing finance policies, housing subsidy policies, taxation, land supply, and affordable housing. Finally, the government should play a guiding role throughout the process, actively supporting the establishment of unified standards, and gradually achieving the ultimate goal of a market-oriented mortgage insurance system through a step-by-step, point-to-point approach.

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