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Carbon Option Product Design and Pricing Study

-- Based on Modified Fractional Brownian Motion Optimized by EGARCH Model

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Abstract

In the context of the "dual-carbon" vision strategy and the development of carbon finance, the development of China's carbon market is very promising, but the system of carbon futures, carbon options and other financial products has not yet been formed. In view of the experience of the European Climate Exchange (ECX) in carbon option contracts, this study aims to design an option product applicable to the Chinese carbon market. The article takes the carbon emission allowances in the national carbon market as the underlying asset of the options, selects the national carbon emission allowance trading price data for the period from July 16, 2021 to December 31, 2022, and uses the EGARCH-fractional Brownian Motion option pricing model to formulate a reasonable call price of the carbon options.

Keywords

Carbon Emission Allowances; EGRACH; Fractional Brownian Motion.

1. Introduction

Climate change caused by global warming poses a great threat to the sustainable development of mankind, and climate change caused by anthropogenic greenhouse gas emissions is a great concern to today's society, and the study of how to reduce emissions has become an urgent issue for countries around the world[1]. According to statistics, the total amount of greenhouse gases, including carbon dioxide, emitted by countries around the world hit a record high in 2018[2].

According to the International Energy Agency (IEA), China's fossil energy carbon emissions have surpassed the United States in 2007, ranking the first in the world, and the total carbon emissions ranked first in the world. In 2016, China's total CO2 emissions amounted to 9,123 million tons, accounting for 27.3 % of the global total[3]. In the face of the global warming trend and enormous pressure from the international community, China has fully integrated emission reduction into its overall strategy for national economic and social development. China's carbon financial market started late, in late 2011, started the pilot work of carbon emissions trading. on June 18, 2013, Shenzhen became the first city to set up a carbon emissions trading platform, filling the blank of China's carbon trading market, laying the foundation for the development of the carbon market in the future, and then gradually in the country's eight provinces and municipalities to set up pilot projects with local characteristics. at the end of 2017, China began to prepare for a unified carbon financial market, and by the end of the year, China began to prepare for a unified carbon financial market. preparing for a unified carbon financial market, and by April 19, 2021, the Guangzhou Futures Exchange, with carbon emissions as the first trading variety, was officially listed. China's carbon financial market has taken shape in less than eight years, with huge market potential and broad prospects, but from the point of view of

ISSN: 2710-0723

trading varieties, China's carbon financial market is characterized by a single trading variety and a serious lack of carbon financial derivatives such as futures and options, which is urgently needed to fill the product gap. Therefore, it is of great strategic significance and practical value to research, develop and design carbon financial derivatives such as carbon options.

Therefore, based on the dual-carbon background and the opportunity of carbon finance development, the article chooses carbon market as the research object, draws on the EUA option design in the EU trading market, researches and designs the carbon option products in line with China's carbon market, and analyzes the carbon option product pricing by taking the carbon quota trading price of the National Carbon Emission Right Exchange as the research data. This is of great practical significance as well as practical significance for filling the blank of domestic carbon financial derivative market and forming the framework of carbon option products with systematization.

2. Literature Review

At present, the design of domestic carbon financial derivatives mainly focuses on carbon futures, with relatively little research on carbon options. In particular, domestic scholars are focusing on the current situation of the domestic carbon trading market, mainly providing suggestions for the construction of the domestic carbon financial market and designing futures treaties. L. Feng et al. (2014) and others constructed three pricing models about carbon futures from the characteristics of carbon emission quotas to provide theoretical references for the pricing of carbon futures contracts in China[6]. R. R. Chen(2018) drew on the design contract of EU carbon futures, designed China's carbon futures contract from the basic components of carbon futures contract, risk control mechanism, delivery time, delivery mode, etc., and used technical analysis to predict the futures price, and finally put forward the initiatives to promote carbon futures[7]. Z. W. Li et al. refer to the carbon financial derivative products in mature foreign markets to design an option contract framework in line with China's carbon market[8].

The option pricing models are B-S option pricing method, binomial tree pricing method, fractional Brownian motion option pricing method and so on. The basis of market effectiveness testing has evolved from the efficient market hypothesis (EMH) to the fractal market hypothesis (FMH).The FMH is derived from the fractal theory and is usually associated with fractal or multiple fractals in the process of detecting the prices of financial assets[9]. Complex systems such as financial markets exhibit highly irregular structures, which are named "fractal" by Mandelbrot[10]. Domestic and foreign scholars through empirical research found that financial asset prices do not follow geometric Brownian motion, but show the characteristics of sharp peaks and thick tails, volatility aggregation, heteroskedasticity, long memory, and fractal[11], which are inconsistent with the most classic B-S model in financial option pricing. The classical B-S formula is based on the assumptions that asset returns follow a normal distribution and volatility is fixed. Therefore, the B-S option pricing model has some limitations in pricing options. Among the common option pricing methods, the most commonly used pricing method still belongs to the fractional Brownian motion option pricing method.

Regarding the characteristics of carbon option price distribution, there are also many research results from foreign scholars. Zhou (2019) used the VAR-VEC model to study the energy price based on the carbon emission trading price data of China Hubei Emission Exchange and concluded that the return series of China's carbon emission price is consistent with the characteristics of the financial time series such as the total amount of volatility, the spiky thick tails as well as the non-normal distribution[12]. Zou (2020) applied the multiple fractal trend inter-correlation analysis method to study the relationship between price and trading volume in the European carbon option market and found that the return distribution in the EU ETS option market exhibited a thick tail phenomenon, which could not satisfy the assumption of

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normal distribution in an efficient market[13]. In conclusion, carbon option prices do not obey the normal distribution and have fractal characteristics such as sharp peaks and thick tails and volatility aggregation. GARCH-like models with unconventional innovations are a popular option pricing method nowadays (Liu et al.)[14][15]. For options, GARCH is better at predicting realized volatility than implied volatility and historical volatility (Chuang et al.)[16]. Currently, scholars at home and abroad determine EUA option prices by using the GARCH model, which calculates the volatility of carbon emission allowance prices and substitutes it into an option pricing model as such (C. Zhang et al, 2015[17]; Z. B. Liu and H.Shan, 2021[18]). The starting point of GARCH models is that the effects of positive and negative error terms on volatility are symmetric, but it has been shown that due to market imperfections such as transaction costs (Aliyev et al. 2020)[19] and different investor responses to good and bad news (Barberiset et al. 1998; Bentes 2018)[20][21], such effects are in financial time series is actually asymmetric. J. X. Liang et al. (2014) proposed to estimate the volatility of financial assets with an improved EGARCH model, proposing to fit the volatility of soybean futures prices based on the introduction of new factor variables and without changing the complexity of the original model, which verified the feasibility and accuracy as well as the applicability of the EGARCH model in estimating volatility of financial assets and overcame the shortcomings of the traditional GARCH model in estimating volatility [22]. X. Y. Wu (2021) predicted the volatility of the Chinese stock market through the EGARCH model, which proved that the EGARCH model can better respond to the leverage effect and reflect the aggregation of volatility[23][24]. Therefore, the EGARCH model may be more suitable for analyzing financial time series.

In summary, domestic and foreign scholars have less product design for carbon options, mainly focusing on carbon futures products, and their design mainly focuses on the basic elements in the contract, and has not yet formed a unified design framework. At present, there are still no carbon financial derivative products such as carbon options and carbon options in China, and the carbon emission right itself does not have the characteristics of real options, but carbon emission quotas have this nature, so the article takes carbon emission quotas as the underlying for the design of China's carbon option products. Since the volatility of the carbon market shows time-varying nature that does not match the historical volatility, and the carbon price return does not obey the normal distribution. Therefore, considering the fractal characteristics of carbon price and the time-varying characteristics of carbon price volatility in pricing is more in line with the real situation of carbon market. In order to improve the accuracy of carbon option pricing, this paper introduces the combination of EGARCH and Fractional Brownian Motion into carbon option pricing research, establishes the carbon option pricing method based on the EGARCH-Fractal Brownian Motion model, and then formulates the reasonable call price of carbon options.

3. Methods

3.1. Fractional Brownian Motion (FBM)

Fractal market theory believes that the capital market is a complex nonlinear system, investors' reaction is inconsistent and rationality is limited, the price change shows fractional Brownian motion law, with fractal nature. As an emerging market, the carbon trading market is affected by the imperfect design of carbon trading market mechanism, incomplete rationality of carbon investors, and uncertainty of governmental carbon related policies. Therefore, theoretically speaking, the carbon trading market is complex and nonlinear. Meanwhile, the Hurst index of carbon option price calculated through empirical research is about 0.5231, which is less than 1, indicating that the carbon trading market is fractal. In summary, the article is based on the fractal market theory and adopts the fractional Brownian motion to study the carbon option price in order to more accurately reflect the market characteristics.

ISSN: 2710-0723

Fractional Brownian motion is a probability space in where is a constant lying between 0 and 1, and fractional Brownian motion with Hurst exponent is a continuous Gaussian process and satisfies the following conditions:

3.1.1. Hurst Exponent

Academics usually use the Hurst index to measure whether the financial time series comply with the random walk process or biased walk process, biased walk process is fractional Brownian motion. The Hurst index was first proposed by H.E. Flutst, a British hydrologist, who proposed to use the analysis method of rescaled extreme deviation (R/S) to calculate the Hurst index.

Hurst has three forms:

(1) If$0.5 < H < 1$, there is a long-term dependence, with a positive correlation between past increments and present increments;

(2) If $0 \leq H < 0.5$, there is anti-persistence, where past increments are negatively correlated with present increments;

(3) If $H = 0.5$, it can be expressed as a standard Brownian motion, which has a random wandering property and the increments are independent of each other.

According to the characteristics of the Hurst index, the financial time series can be described using fractional Brownian motion when the Hurst index is not equal to 0.5.The Hurst index can be obtained by the R/S analysis method, and the following is the basic principle:

Let the corresponding time series obtained at moments t1, t2, ..., tN be ξ1, ξ2, ..., ξN, and the number of time series be N. Divide N into A subsequences, each with n observations. Label the subsequence as Ia, where a = 1, 2, 3...A. Each element in Ia is labeled as Nk,a, k=1, 2, 3…n.

Then the rescaled polar deviation averaged over A subintervals for partition length n is:

$$(R/S)_{\ n} = \frac{1}{A}\sum_{a=1}^{A}(R_{Ia}/S_{Ia})$$

Among them:

R_{Ia}defined as the extreme deviation,

$$R_{Ia} = max(X_{k,a}) - min(X_{k,a})$$

S_{Ia}defined as the standard deviation,

$$S_{Ia} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(N_{k,a} - e_a)^2}$$

$e_a$denotes the average value of the elements contained in the sequence,

$$e_a = \sum_{k=1}^{n}\frac{N_{k,a}}{n}$$

$X_{k,a}$denotes the cumulative mean deviation,

ISSN: 2710-0723

$$X_{k,a} = \sum_{k=1}^{n} N_{k,a} - e_a$$

3.1.2. Fractional Brownian Motion

The fractal Brownian motion option pricing model is used with the following five assumptions:

(1) Option prices are determined primarily by the stock price and the risk-free rate;

(2) There are no taxes or transaction costs in the market;

(3) The risk-free rate is a constant;

(4) The stock price is consistent with a fractal Brownian motion;

(5) Options are European style options.

In this paper, it is assumed that the options under study are call options. Suppose a European call option has an exercise price of K, an expiration date of T, a stock price of ST at any time t (0 ≤ t ≤ T), a risk-free rate of r, and a volatility of σ. Then the price of the call option, C(t, ST), at time t is:

$$\mathrm{C(t, S_T) = S_T N(d_1) - K e^{-r(T-t)} N(d_2)}$$

$$d_1 = \frac{ln(\frac{S_1}{K}) + (r + \frac{\sigma^2}{2})(T^{2H} - t^{2H})}{\sigma\sqrt{(T^{2H} - t^{2H})}}$$

$$d_2 = d_1 - (T^{2H} - t^{2H})$$

N(·)denotes the cumulative standard normal distribution function.

3.2. EGARCH Model

The estimation of time series volatility parameters in fractional Brownian motion is usually described by historical volatility, but because the price volatility of carbon options presents the characteristics of sharp peaks and thick tails and volatility aggregation, which is different from the assumptions of traditional regression models, the article chooses to adjust the volatility by using the EGARCH model. Compared with the GARCH model, the EGARCH model is more dynamic and can better reflect the leverage effect of the price volatility of carbon emission rights, improve the fit of the volatility measurement model to the price volatility, and enhance the accuracy of the option pricing of carbon emission rights.

In 1982, Engle proposed the ARCH (Autoregressive Conditional Heteroskedasticity) model for describing volatility in time series data, a method for modeling heteroskedasticity (heteroskedasticity) in time series data. In the ARCH (q) model, let:

$$y_t = \varepsilon_t \cdot \sigma_t$$

where σ_t represents volatility and ε_t represents white noise with zero mean and finite variance. In addition, the conditional variance $\mathrm{V(y_t \mid y_{t-1}) = \sigma_t^2}$ of the series at each moment has the following autoregressive process:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \cdot y_{t-i}^2$$

$$\alpha_i \geq 0_{i=1\ldots q}, \alpha_0 > 0$$

ISSN: 2710-0723

The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, proposed by Bollerslev (1986) in 1986, recognizes the difference between unconditional and conditional variance and allows the conditional variance to vary over time as a function of past error. It is defined as follows:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \cdot y_{t-i}^2 + \sum_{i=1}^{p} \beta_i \cdot \sigma_{t-i}^2$$

Where:

$$\alpha_i \geq 0_{i=1\dots q}, \beta_i \geq 0_{i=1\dots p}, \alpha_0 > 0, p \geq 0, q > 0, \sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i < 1$$

Unlike traditional ARCH models, EGARCH models allow the response of volatility to be nonlinear and therefore capture the characteristics of volatility more flexibly. In the EGARCH(p,q) model, the conditional variance is defined as follows:

$$r_t = \mu_t + \alpha_t$$
$$\alpha_t = \sigma_t \cdot \varepsilon_t$$
$$\ln(\sigma_t^2) = \omega + \sum_{j=1}^{m} \beta_j \ln(\sigma_{t-j}^2) + \sum_{i=1}^{n} \alpha_i \cdot \left(\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma_i \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right)$$

where α_i denotes the leverage effect, $\alpha i \neq 0$ asymmetric effects are present.

4. Design of Carbon Option Products

The Intercontinental Exchange of the United Kingdom is currently the largest carbon futures and carbon options exchange in Europe, and its carbon options contract specifications as well as trading model are relatively mature. The article will draw on its relevant experience and fully consider China's actual situation when designing China's carbon option products, so as to ensure the rationality and operability of the product design.

4.1. Carbon Option Product Underlying Design

The article studies the carbon option contract in the national carbon market and makes reasonable pricing for it, thus choosing the carbon emission quota in the national carbon market as the underlying of the contract.

4.2. Carbon Option Product Contract Element Design

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4.2.1. Transaction Code

Usually option contracts have a fixed form of contract code, according to the regulations, the article designed the carbon option contract code in the following form: QGEA + month + C / P + strike price. Where C stands for call option and P stands for put option.

ISSN: 2710-0723

4.2.2. Trading Units and Offer Units

At present, most of the participants in the domestic carbon market are enterprises as well as institutional investors, thus a larger trading unit is chosen, and the article sets the trading unit of carbon options as "1,000 tons of carbon dioxide/lot".

4.2.3. Minimum Change Unit

In the current international carbon emission system, the minimum unit of change of carbon option contract set by the European carbon market is 0.005 euro (0.036 yuan after exchange rate conversion), and the minimum unit of change set by the U.S. regional greenhouse gas action is 0.01 U.S. dollar (0.064 yuan after exchange rate conversion). In view of the fact that China's carbon market is in the early stage, in order to ensure liquidity and stability, a smaller unit of change is chosen, and a moderate amount of adjustment is made, and the minimum unit of change in this thesis is set at $0.03/ton, which means that the minimum value of change for an option contract is $30.

4.3. Design of Risk Control Elements for Carbon Option Products

4.3.1. limit up

The up-and-down range refers to the range of price movements that are permissible for an option contract to trade on the exchange. At present, EU EUA options and US CCA options do not set up the stop range. Given that China's carbon market started late and the market is not yet mature, the absence of up and down stops may bring huge market risks, so it is necessary to design up and down stop ranges. Table 1 analyzes the daily fluctuation range of carbon emission price from July 16, 2021 to December 31, 2022, and the daily fluctuation range of carbon emission price from July 16, 2021 to December 31, 2022 is analyzed.

Table 1. Absolute value statistics of daily price volatility of national carbon emission allowances

absolute value	(0,2%]	(2%,3%]	(3%,5%]	(5%,7%]	(7%,10%]
percentage	83.14%	6.74%	6.46%	2.53%	1.12%
Cumulative share	83.14%	89.89%	96.35%	98.88%	100%

According to the data reality of the absolute value of daily price volatility of national carbon emission allowances in Table 1, during the data observation period, the upward and downward ranges of carbon prices were maintained within 10%, especially the cumulative percentage of trading days with volatility ranging from 3% to 5% reaches 96.35%, which covers most of the trading days of the price changes. Therefore, taking into account the coverage of the upward and downward range and the need for risk control, and based on the practice that the upward and downward range of option products in the domestic securities market is usually set at 3%-5%, the article sets the upward and downward range of carbon option contracts at 5%.

4.3.2. Margin System

Margin is an important system used in options contracts to control the risk of trading, such a system can provide performance guarantee for both parties to facilitate the smooth running of the transaction. The margin is set as follows:

Margin for opening an obligatory position in call options = [Pre-contract settlement price + Max (12% × the previous closing price of the underlying contract - the virtual value of the call option, 7% × the previous closing price of the underlying contract)] × contract unit.

Put Option Obligation Position Opening Margin = Min [Pre-Contract Settlement Price + Max (12% x Pre-Closing Price of the underlying contract - Put Option Dummy, 7% x Strike Price), Strike Price] x Contract Unit.

ISSN: 2710-0723

4.4. Carbon Option Product Delivery Element Design

4.4.1. Contract Delivery Month

Considering that carbon emission allowance is a right that can be traded at any time, and in order to be in line with the international community in the future, the design of the delivery month is based on the international carbon option contract, which is March, June, September and December.

4.4.2. Last Trading Day and Expiration Date

The last trading day is the last trading day for a certain futures contract to be traded in the delivery month, after which open futures contracts must be physically delivered or cash delivered as required. The article sets the last trading day and expiration date of carbon emission allowance option contracts in the national carbon market as the fourth Wednesday of the expiration month (postponed in case of statutory holidays), so as to allow investors to make deliveries.

4.4.3. Exercise Method

Carbon options can be categorized into American and European options based on the manner in which the rights are exercised. Given the immaturity of the domestic carbon market, European-style options are more appropriate to ensure market stability and risk control.

Table 2. Carbon Emission Allowance Option Contracts

contract element	concrete content
the object	Carbon Formula Allowance(QGEA)
Contract Type	Call Options, Put Options
Trading Units	1000 tons of carbon dioxide
Offer Units	Yuan (RMB) per ton
Minimum change unit	0.03 yuan/ton
limit up	5%
contract month	March, June, September, December
trading hours	Monday-Friday 9:00-11:30 a.m.; 13:30-15:00 p.m.
Last trading day	Fourth Wednesday of the expiration month
expiration date	same Last trading day
Exercise method	Euclidean
Transaction Code	Call Option: QGEA + month + C + strike price Put Option: QGEA + month + P + strike price

5. Carbon Option Product Pricing

5.1. Sample Selection and Model Construction

The national carbon market is more liquid than other markets in terms of the degree of openness and trading scale, and price continuity and price discovery show strong advantages, so the national carbon market is chosen as the research object. The article selects the national carbon emissions trading price data from July 16, 2021 to December 31, 2022, with a total of 356 data, as the data source of this article. This time period, as the first complete trading year after the establishment of the national carbon market, includes the development, adjustment and change of the market, and is representative and complete. This data source is obtained from the Shanghai Environmental Energy Exchange.

ISSN: 2710-0723

5.2. Fractional Brownian Motion Parameter Estimation

5.2.1. Strike Price(K)

The exercise price of a carbon option is the price actually paid by the holder of the carbon option at expiration. The exercise price of an option agreement is selected as the average of the prices of its underlying asset over the life of the option and is rounded to the nearest whole number, i.e., 54.

5.2.2. Expiry Date(T)

The expiration date of a carbon option is the period of time for which the carbon option holder can hold the carbon option as specified inside the contract when the contract related to the carbon option is made. The article selects a one-year maturity.

5.2.3. Arbitrary Time(t)

Carbon option at-will means any point in time during the term of the carbon option from issuance to expiration, $0 \le t \le T$.

5.2.4. Stock Price(S_t)

S_t denotes the price of the underlying asset at any given time. Options are underpinned by carbon futures, which are used here as a proxy for stock prices.

5.2.5. Risk-free Interest Rate(r)

The risk-free interest rate is the rate of return that an investor can earn by investing in a risk-free program. For the article, the one-year Treasury rate of 2.24% was selected for 2021.

5.2.6. Volatility(σ)

In this article, the EGARCH model is chosen for the prediction of dynamic volatility, and in the following, the EGARCH(1,2) model will be built to predict the volatility for the 30-day forecast period. Considering that the volatility used in the process of fractal Brownian motion price movement is annualized volatility, the article will adjust the predicted volatility to annualized volatility:

$$\text{Annualized Volatility} = \text{Daily Volatility} \times \sqrt{\text{Number of trading days per year}}$$

5.2.7. Hurst Exponent(H)

In this article, the EGARCH model is chosen for the prediction of dynamic volatility, and in the In order to verify whether the carbon financial asset price series conforms to the basic assumption of fractal Brownian motion, the article adopts the R/S analysis method to calculate the Hurst index of the carbon financial asset price series. After calculating the price series of carbon financial assets during the period from 2021.7.16 to 2022.12.31, the Hurst index is obtained as 0.5231.This index is obviously not equal to 0.5, which indicates that the price series of carbon financial assets has a long term memory, i.e. there is a fractal property. Therefore, the comprehensive analysis results can be obtained that the fluctuation process of carbon financial asset prices can be described by fractional Brownian motion.

5.3. Forecasting Volatility Using EGARCH

In order to better test the distributional characteristics of carbon options, the sample data are processed and analyzed. Firstly, the carbon option price series are analyzed by descriptive statistics and unit root test. Then the volatility model is established and the residual series are tested for ARCH effect. Finally, the calculation of Hurst index is carried out. In order to make

ISSN: 2710-0723

the test better, this paper carries out the logarithmic operation on the daily trading data of the national carbon market to get the logarithmic yield, that is:

$$R = 100 \times \left((y_t) - (y_{t-1})\right)$$

5.3.1. Sequential Smoothness Test

Based on the observation, the volatility of the original price series of carbon emission allowances is significantly clustered, as shown in Figure 1. After that, in order to further determine whether the carbon option price series is smooth or not, a unit root test is performed. In view of the better performance of ADF test, this paper adopts the ADF test to conduct the unit root test of the carbon option price series, and the running results are shown in Table 3, which passes the significance test of the smoothness of the series at the 1% significance level.

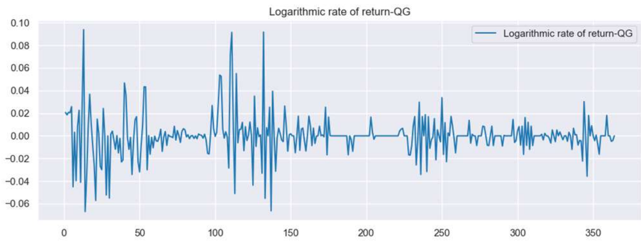

Figure 1. Timing diagram of the original price sequence

Table 3. Carbon Emission Allowance Option Contracts

Variable	ADF test	P-value	Conclusion
QGEAt	-6.891273***	0.0000	stable

Note: "***" indicates significant at the 1% significance level.

5.3.2. General Descriptive Statistical Analysis

Figure 2 demonstrates the general descriptive statistics of QGEAt. It can be seen that the skewness of the QGEAt series is 0.73, which is greater than 0, indicating that the distribution of the series has a long right trailing tail; the kurtosis is 9.56, which is higher than the kurtosis value of 3 for the normal distribution, indicating that the series exhibits the characteristics of spiking and thick-tailed. Therefore, QGEAt as a whole exhibits a non-normal characteristic of spiky and thick-tailed clusters, which needs to be corrected using the mean equation and the GARCH-like model.The P-value is 0, thus rejecting the null hypothesis at the level of significance, i.e., the series does not conform to the normal distribution.

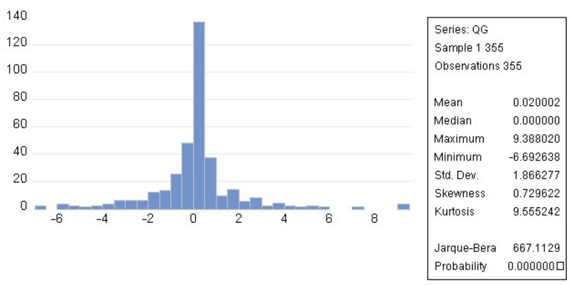

Figure 2. Results of general descriptive statistics

5.3.3. Corrected Series Using Mean Value Equations

The previous section has verified that the logarithmic yield series of carbon financial asset price is a smooth series, further examined the autocorrelation and partial autocorrelation of the series, and then obtained the residual series after mean regression of the carbon financial asset

ISSN: 2710-0723

price yield series. By calculating the AIC of the autoregression of carbon financial asset price return of different orders, according to the principle of AIC minimization, it is found that the fitting effect of ARMA(2,4) is optimal, as shown in the heat map below. After ARMA(2,4) model fitting the residual trend is shown in the figure, it can be found that the residual sequence of fluctuations in the aggregation phenomenon, the preliminary judgment of the existence of arch effect. According to the regression parameters, the mean equation ARMA(2,4) of QGEAt is constructed, see equation (1).

Figure 3. Heat map on AIC

Table 4. Carbon Emission Allowance Option Contracts

	coef	std err	z	P>|z|	[0.025	0.975]
const	0.0003	0.001	0.283	0.777	-0.002	0.002
ar.L1	-0.0489	0.084	-0.582	0.560	-0.213	0.116
ar.L2	0.7471	0.097	7.722	0.000	0.557	0.937
ma.L1	-0.0617	0.092	-0.673	0.501	-0.241	0.118
ma.L2	-0.9431	0.095	-9.938	0.000	-1.129	-0.757
ma.L3	0.0387	0.037	1.047	0.295	-0.034	0.111
ma.L4	0.2974	0.034	8.623	0.000	0.230	0.365

$$QGEA_t = 0.0003 - 0.0489QGEA_{t-1} + 0.7471QGEA_{t-2} - 0.0617\epsilon_{t-1} - 0.9431\epsilon_{t-2} + 0.0387\epsilon_{t-3} + 0.2974\epsilon_{t-4} + \epsilon_t \quad \text{Equation} \qquad (1)$$

where $QGEA_t$ is the observation at time t and ϵ_t is the white noise error at time point t.

As can be seen from Table 4, the coefficients of this mean equation are all significant, the model is well fitted, and the residual series are tested to be free of serial correlation. Therefore, the correction of QGEAt with ARMA(2,4) model is effective. After ARMA(2,4) model fitting.

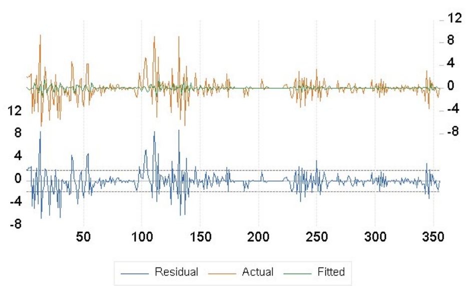

Figure 4. Residual Sequence Plot

ISSN: 2710-0723

The residual trend after the ARMA(2,4) model is shown in Fig. 4, which can be found that there is a fluctuation aggregation phenomenon in the residual series, and it is preliminarily judged that there is an ARCH effect.

5.3.4. ARCH Effect Test

In order to confirm the existence of ARCH effect in the financial asset price return series, this paper adopts the ARCH-LM test, which is the most commonly used in the ARCH effect test methods, to test the residual series after ARMA(2,4) fitting, and the specific test results are shown in Table 5. p-value is less than 0.05, which rejects the original hypothesis, i.e., there exists the ARCH effect, which indicates that the residual series after ARMA(2,4) fitting exists the ARCH effect. residual series exists ARCH effect.

Table 5. ARCH test results

Heteroskedasticity Test: ARCH			
F-statistic	13.69482	Prob.F	0.0000
Obs*R-squared	37.16862	Prob.Chi-square	0.0000

5.3.5. Estimated Volatility of the Underlying Asset

(1) Building the EGARCH model

The conditional heteroskedasticity ARCH-LM test has been carried out in the previous section and found that there is an ARCH effect in the residual series. Therefore, in this paper, an EGARCH-like model is used to estimate the volatility of the underlying asset. According to the AIC information criterion, ARMA(2,4)-EGARCH(1,2) model is chosen to be built to finally obtain the variance equation:

$$\ln(\sigma^2) = 0.1724 + 1.2124|r_{t-1} - 0.1336r_{t-1}| + 0.6543\ln(\sigma_{t-1}^2) + 0.3193\ln(\sigma_{t-2}^2)$$

Table 6. Parameter list of EGARCH (1, 1)

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.1724	0.463	0.373	0.709	[-0.735, 1.079]
alpha[1]	1.2124	0.353	3.436	5.902e-04	[0.521, 1.904]
gamma[1]	-0.1336	0.136	-0.986	0.324	[-0.399, 0.132]
beta[1]	0.6543	0.250	2.620	8.785e-03	[0.165, 1.144]
beta[2]	0.3193	0.230	1.389	0.165	[-0.131, 0.770]

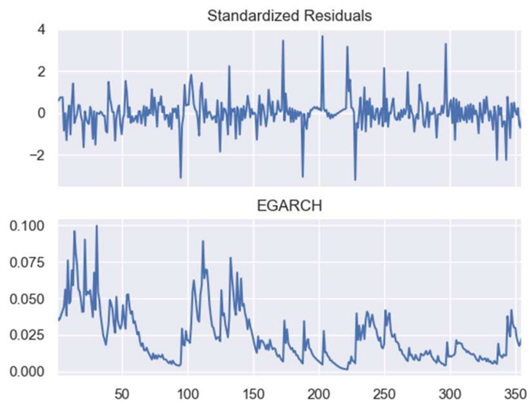

Figure 5. EGARCH model fitted volatility plot

ISSN: 2710-0723

Table 7. ARCH test results

Heteroskedasticity Test: ARCH			
F-statistic	0.296509	Prob.F	0.8279
Obs*R-squared	0.897503	Prob.Chi-square	0.8260

Among them, β reflects the dependence of the current volatility on the previous period's volatility, and its value is close to 1 and the coefficient is significant, which indicates that the previous period has a more lasting impact on the current period's price, and the coefficient is strictly smaller than 1, which proves that the volatility has a certain degree of predictability. In general, the market is more sensitive to negative news, where the coefficient of γ is -0.13 and the asymmetric term is less than zero, which can better reflect the leverage effect. Although the impact of negative shocks on positive shocks has a greater impact on the price of carbon emission rights, but because the domestic carbon trading market is in the early stage, the relevant policies will also have a greater impact, for example, in 2021, after the Ministry of Ecological and Environmental Protection issued the relevant documents, the number of carbon emission rights transactions clearly showed an upward trend in the market to promote the price to appear to rise further, which proves that the carbon emission rights return does There is a leverage effect, and carbon emission rights investors can base their investment strategy on this effect.

(2) Forecasting volatility through the EGARCH model

The volatility for the 30-day forecasting period is obtained by predicting according to the prediction formula of the EGARCH(1,2) model, and the prediction results are shown in Table 8.

Table 8. Projected volatility

Date	Predicted volatility (%)	Date	Predicted volatility (%)
1	0.8603052	16	1.1661547
2	0.9021231	17	1.0897389
3	0.6914782	18	1.1062367
4	0.8735594	19	3.7820307
5	0.6634717	20	2.8274622
6	0.7149138	21	2.3906825
7	0.5654308	22	4.2088423
8	0.5451848	23	3.2819746
9	0.7315918	24	3.0086272
10	0.6006426	25	2.9740885
11	0.5195141	26	2.3151042
12	1.906146	27	1.9787865
13	1.1283461	28	1.7557332
14	1.0694204	29	2.0044038
15	0.946098	30	2.2979754

5.4. Fractional Brownian Motion Pricing Results

After analyzing and calculating, the parameter values of fractal Brownian motion such as execution price, expiration date, arbitrary time, stock price, risk-free interest rate, volatility and Hurst index are obtained above, from which the predicted price of carbon option for the 30-day prediction period can be calculated. The prediction results are shown in Table 9.

ISSN: 2710-0723

Table 9. Expected price of carbon options

Date	Forecast price	Date	Forecast price
1	3.21	16	1.87
2	2.64	17	0.84
3	2.97	18	0.84
4	2.91	19	1.43
5	2.82	20	0.31
6	2.34	21	0.64
7	2.10	22	2.16
8	2.21	23	1.31
9	1.23	24	1.24
10	2.33	25	1.71
11	2.38	26	1.94
12	1.06	27	2.57
13	1.46	28	1.70
14	1.64	29	3.45
15	1.83	30	2.00

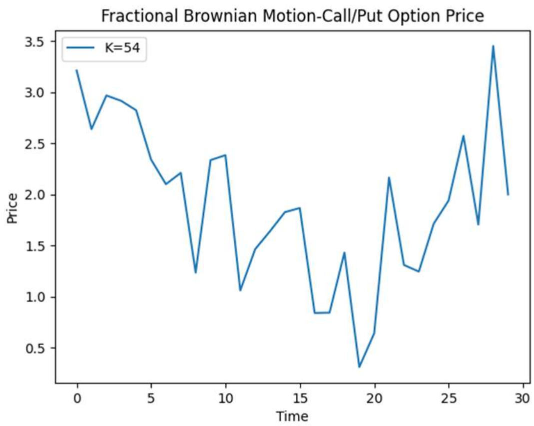

Figure 6. Fractional Brownian motion predicts option price graphs

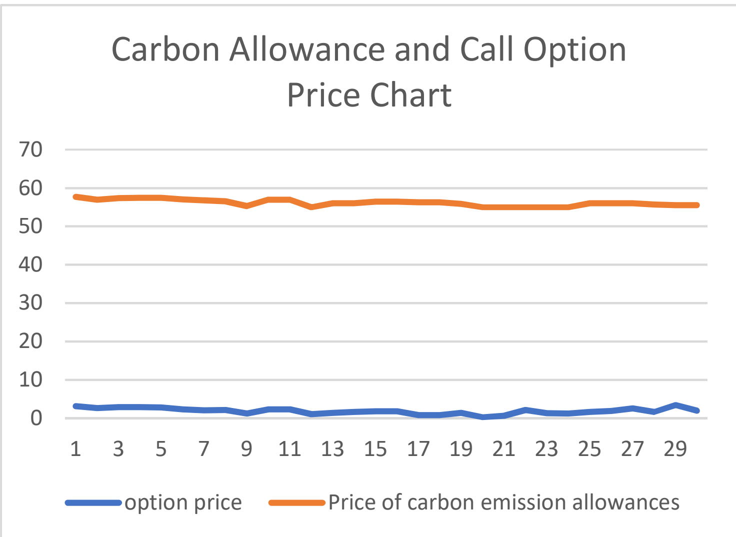

Figure 7. Trend of Carbon Allowance and Option Prices

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As can be seen from figure 7, when the time t is short, the trend of the option price and the underlying asset price is basically consistent, and with the passage of time, the intrinsic value toward the expiration date converges, and the time value eventually converges to 0, which indicates that this paper's pricing of carbon options is reasonable. Moreover, when the underlying asset spot price oscillation is large, the oscillation of the carbon option price also intensifies, which fully shows the transmission effect of the underlying asset price to the option price, in line with the theory and reality.

Finally, taking 2022 as an example, this paper calculates that the initial reasonable price of one-year European carbon allowance call option is RMB 3.21/tone according to the simulated pricing process of the set Fractal Brownian Motion, i.e., the premium of RMB 3,210 needs to be paid for buying one lot of European carbon allowance call option. In summary, the design of the pricing of carbon option products in China is completed.

6. Conclusion

Facing the great pressure of emission reduction, China, as a responsible big country, shoulders the heavy responsibility of emission reduction. Based on the requirements of "carbon peak, carbon neutral" and the vision goal of energy saving and emission reduction in 2035, according to the reality of the domestic carbon financial market and the development of the international carbon market, the article selects the blank carbon quota as the commodity underlying, and analyzes and researches the favorable factors for the launch of China's carbon options, and researches and designs the carbon option products suitable for China. The article analyzes and researches the favorable factors for launching carbon options in China, and then researches and designs the suitable carbon option products for China, with a view to providing reference for the product innovation of China's carbon market. By selecting the national carbon emission allowance trading price data as the research object, the article constructs ARMA(2,4)-EGARCH(1,2) model to estimate the volatility of the underlying asset, and combines the fractal Brownian motion to formulate the reasonable initial price of the option. By actively promoting the construction of carbon trading pricing mechanism, carbon financial system and carbon trading market, we provide a fair, equitable and open dialogue mechanism for China's carbon trading, and enhance China's discourse power in global carbon trading pricing, so as to better realize the saving of resources, and promote the country's environmental protection and sustainable development.

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