

Time of Fragmentation in Heavy Ion Collisions near the Fermi Energy

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Abstract

We simulated $^{64}\text{Ni} + ^{58}\text{Ni}$ collisions at an incident energy of $E/A = 40$ MeV by using an antisymmetrized molecular dynamics (AMD) model with employing a statistical decay code. The primary fragment yields were analyzed, and a power-law distribution was shown in the mass range $1 \leq A \leq 30$. $\tau^{prim} = 2.3$ was treated as the signature of the reaction system ($^{64}\text{Ni} + ^{58}\text{Ni}$) at the critical point, and suggested that the fragments formed at 55-75 fm/c while the disassembling system at/or near its critical point.

Keywords

Fragmentation Time; Critical Point; Heavy-ion Collision; Fermi Energy.

1. Introduction

The physical picture about intermediate heavy-ion collisions is that, at the early stage of the collision, two heavy-ion are compressed and get excited with high energy, then this hot composite nuclei system is expanded to very lower density and evaporated various light particle or formed heavy fragments by the multifragmentation process [1].

In multifragmentation process, the measurements of fragment isotopic yield distribution can provide an interesting probe for extracting the properties of nuclear matter, i.e., the possible occurrence of the nuclear liquid-gas phase transition. There are many excellent types of research to find the signals of the liquid-gas phase transition in experimental data since the first claim [2,3]. Such as using Fisher droplet model analysis to probe the liquid-gas phase transition [4-6], and a lot of signal of measures for the nuclear liquid-gas phase transition were demonstrated in theoretical modeling [7,8]. However, those conclusions are still opening and requires more careful arguments.

One of the difficulties to compare theoretical calculations with experimental data is that the information of fragments detected from experiments were the final ground state. In our previous works, the mass distribution and the ratio of the symmetry energy coefficient to the temperature were extracted from the experiments, those observables are significantly affected by the sequential decay processes [9]. After correcting the experimental observed isotopes yields by the known secondary decay processes shows a power-law distribution and their exponent depends on the $I = N - Z$ value of all isotopes. A simple statistical model calculation indicates that these I -dependent exponents result from the primary isotope distribution with the critical exponent $\tau \sim 2.3$ [10].

With the anti-symmetrized molecular dynamics (AMD) model, the comparison of heavy-ion collision and equilibrium ensemble of the same reaction system shown the excitation energy and the mass distribution of the same collision system was well reproduced by the equilibrium assumption at early reaction stage, which indicated the system may reach an equilibrium state, even though the system undergo a rapid dynamical expansion [11]. Therefore, it is significant to investigate the mass distribution of heavy-ion collisions at an early stage, using model simulations by the AMD.

This research aim to study when did the nuclei system reached the critical point, and the fragments (or clusters) started to form. We applied AMD to simulate the dynamic process of the heavy-ion system at Fermi energy and analyzed the mass yield of fragments in the final ground state, and then compared with modified Fisher model to determine whether the system at a critical point or not, and what's the fragmentation time in this collision system.

This paper is organized as follows: Firstly, we explain the main framework of the used AMD and GEMINI model for theoretical modeling. Then, we show the AMD simulation details in $^{64}\text{Ni} + ^{58}\text{Ni}$ collision at 40 A MeV. After that, we discuss the time of fragments formed in this system. Finally, some conclusions are given, and some possible future works are suggested.

2. AMD and GEMINI Model

2.1. The AMD Model

In this section, we introduce a brief description of the framework of AMD to solve the quantum many-nucleon system problems in the nuclear collision dynamical process.

The A-nucleon quantum state $|\Psi(t)\rangle$ followed the time-dependent many-body Schrödinger equation, which could be represented by a superposition of various reaction channels as [12-14].

$$|\Psi\rangle = c_1|\Phi_1\rangle + c_2|\Phi_2\rangle + c_3|\Phi_3\rangle + \dots$$

AMD uses a Slater determinant of Gaussian wave packets for the reaction channel wave function to treat the fermionic nature of nucleons [6].

$$\langle \mathbf{r}_1 \dots \mathbf{r}_A | \Phi(Z) \rangle = \det_{ij} [\varphi_{Z_i}(\mathbf{r}_j) \chi_{\alpha_i}(j)]$$

where $\chi_{\alpha} = p \uparrow, p \downarrow, n \uparrow, \text{ or } n \downarrow$ represents the spin-isospin states of the nucleon, and the spatial wave functions φ_Z of nucleons are given by:

$$\langle \mathbf{r} | \varphi_Z \rangle = \left(\frac{2\nu}{\pi} \right)^{3/4} \exp \left[-\nu \left(\mathbf{r} - \frac{\mathbf{Z}}{\sqrt{\nu}} \right)^2 + \frac{1}{2} \mathbf{Z}^2 \right]$$

where the complex variables $\mathbf{Z} \equiv \{\mathbf{Z}_i; i = 1, \dots, A\}$ represents the centroids of each wave packets. Usually chosen the width parameter $\nu = 0.16 \text{ fm}^{-2}$ to all the wave packets, which has been adjusted to reasonably describe the ground states of nuclei such as ^{16}O .

The time evolution of the centroid \mathbf{Z} , which parameterizes the quantum state $|\Psi(t)\rangle$, is determined by the time-dependent variational principle. The dynamical evolution starting from the given initial state to the final state of two colliding nuclei could be determined by the AMD equation of motion together with the stochastic nucleon-nucleon collisions.

2.2. The GEMINI Model

The GEMINI model is a statistical decay model, which calculates the decay of excited compound nuclei by sequential binary decays, and involves all sorts of binary divisions include light-particle emission, fission process, intermediate-mass fragment (IMF) emission, and even γ -ray emission. GEMINI employs a Monte Carlo technique to simulate the decay chains of individual mother nuclei by calculating the decay width for possible decay channel until the daughter nuclei are unable to undergo further decay [15].

Light-particle emission. For light-particle fragments with $Z \leq 2$ (n, p, d, t, 3He, α and 6He), the evaporate decay width is calculated using the Hauser-Feshbach formalism [16]. The decay width for the emission of a light particle i from compound nuclei of excitation energy E^* and spin S_{CN} is calculated by:

$$\Gamma_i^{HF} = \frac{1}{2\pi\rho(E^*, S_{CN})} \int d\varepsilon \sum_{S_d=0}^{\infty} \sum_{J=|S_{CN}-S_d|}^{S_{CN}+S_d} \sum_{\ell=|J-S_i|}^{J+S_i} T_\ell(\varepsilon) \rho(E^* - B_i - \varepsilon, S_d)$$

where E^* is the excitation energy of compound nuclei. The S_d is the spin of residual nuclei. The S_{CN} , S_i , ℓ , J , ε , B_i and T_ℓ are the compound nuclei spin angular momentum, light particle i spin angular momentum, orbital angular momentum, total angular momentum, kinetic energy, binding energy and transmission coefficient of the emitted light particle. The ρ and ρ_{CN} are the density of residual nuclei and initial compound nuclei.

Fission and IMF emission. The decay width for heavier fragments emission is calculated using the transition state of Moretto formalism [17]:

$$\Gamma_{Z,A} = \frac{1}{2\pi\rho_{CN}(E^*, S_{CN})} \int d\varepsilon \rho_{sad} [E^* - B_{Z,A}(S_{CN}) - \varepsilon]$$

where ρ_{sad} and $B_{Z,A}(S_{CN})$ are the level density and spin-related energy (the deformation energy plus rotation energy) of the saddle-point configuration, and ε is the kinetic energy of the translational degree of freedom. After modified the shell effect into the above equation by correct mass asymmetry and angular momentum, better result is expected.

γ -ray emission. The compound nuclei may undergo γ -ray emission while the excitation energy is very low. The decay width for γ -ray emission is calculated by Blatt and Weisskopf formalism [18]:

$$\Gamma_\gamma^\ell = F_\ell \frac{18(\ell+1)}{\ell(\ell+3)^2 [(2\ell+1)!!]^2} \frac{e^2}{\hbar c} \left(\frac{R}{\hbar c}\right)^{2\ell} \frac{1}{D_0} \sum_{S_d=|S_{CN}-\ell|}^{S_{CN}+\ell} \int \varepsilon^{2\ell+1} \frac{\rho(E^* - \varepsilon, S_d)}{\rho(E^*, S_{CN})} d\varepsilon$$

where ε is γ -ray energy, $R = 1.2 A^{1/3}$ fm, and $D_0 = 1$ MeV. The GEMINI model only considered the electric dipole and quadrupole moment.

3. Model Simulations

As the AMD model may simulate the equilibrium ensemble, we apply it to model $^{64}\text{Ni} + ^{58}\text{Ni}$ collisions at 40 A MeV, and perform the simulation in a usual way. The time $t = 5$ fm/c was set at which the projectile and the target touch each other, then calculate the time evolution up to $t = 300$ fm/c. The Gogny force was utilized in AMD, and about 13000 events independently carried out by the simulations. The fragments were identified by a coalescence radius in phase space with different R_c values of 2.0, 2.25 and 2.5, which correspond to values of 2.0 fm, 2.25 fm and 2.5 fm when only the coordinate space was taken into account in the coalescence.

The sequential decay of excited nuclei after freeze-out of the compound system takes a very long time, and they are moving out without strong interaction with each other. It is a better decision to use a statistical decay code taking account of the sequential process, including emission of nucleons, nuclei and γ -rays. In this work, we used GEMINI [19] code to calculate the secondary decay, which based on the sequential binary decay model.

4. Discussion

We use AMD to reproduce the primary fragments. We were then using the results of the modified Fisher model to interpret our calculations.

In the modified Fisher model (MFM), the fragment yield as a function of mass number A is given by: [20,21].

$$Y(A) = y_0 A^{-\tau} e^{-\beta \Delta\mu A}$$

where y_0 is a normalization constant. The factor $A^{-\tau}$ derived from the entropy of produced fragments. The quantities β is the inverse temperature, and $\Delta\mu = F(I/A)$ is the difference between neutron and proton chemical potential, and $F(I/A)$ is the free energy of clusters (fragments). At the critical point of the compound hot nuclei, nearly leaves only the exponential term, so that the power-law emerges: [22].

$$Y(A) = y_0 A^{-\tau}$$

According to the equation given above, the observation that mass distribution of fragments obey a power-law behavior in fragment mass number A , and $\tau^{prim} = 2.3$ was calculated by mean-field theory. Therefore $\tau^{prim} = 2.3$ was recognized as possible evidence for compound nuclei at, or very near, the critical point, but precisely how this state is attained still needed more researches. Away from the critical point, the yield was expected to be damped exponentially.

The theoretical simulated fragments of the reaction system ($64\text{Ni} + 58\text{Ni}$ at 40 A MeV) from each time were fit with this expression. As Figure 1 demonstrated, a log-log plot on the fit is successful over three orders of magnitude with two free parameters for each R_c in the left column. The value $\tau^{prim} = 2.3$ was obtain at the time between 55-75 fm/c after the projectile and the target touch each other. Therefore, it seems that the fragmentation of this reaction is at or near its critical point at this period, and 55-75 fm/c may be the "critical time". In the right column, the fitting values of τ^{prim} gradually decrease from 1.7 to 1.3 as the reaction time moves on, indicating that away from the critical time, the power-law behavior doesn't show up, making the worsen agreement between the data points and power-law line.

Fisher's theory treats the thermodynamic limit of infinite volume and infinite particle number. When we apply the MFM to a heavy-ion collision system of finite particle number in the compound nucleus volume, a bimodal distribution presents as shown in Figure 1. For mass number $A=1$ to $A=30$, which are much lighter than the target, shows the power-law behavior for fragments. But for mass number A great than 30, tow rises appeared in the fragment yield near the projectile mass, target mass, or projectile plus target mass.

In all R_c cases, the power-law distribution characterize the fragments mass distributions at the critical point, which implies that all the fragments were formed at an certain time. And as fragment formed sequentially, the larger fragments yield would be progressively smaller. As a result, the fragments were all form at 55-75 fm/c. After this fragmentation time, the system underwent a dynamic decay and then cooled by breaking the fragments as well as by emitting light fragments and nucleons.

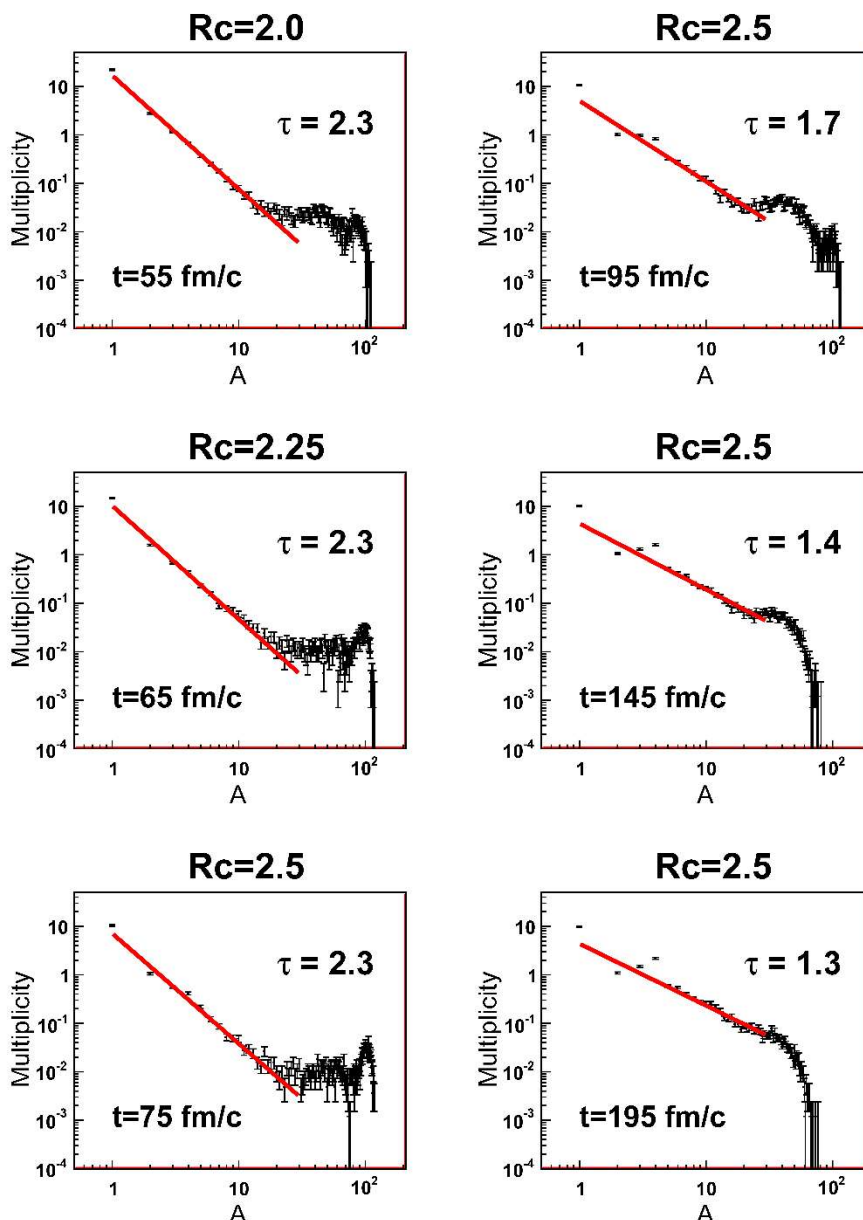


Figure 1. The mass distribution of fragments at each reaction time $t = 55-195$ fm/c obtained from the central ($b_{imp} < 5$ fm) reaction $64\text{Ni} + 58\text{Ni}$ at 40 MeV/nucleon. The lines are power-law fits. The values of R_c for identification fragments were shown.

5. Conclusion

In summary, fragment formation is viewed as occurs at a critical point of a compound nucleus resulting from the interaction of heavy-ion collision, and the fragment yield appears in a proportion of $A^{-2.3}$. From this point of view, the fragment may form at 55-75 fm/c. If this description is correct, it should be possible to use the formation time of fragments to explore the shape and density of the reaction system at the critical point.

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