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Tooth Surface Generation and Meshing Contact Analysis of Conjugate Face Gear

Wang Li[a], Yufeng Gao[b], Yuxin Wang[c]

School of Mechanical Engineering, North China University of Science and Technology, Tangshan 063210, China

[a]lw201906@ncst.edu.cn, [b]gyf00331@163.com, [c]15612539816@163.com

Abstract

This article establishes a spatial coordinate system for conjugate face gear transmission based on the principles of differential geometry and gear spatial meshing. The tooth surface equation of the conjugate face gear is derived using the gear hobbing method, and the tooth contact characteristics during the meshing process of the conjugate face gear are studied based on the theory of point contact face gear. The research results indicate that the contact trace position on the conjugate face gear tooth surface is biased towards the small end of the face gear tooth under standard installation conditions.

Keywords

Conjugate Face Gear; Point Contact; Tooth Contact Analysis; Transmission Error; Contact Trajectory.

1. Introduction

Compared with the traditional bevel gear transmission and cylindrical gear transmission, the face gear transmission has the advantages of a compact structure, a large transmission ratio, high power density, and high contact ratio, making it widely used in aviation and automobile industries [1-4]. Contact analysis of gear teeth can provide an important basis for improving the meshing performance of face gear pairs. Therefore, scholars have conducted extensive research on the contact analysis of gears.

Wang Jin [5] proposed constructing the real tooth surface of gears through the grid method, laying a foundation for the research work simulating the meshing transmission of gears with real tooth surfaces. Litvin [6-8] presented various face gear meshing forms, such as face gears meshing with cylindrical gears and face gears meshing with helical gears. They also conducted contact analysis and stress analysis of various face gears. Li Daqing [9], based on the principle of gear grinding, analyzed the influence of tooth profile parameters on the meshing performance of face gear pairs. It was shown that the position and direction of the contact trajectory can be influenced by altering tooth shape parameters, thereby improving the meshing performance of face gears. Zhou Ruchuan [10] studied the transmission performance of the conical face gear pair, conducted contact analysis and stress analysis of the teeth, and found that the conical face gear pair has good tolerance to installation errors. Zhao Yulong [11] studied the tooth surface contact analysis during the meshing process of orthogonal arc tooth surface gears, and obtained the variation law of the principal curvature at the contact point in arc tooth surface gear transmission. Chu Liyang [12] studied the topological modification design method of the face gear tooth surface, conducted tooth contact analysis and finite element meshing simulation analysis on the modified tooth surface. By adjusting the modification coefficients appropriately, different contact performance requirements can be met. Tang Jinyuan [13] established a contact analysis model of the crowned gear with installation errors, analyzing the impact of installation errors and crowned parameters on the contact trajectory.

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This article derives the tooth surface equation of conjugate face gears, establishes a mathematical model of the gear teeth using MATLAB software, and performs meshing contact analysis on the conjugate face gear pair with standard installation.

2. Tooth Surface Equation

The meshing of conjugate face gear pairs differs from traditional gear pair meshing. The generation of the tooth surface of conjugate face gear is illustrated in Figure 1, where an internal gear cutter meshes with the inner conical face gear, and an external gear cutter with the same parameters meshes with the outer conical face gear. Based on the principle of tooth surface conjugation, the formation of the tooth surface of a face gear is a process where a gear shaping cutter (involute cylindrical spur gear) shapes the gear blank according to a given transmission ratio. The tooth surface of the face gear is formed by the envelope of the tooth surface of the gear shaping cutter.

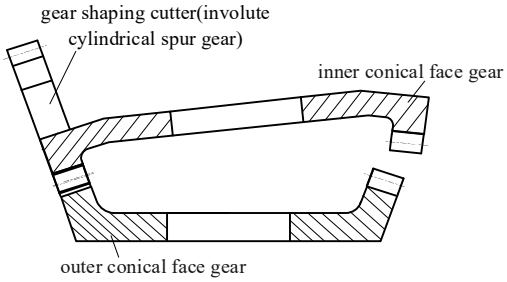


Figure 1. Schematic diagram of conjugate face gear development

2.1 Tooth Surface Equation of Gear Shaping Cutter

The involute tooth profile and parameters of the gear shaping cutter are shown in Figure 2. In the dynamic coordinate system of the gear shaping cutter $S_s(o_s, x_s, y_s, z_s)$, based on geometric relationships, the tooth surface equation of the involute cylindrical spur gear cutter can be expressed as:

$$\mathbf{r}_s(\theta_s, u_s) = \begin{bmatrix} r_{bs}\left[\sin(\theta_s+\theta_{s0}) - \theta_s\cos(\theta_s+\theta_{s0})\right] \\ -r_{bs}\left[\cos(\theta_s+\theta_{s0}) + \theta_s\sin(\theta_s+\theta_{s0})\right] \\ u_s \\ 1 \end{bmatrix} \tag{1}$$

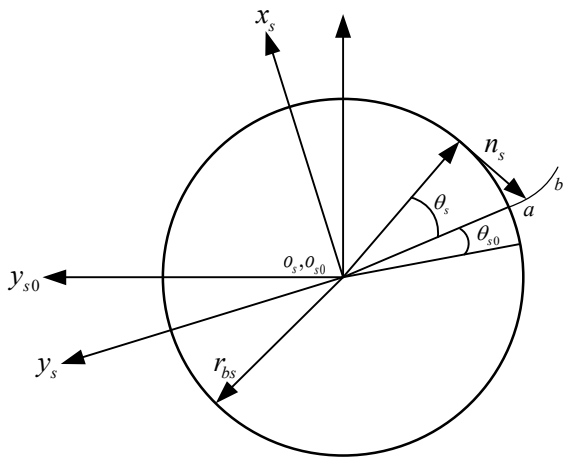


Figure 2. Schematic diagram of involute gear cutter

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In the formula, θ_s is the angular parameter of any point on the involute of the cutter; u_s is the parameter of the axis direction of z_s the gear shaping cutter; r_{bs} is the base circle radius of the gear shaping cutter; From equation (1), the unit normal vector of the tooth surface of the gear shaping cutter can be obtained as:

$$\boldsymbol{n}_s(\theta_s)=\frac{\frac{\partial r_s}{\partial \theta_s}\times\frac{\partial r_s}{\partial u_s}}{\left|\frac{\partial r_s}{\partial \theta_s}\times\frac{\partial r_s}{\partial u_s}\right|}=\begin{bmatrix}-\cos(\theta_s+\theta_{s0})\\-\sin(\theta_s+\theta_{s0})\\0\end{bmatrix} \tag{2}$$

2.2 Tooth Surface Generation

The tooth surface $\sum_i$ of the face gear is enveloped through generative machining from the tooth surface $\sum_s$ of the gear shaping cutter. The tooth surface equation of the face gear is obtained by multiplying the tooth surface equation of the gear shaper with the matrix equation of coordinate transformation. The principle of its generative processing is shown in Figure 3, γ is the angle of intersection between the outer conical face gear and the gear shaping cutter.

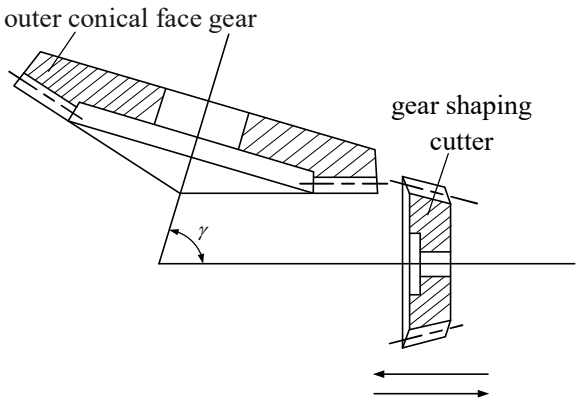


Figure 3. Principle of generating and machining external conical face gear

From Figure 1, it can be seen that the outer conical face gear is obtained by external cutting with the real gear shaping cutter, while the inner conical face gear is obtained by internal cutting with the hypothetical gear shaping cutter, as shown in Figures 4 and 5. β_1 and β_2 is the pitch cone angles of the outer conical face gear and the inner conical face gear, γ_s is the pitch cone angle of the gear shaping cutter, γ_1 is the angle between the shaft of the external conical face gear and the gear shaping cutter, γ_2 is the angle between the inner conical face gear and the hypothetical gear shaping cutter.

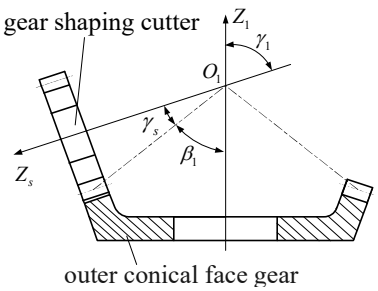


Figure 4. Coordinate system for generating external conical face gear

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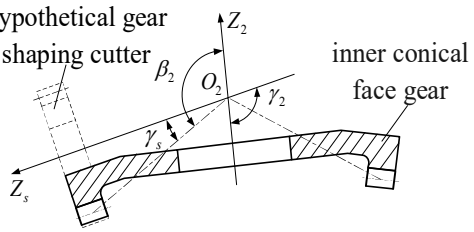


Figure 5. Coordinate system for generating internal conical face gear

Based on the principle of spatial gear meshing, establish a spatial coordinate system for conjugate face gear transmission as shown in Figure 6. Where $S_0(o,x,y,z)$ and $S_{i0}(o_{i0},x_{i0},y_{i0},z_{i0})$ are fixed coordinate systems fixed to the gear shaping cutter and the conjugate face gear. $S_s(o_s,x_s,y_s,z_s)$ and $S_i(o_i,x_i,y_i,z_i)$ are respectively the moving coordinate systems fixedly connected to the gear shaping cutter and the conjugate face gear. Among them , ϕ_s and ϕ_i The instantaneous rotation angles of the gear shaping cutter and the conjugate face gear are respectively. The coordinate origins of the four coordinate systems mentioned above coincide. The subscript i takes 1 and 2 respectively to represent the outer conical face gear and the inner conical face gear, the same below.

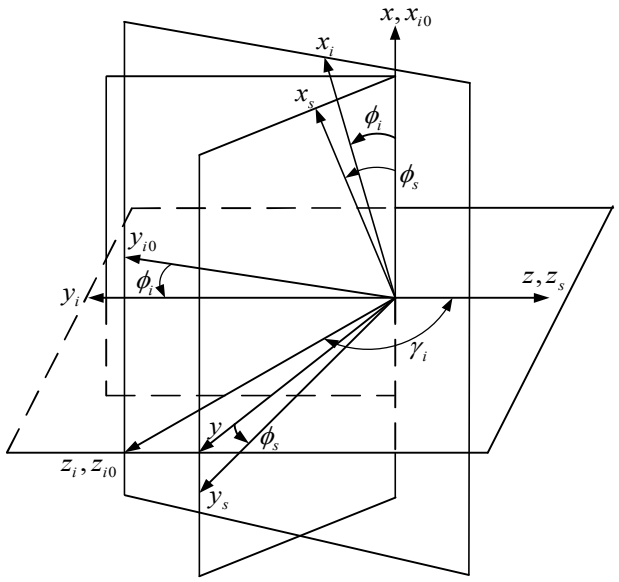


Figure 6. Spatial Coordinate System of Conjugate face Gear Transmission

conjugate face gear tooth surface $\sum_i$ the equation can be expressed as:

$$\left.\begin{aligned} \boldsymbol{r}_i(\phi_s,\theta_s,u_s) &= \boldsymbol{M}_{is}(\phi_s)\cdot\boldsymbol{r}_s(\theta_s,u_s) \\ f_s(u_s,\theta_s,\phi_s) &= \boldsymbol{n}_s\cdot\boldsymbol{v}^{(s,1)}=0 \end{aligned}\right\} \tag{3}$$

In the formula, $\boldsymbol{r}_i$ is the position vector of the conjugate face gear tooth surface, $\boldsymbol{M}_{is}$ is the coordinate transformation from coordinate system S_s to coordinate system S_i , $\boldsymbol{M}_{is}=\boldsymbol{M}_{ii0}\boldsymbol{M}_{i0s0}\boldsymbol{M}_{s0s}$, $f_s(u_s,\theta_s,\phi_s)=0$ it is the meshing equation during the generation motion process; $\boldsymbol{v}^{(s,1)}$ is the relative velocity vector between the gear cutter and the face gear at the meshing point.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202409_10(9).0001

Moving coordinate system of gear shaping cutter $S_s(o_s, x_s, y_s, z_s)$ to the conjugate face gear tooth surface dynamic coordinate system $S_i(o_i, x_i, y_i, z_i)$ coordinate transformation to:

$$\boldsymbol{M}_{is} = \boldsymbol{M}_{ii0}\boldsymbol{M}_{i0s0}\boldsymbol{M}_{s0s} = \begin{bmatrix} \cos\phi_i\cos\phi_s + \sin\phi_i\sin\phi_s\cos\gamma_n & -\cos\phi_i\sin\phi_s + \sin\phi_i\cos\phi_s\cos\gamma_n & -\sin\phi_i\sin\gamma_n & 0 \\ -\sin\phi_i\cos\phi_s + \cos\phi_i\sin\phi_s\cos\gamma_n & \sin\phi_i\sin\phi_s + \cos\phi_i\cos\phi_s\cos\gamma_n & -\cos\phi_i\sin\gamma_n & 0 \\ \sin\phi_s\sin\gamma_n & \cos\phi_s\sin\gamma_n & \cos\gamma_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

In the formula, for outer conical face gear, $\gamma_n = \pi - \gamma_i$; for inner conical face gear, $\gamma_n = \gamma_i$.

The unit normal vector of the conjugate face gear tooth surface is:

$$\boldsymbol{n}_i(\theta_s, \phi_s) = \boldsymbol{L}_{is} \bullet \boldsymbol{n}_s \quad (5)$$

In the formula, $\boldsymbol{L}_{is}$ is the submatrix formed by the first three rows and first three columns of the coordinate transformation matrix $\boldsymbol{M}_{is}$.

According to the parameters shown in Table 1, the mathematical model of the conjugate face gear teeth can be obtained by substituting the working teeth of the face gear as shown in Figure. 7.

Table 1. Design parameters of conjugate face gears

Design parameters	Parameter values
Number of teeth for gear shaping cutter	18
Number of teeth on outer conical face gear	63
Number of teeth on inner conical face gear	65
modulus	2.5
Pressure angle(°)	20
Intersection angle between outer conical face gear and gear cutting tool shaft(°)	66.360
Intersection angle between inner conical face gear and gear cutting tool shaft(°)	111.140

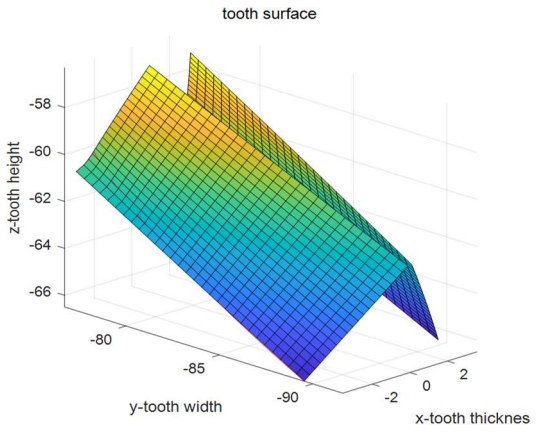


Figure 7. Schematic diagram of conjugate face gear tooth surface

ISSN: 2414-1895 DOI: 10.6919/ICJE.202409_10(9).0001

3. Contact Analysis of Conjugate Face Gear

During the meshing process of conjugate face gear pairs, the condition of continuous tangent contact between the two tooth surfaces is met. Therefore, in the same fixed coordinate system, there is a common contact point between the two tooth surfaces at any time, and there is a common normal at the common contact point, as shown in Figure 8. Based on the position vector equation and normal vector equation of two gear tooth surfaces, a system of contact analysis equations for conjugate face gear teeth can be established.

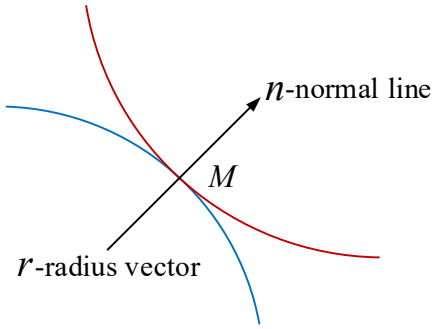
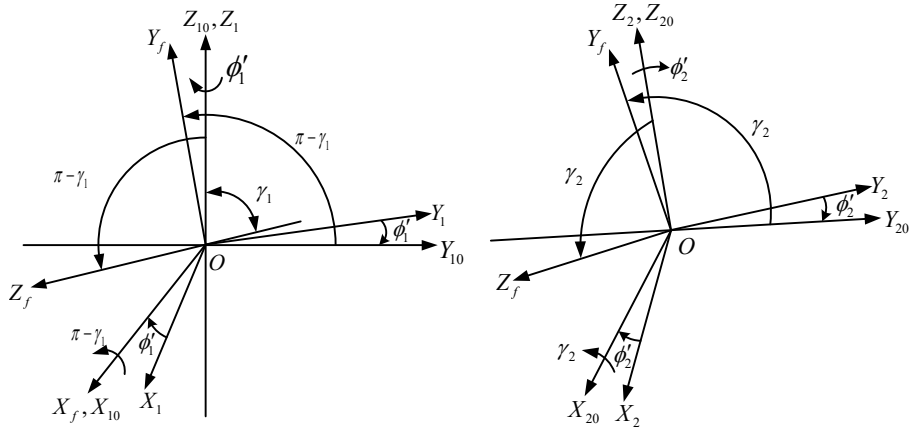


Figure 8. Moment of tooth contact point

3.1 Coordinate System for Gear Tooth Contact Analysis

Establish the coordinate system for conjugate face gear contact analysis as shown in Figure 9,set coordinate system S_1 and S_2 the dynamic coordinate systems of the outer conical face gear and the inner conical face gear of the conjugate face gear pair, respectively. S_{10} and S_{20} the fixed coordinate systems for the outer conical face gear and the inner conical face gear, respectively. S_f is an auxiliary fixed coordinate system. The outer conical face gear and the inner conical face gear rotate around the z_1 -axis and z_2 -axis respectively, with the angles of rotation being ϕ_1' and ϕ_2'. In a fixed coordinate system S_f ,the outer conical face gear meshes with the inner conical face gear.



(a) Coordinate system for contact analysis of outer conical face gear

(b) Coordinate system for contact analysis of inner conical face gear

Figure 9. Coordinate System for Conjugate face gear Contact Analysis

3.2 Tooth Contact Analysis

Outer conical face gear tooth surface Σ_1 the tooth surface equation and corresponding normal vector in a fixed coordinate system S_f it can be expressed as:

ISSN: 2414-1895 DOI: 10.6919/ICJE.202409_10(9).0001

$$\begin{cases} \vec{r}_f^{(1)}(\theta_{s1},\phi_{s1},\phi_1') = \boldsymbol{M}_{f1}\bullet\vec{r}_1(\theta_{s1},\phi_{s1}) \\ \vec{n}_f^{(1)}(\theta_{s1},\phi_{s1},\phi_1') = \boldsymbol{L}_{f1}\bullet\vec{n}_1(\theta_{s1},\phi_{s1}) \end{cases} \tag{6}$$

In the formula, $\boldsymbol{M}_{f1}$ is the matrix transformation of the outer conical face gear tooth surface Σ_1 from its own dynamic coordinate system S_1 to a fixed coordinate system S_f. θ_{s1} and ϕ_{s1} are respectively the spanning angle and self turning angle of the gear shaper cutter that forms the tooth surface Σ_1 of the outer conical face gear. $\boldsymbol{L}_{f1}$ is the coordinate transformation matrix $\boldsymbol{M}_{f1}$ submatrix composed of elements from the first three rows and columns.

$$\boldsymbol{M}_{f1} = \boldsymbol{M}_{f10}\bullet\boldsymbol{M}_{101} = \begin{bmatrix} \cos\phi_1' & -\sin\phi_1' & 0 & 0 \\ \cos\gamma_1\sin\phi_1' & \cos\gamma_1\cos\phi_1' & \sin\gamma_1 & 0 \\ -\sin\gamma_1\sin\phi_1' & -\sin\gamma_1\cos\phi_1' & \sin\gamma_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Inner conical face gear tooth surface Σ_2 the tooth surface equation and corresponding normal vector in a fixed coordinate system S_f it can be expressed as:

$$\begin{cases} \vec{r}_f^{(2)}(\theta_{s2},\phi_{s2},\phi_2') = \boldsymbol{M}_{f2}\bullet\vec{r}_2(\theta_{s2},\phi_{s2}) \\ \vec{n}_f^{(2)}(\theta_{s2},\phi_{s2},\phi_2') = \boldsymbol{L}_{f2}\bullet\vec{n}_2(\theta_{s2},\phi_{s2}) \end{cases} \tag{8}$$

In the formula, $\boldsymbol{M}_{f2}$ is the matrix transformation of the inner conical face gear tooth surface Σ_2 from its own dynamic coordinate system S_2 to a fixed coordinate system S_f. θ_{s2} and ϕ_{s2} are respectively the spanning angle and self turning angle of the gear shaper cutter that forms the tooth surface Σ_2 of the inner conical face gear. Similarly, $\boldsymbol{L}_{f2}$ is a submatrix composed of the first three rows and the first three columns of the coordinate transformation matrix $\boldsymbol{M}_{f2}$.

$$\boldsymbol{M}_{f2} = \boldsymbol{M}_{f20}\bullet\boldsymbol{M}_{202} = \begin{bmatrix} \cos\phi_2' & -\sin\phi_2' & 0 & 0 \\ \cos\gamma_2\sin\phi_2' & \cos\gamma_2\cos\phi_2' & \sin\gamma_2 & 0 \\ -\sin\gamma_2\sin\phi_2' & -\sin\gamma_2\cos\phi_2' & \cos\gamma_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

According to the principle of gear meshing, to ensure continuous contact between two engaging tooth surfaces, their position vectors and unit normal vectors at the meshing contact point must coincide at any times in the same coordinate system, that is:

$$\begin{cases} \vec{r}_f^{(1)}(\theta_{s1},\phi_{s1},\phi_1') = \vec{r}_f^{(2)}(\theta_{s2},\phi_{s2},\phi_2') \\ \vec{n}_f^{(1)}(\theta_{s1},\phi_{s1},\phi_1') = \vec{n}_f^{(2)}(\theta_{s2},\phi_{s2},\phi_2') \end{cases} \tag{10}$$

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Because $\left|\vec{n}_f^{(1)}\right| = \left|\vec{n}_f^{(2)}\right| = 1$,so the above equation is a nonlinear system of equations with 6 unknowns and 5 equations. According to the existence theorem of implicit functions, assuming the corner ϕ_2' as the input parameter, the other 5 unknown parameters can be solved. During the entire meshing process, the position vector and normal vector of the meshing contact point can be determined through the TCA program, and then the position of the conjugate face gear meshing contact point on the tooth surface under standard installation can be solved, as shown in Figure 10.

From Figure 10, it can be seen that the conjugate face gear transmission is a point contact, and the contact trace formed by the instantaneous meshing points of the two surface gears during meshing transitions from the tooth top to the tooth root, which is a curve inclined in the direction of tooth width and clearly leans towards the small end of the surface gear teeth.

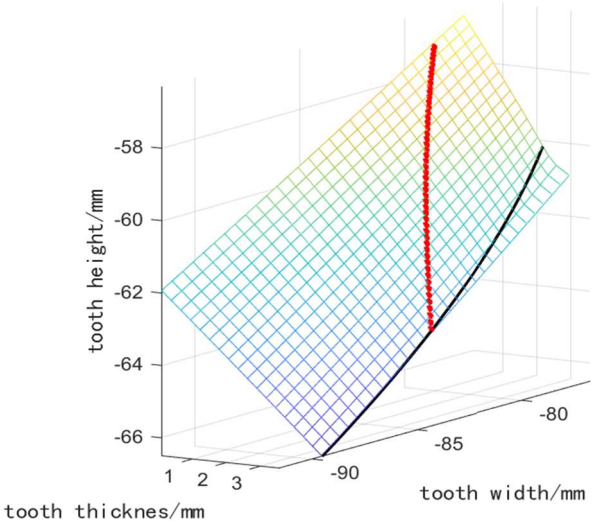


Figure 10. Contact trace of gear tooth surface

4. Conclusion

(1) By creating a generating coordinate system for conjugate face gears and based on the meshing principle, the tooth surface equation of the face gear was derived, and the mathematical model of the face gear was obtained in MATLAB software.

(2) Without considering installation errors, the contact trajectory of the two gears can be solved. By visualizing the meshing points, it can be observed that the position of the meshing points transitions from the tooth top to the tooth root, near the small end of the face gear teeth, approximating a curve inclined in the direction of tooth width, which cannot fully utilize the full tooth width of the face gear.

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