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Fractional-order Modeling of Constant Temperature Water Bath System based on Particle Swarm Optimization

Zishuo Wang[1], Ruipeng Lu[1], Xingquan Gao[1,2, *]

[1] School of Information and Control Engineering, Jilin Institute of Chemical Technology, Jilin 132022, China

[2] School of Information Engineering, Jilin Industrial Vocational and Technical College, Jilin 132013, China

Abstract

This paper proposes a fractional-order modeling method for a constant temperature water bath based on particle swarm optimization. For the fractional-order and unknown parameters of the constant temperature water bath model, different particle swarm optimization ranges are set, and the fractional-order unknown parameters are simultaneously identified through optimization iteration, and experimentally verify the effectiveness of the model through experiments. The comparison results show that the fractional-order model has a higher fitting degree than the integer order model, is closer to the actual output response curve of the constant temperature water bath system, and can better characterize the dynamic change characteristics of the constant temperature water bath temperature.

Keywords

Constant Temperature Water Bath; Fractional-order Modeling; Particle Swarm Optimization; Parameter Identification.

1. Introduction

In recent years, fractional-order calculus theory has attracted the attention of many scholars. The reason is that fractional-order differential equations are widely used in practical systems, such as thermal systems[1], power systems[2], financial systems[3] and hyperchaos system[4]. Therefore, studying fractional order theory is an important topic.

The constant temperature water bath is a typical temperature control device. It has the characteristics of unidirectional temperature rise and large hysteresis. It is widely used in petrochemical industry, biological cell culture and environmental protection, scientific research institutes and other laboratories for reagent distillation, concentration and other heating processes. This system has the characteristics of nonlinearity and strong coupling, and it is difficult to accurately describe it with traditional integer-order models. Fractional-order calculus has global correlation, asymptoticity and memorability, which makes the fractional-order system model closer to the real system and more accurate in description than the integer-order system. In 2017, Zhang et al[5]. established a fractional-order model of an industrial electric furnace, using an improved extended non-minimum state space fractional-order model predictive control method, and verified that the fractional-order model can fit actual data more accurately than the integer-order model. In 2022, Zhao et al[6], in order to improve the power tracking performance and meet the requirements of high fossil energy utilization, established a fractional-order model of the marine boiler-steam turbine system. In 2023, D Grabowski et al[7]. established a fractional-order electric arc furnace model by using the Hammerstein-Wiener model, which proved that the fractional-order electric arc furnace model can reduce errors and thus

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more accurately reflect the phenomena occurring in the furnace. Research results in many fields show that fractional-order modeling has high accuracy, laying a solid foundation for the research of accurate models. This paper conducts research on fractional-order modeling of a constant temperature water bath system, and applies fractional-order theory[8] to the modeling. The mathematical model of the constant temperature water bath system is established using fractional-order thermodynamic equations. In order to improve the fitting degree of the system modeling, the unknown parameters and orders of the fractional-order model were simultaneously identified through the particle swarm optimization (PSO)[9] with intelligent optimization iteration, and finally the accuracy of the model was verified through experiments.

This article is organized as follows. In Section 2, a fractional-order model of the constant temperature water bath temperature system is established. In Section 3, particle swarm optimization is applied to parameter identification of fractional-order systems. In Section 4, experimental verification is given. The conclusions are summarized in Section 5.

2. Fractional Order Model of Constant Temperature Water Bath System

Fig. 1 is a structural diagram of the constant temperature water bath temperature system. The system consists of an inner and outer pot body, a heating device, a platinum thermal resistance temperature sensor, a temperature transmitter and a stirring rotor. Fig. 2 is the schematic diagram of the constant temperature water bath system. The computer controls the thyristor voltage regulating module by inputting control signals from the data acquisition card to set the input power and drive the heating device to heat the pot. The heating device is located in the outer pot. The device heats the liquid in the outer pot through the heating rod and through the coupling effect Conductive heat energy heats the liquid in the inner pot. Pt100 resistive temperature sensors are placed in the inner pot and outer pot respectively to monitor the real-time temperature changes inside the pot, and convert the temperature data into a voltage signal readable by a data acquisition card through a temperature transmitter to achieve a constant temperature water bath system temperature Collection of data.

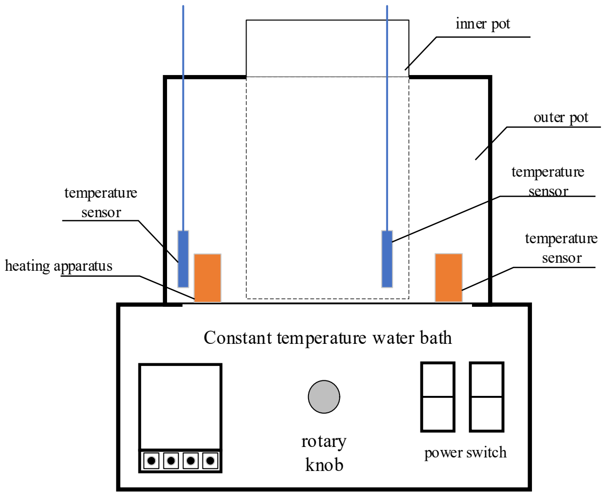

Fig. 1 System overall architecture

The constant temperature water bath temperature system has strong coupling, multi-variable, nonlinear and other characteristics. According to the conservation of energy and the theorem of thermodynamics, it can be known:

$$W = W_1 + W_2 \tag{1}$$

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$$\begin{cases} c_{1\ t0}D_t^{\gamma 1}\beta_1 + k_1\beta_1 = W_1 \\ c_{2\ t0}D_t^{\gamma 2}\beta_2 + k_2\beta_2 = W_2 \end{cases} \tag{2}$$

Where W is the total energy provided by the heater. Suppose the total energy obtained by the outer pot is W_1 and the total energy obtained by the inner pot is W_2, c_1 and c_2 are the heat capacities of the liquid in the water bath. The heat dissipation coefficients of the inner pot are k_1 and the outer pot. The heat dissipation coefficient of the pot is k_2, and the pot temperatures of the inner and outer pots are β_1, β_2.

Apply Laplace transform to formula (2), we get:

$$\begin{cases} c_1 s^{\gamma 1}\beta_1(s) + k_1\beta_1(s) = W_1 \\ c_2 s^{\gamma 2}\beta_2(s) + k_2\beta_2(s) = W_2 \end{cases} \tag{3}$$

Be:

$$G_1(s) = \frac{\beta_1(s)}{W_1} = \frac{k_1}{T_1 s^{\gamma 1} + 1} \tag{4}$$

$$G_2(s) = \frac{\beta_2(s)}{W_2} = \frac{k_2}{T_2 s^{\gamma 2} + 1} \tag{5}$$

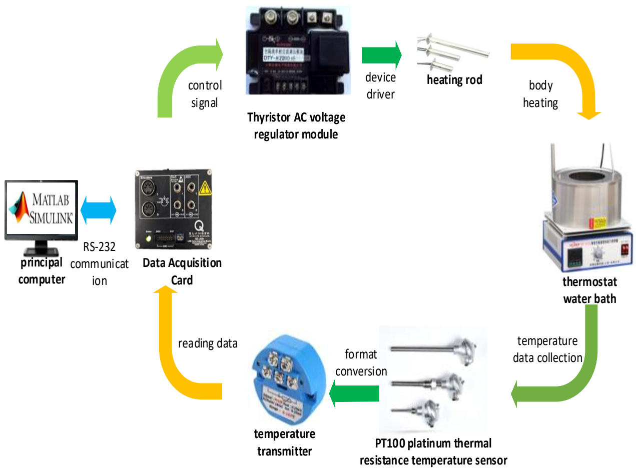

Fig. 2 Schematic diagram of thermostatic water bath system

Therefore, the transfer function of the inner and outer pots of the constant temperature water bath can be obtained by connecting two first-order inertia links in series. However, considering the time delay in the actual process, the transfer function of the outer pot of the simulated constant temperature water bath is expressed as shown in Equation (6), The transfer function of the inner pot is shown in equation (7), where K and τ are the static gain and lag time, and T_1 and T_2 are time constants.

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$$G_1(s)=\frac{K_1}{(T_1s^{\gamma 1}+1)(T_2s^{\gamma 2}+1)}\cdot e^{-\tau_1 s} \tag{6}$$

$$G_2(s)=\frac{K_2}{(T_1s^{\gamma 1}+1)(T_2s^{\gamma 2}+1)}\cdot e^{-\tau_2 s} \tag{7}$$

3. Monitoring System System Design based on Edge Computing

The particle swarm optimization is highly robust in solving nonlinear and non-differentiable problems derived from social psychology. Different from evolutionary algorithms based on the principle of survival of the fittest, the motivation of PSO is to simulate the social behavior of a flock of birds. The goal of this algorithm is to maximize or minimize the fitness value through iteration. After the iteration ends, the current optimal position of the group corresponds to the optimal solution to the optimization problem. At the beginning of the PSO algorithm, particle velocity and position information need to be randomly initialized, and then the optimal solution is found through iteration. During the iteration process, the particles update their speed and position through two extreme values, which correspond to the optimal solutions found in each iteration process, namely the individual optimal solution $pbest$ and the group optimal solution $gbest$.

Therefore, it is assumed that in the optimization space of a given dimension, there are particles, where the position and velocity of the particle in the dimensional search space are $X_i=(x_{i1},x_{i2},x_{i3},...,x_{id})$ and $V_i=(V_{i1},V_{i2},V_{i3},...,V_{id})$ respectively.

During each iteration, the particle velocity and position are modified according to:

$$V_{id}(t+1)=\omega V_{id}(t)+c_1r_1\times(P_i(t)-X_{id}(t))+c_2r_2\times(P_g(t)-X_{id}(t)) \tag{8}$$

$$X_{\mathrm{id}}(t+1)=X_{id}(t)+V_{id}(t+1) \tag{9}$$

Among them, ω is the inertia weight, c_1 and c_2 are normal numbers, r_1 and r_2 are random numbers obtained from the uniform random distribution function on the interval [0,1]. P_{g} represents the optimal position of the group among all particles in the particle swarm, and P_i refers to the optimal position of individual particles in the population.

The individual extreme value P_i of the self-cognition item is updated according to the inverse method of equation (10).

$$P_i=\begin{cases}P_i, f(x(t+1))\geq f(P_i)\\ x(t+1), f(x(t+1))\leq f(P_i)\end{cases} \tag{10}$$

The particle swarm algorithm updates the individual extreme value P_i by optimizing the individual optimal value of the particle; similarly, the optimization of the global extreme value P_{g} is updated by the fitness function value of the historical optimal position of the particle.

The PSO algorithm is applied to the constant temperature water bath fractional-order system identification process to achieve fractional-order system identification. The specific identification algorithm flow chart is shown in Fig. 3. The implementation steps are as follows:

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Choose the objective function. Add the unit step signal to the actual system and get the output response $y(t)$.

Determine the parameters to be identified, including model parameters and fractional orders, and determine the cut-off conditions. As the condition, select the maximum number of iterations or the objective function value.

Algorithm initialization. That is, the relevant parameters of the algorithm are randomly initialized, and the particle population is randomly generated at the same time, and the corresponding objective function value of each particle is calculated.

Update its own speed and position, record the corresponding speed and position, and calculate the objective function value.

Update the individual optimal and global optimal, and perform boundary processing.

The algorithm ends and the identification result is output.

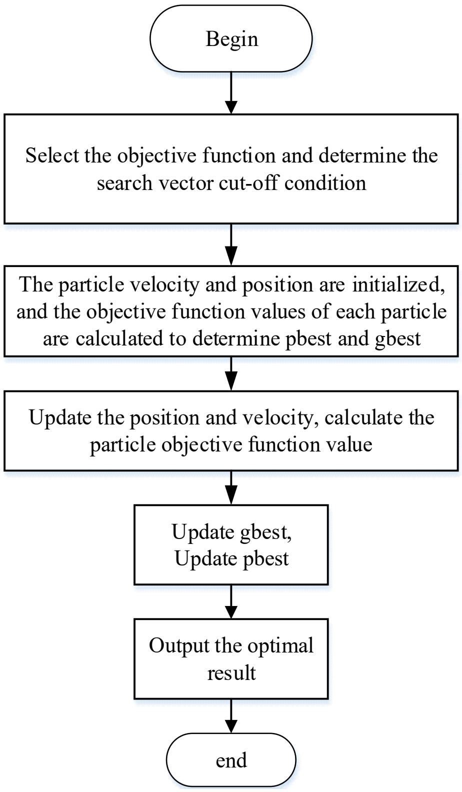

Fig. 3 Process of identification algorithm

4. Experimental Verification

In this paper, under the condition of $20^{\circ}C$ closed room temperature, the temperature change value of constant temperature water bath is collected by giving a fixed control voltage signal, and the temperature data is automatically collected every 0.1 seconds through the data acquisition card. The sampling time is 100 minutes, and a total of 60,000 data points are collected, and the average value is taken for every 10 data points. Each feature has 6000 data points. The voltage is selected as input, the temperature change data of constant temperature water bath as output, Equation (7) is used as the transfer function model for identification, and the error function is used as the index to judge the degree of fitting. The smaller the error, the smaller the degree of fitting, which can describe the characteristics of the system more accurately and lay the foundation for subsequent accurate control. The error index function is:

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$$J = \sum_{i=1}^{n} \frac{1}{2}(y_i - \hat{y})^2 \tag{11}$$

Where: $\hat{y}$ is the predicted value; y_i is the true value; N is the sample population.

This paper uses MATLAB software. The physical picture of the constant temperature water bath system is shown in Fig. 4. Select the temperature data under the determined voltage, identify the fractional order unknown parameters synchronously through the intelligent optimization iterative PSO algorithm, and constantly update and optimize the model parameters of the constant temperature water bath system. The number of population was 50. The maximum number of iterations is 1000, the tolerance of the function value is 0.01 and the learning factor is 1.5. In this paper, 4.1V and 4.3V input voltage data are taken as examples for identification.

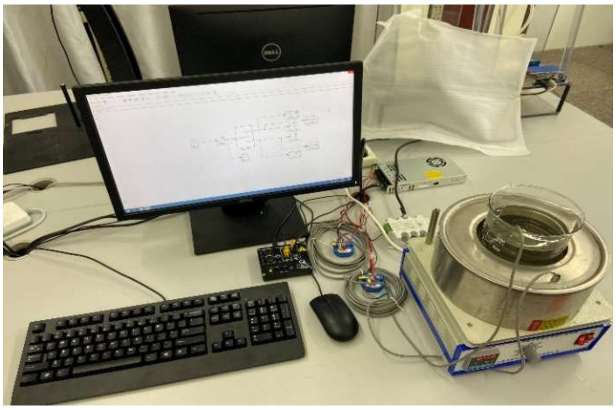

Fig. 4 Physical picture of constant temperature water bath system

In this paper, the identification results of a single input model with voltage as input and temperature of constant temperature water bath as output are compared with experimental data. The identification curve of 4.1V inner pot model is shown in Fig. 5. The identification curve of 4.3V inner pot model is shown in Fig. 6.

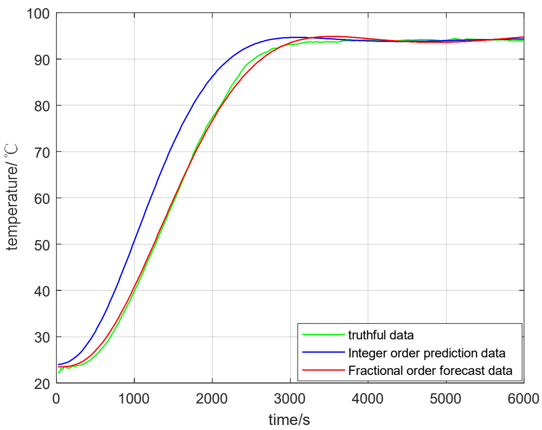

Fig. 5 Identification curve of 4.1V inner pot model

When the 4.1V voltage is used as input, the identified inner pot transfer function is shown in Formula (12):

$$G(s)=\frac{16.738}{1.4493e07s^{2.4194}+2456.2s^{1.0519}+1}e^{-60s} \tag{12}$$

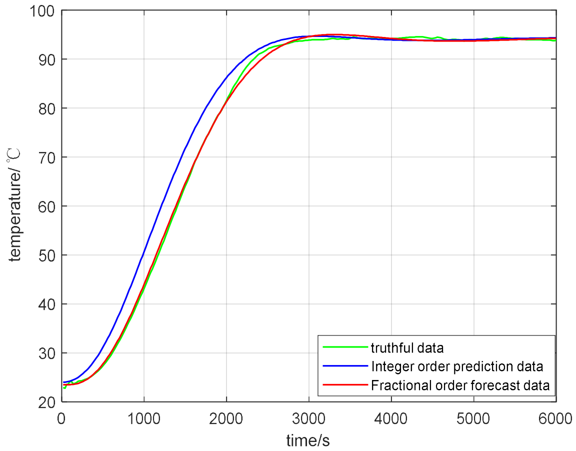

Fig. 6 Identification curve of 4.3V inner pot model

When the 4.3V voltage is used as input, the identified inner pot transfer function is shown in Formula (13):

$$G(s)=\frac{16.101}{1.2492e07s^{2.4459}+3051s^{1.1024}+1}e^{-60s} \tag{13}$$

As can be seen from Fig. 5 and Fig. 6, by comparing the output value of the model obtained by the identification of fractional order model and integer order model with the actual output value, it shows that the identified fractional order model has higher accuracy.

Error analysis is carried out according to the data in Fig. 5 and Fig. 6. The error function is as follows:

$$e_{MAE}=\frac{1}{N}\sum_{i=1}^{n}\left|\hat{y}_i-y_i\right| \tag{14}$$

Where: e_{MAE} is the average absolute error value.

$$e_{MSE}=\frac{1}{N}\sum_{i=1}^{n}(\hat{y}_i-y_i)^2 \tag{15}$$

Where: e_{MSE} is the mean square error value.

$$e_{RMSE}=\sqrt{\frac{1}{N}\sum_{i=1}^{n}(\hat{y}_i-y_i)^2} \tag{16}$$

Where: e_{RMSE} is the root mean square error value.

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The error curve of 4.1V inner pot model identification is shown in Fig. 7:

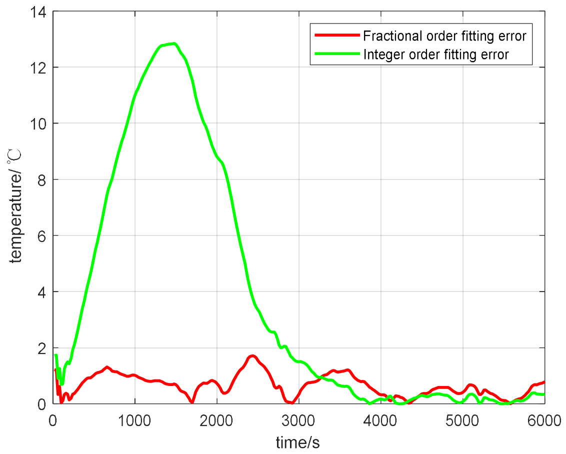

Fig. 7 4.1V inner pot model identification error curve

The error curve of 4.3V inner pot model identification is shown in Fig. 8:

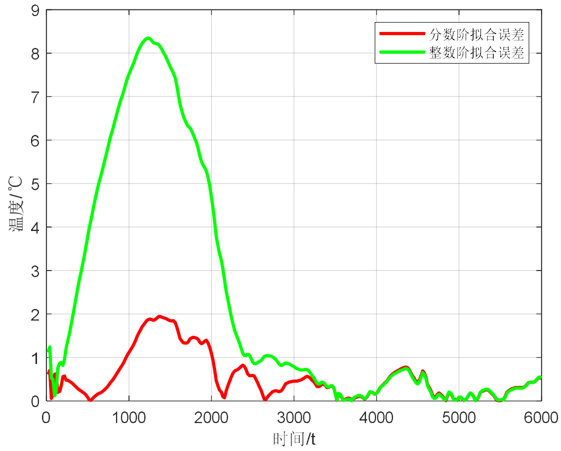

Fig. 8 4.3V inner pot model identification error curve

The model identification error index is shown in Table 1.

Table 1. Model identification error index

	MAE	MSE	RMSE
4.1V integer order mode	0.984	1.318	1.148
4.1V fractional-order model	0.582	0.695	0.833
4.3V integer order mode	0.993	1.598	1.264
4.3V fractional-order model	0.392	0.509	0.713

Compared with the integer-order model, the MAE, MSE and RMSE values of the fractional-order model at 4.1V are reduced by 0.402, 1.308 and 0.244 respectively; the MAE, MSE and RMSE values

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of the fractional-order model at 4.3V are reduced respectively compared with the integer-order model. 0.446, 1.024 and 0.507 smaller. The data shows that the fractional-order temperature model is superior to the integer-order model in terms of absolute error, variance and root mean square error.

5. Conclusion

In view of the complex mathematical mechanism of the constant temperature water bath and the inaccurate model fitting, this paper studies and establishes a fractional order model of the temperature system of the constant temperature water bath. In order to improve the accuracy of the model, the particle swarm optimization algorithm was used to identify the fractional model parameters. First, the experimental data is preprocessed, and then the particle swarm optimization algorithm is used to identify the fractional order model. Finally, the obtained fractional order model is compared with the constant temperature water bath temperature model of the integer order model and the performance of the model is calculated. The results show that the proposed method in this article The model fitting degree achieved by this method reaches 96.18%, which shows that compared with the integer-order model, it is superior in prediction absolute error and prediction variance. The method proposed in this article has higher fitting accuracy and is of great significance for improving the production efficiency and control accuracy of constant temperature water baths.

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