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Application of Markov Processes in Traffic Signal Control

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Abstract

This paper explores the application of Markov processes in traffic signal control systems, utilizing Markov chains to model vehicular queuing systems, thereby optimizing traffic signal control strategies and improving traffic flow efficiency. The paper first provides an overview of the theoretical foundation of Markov processes and subsequently models the vehicle queuing problem within the traffic signal control system as a birth-death process, establishing a queue model based on Markov chains. Through the steady-state analysis of the queuing system, the paper investigates the impact of signal timing adjustments on long-term system behavior. Experimental results reveal that the dynamic signal control strategy based on Markov processes can effectively adjust signal timing according to real-time traffic conditions, significantly mitigating vehicle queue lengths during peak periods. The findings provide theoretical support for intelligent traffic management and hold promise for broad applications.

Keywords

Markov Process; Markov Chain; Traffic Signal Control; Queuing Problem; Birth-Death Processes.

1. Introduction

A Markov process is a stochastic process that satisfies the Markov property, meaning that the probability distribution of future states depends only on the current state and is independent of past states. This characteristic is referred to as "memorylessness." Markov processes are commonly used to describe the dynamic evolution of random variables across a sequence of states[1]. In a discrete Markov process, the state space is finite, and the transition probabilities between states can be described using a Markov chain. Starting from an initial state, the system transitions between different states based on the transition probabilities, forming a sequence of states. Due to its flexibility and memorylessness, the Markov model is highly versatile in the modeling and analysis of stochastic systems and has thus found broad applications in various fields[2].

For instance, in the analysis of internet user behavior, the Markov chain can be used to model the transition users make between different web pages or networks. By observing the user's browsing history, the Markov model can provide more accurate predictions of future browsing paths, thereby optimizing personalized recommendation services. In the financial sector, Markov models are commonly applied to asset pricing, risk assessment, and credit rating. Additionally, in economics, Markov models are widely used to analyze the cyclical transitions of economic phenomena, such as GDP growth, inflation, and deflation forecasts.

In biomedical science, Markov processes are widely used in scenarios such as gene prediction. Liang et al.[3] utilized a bidirectional Markov model to capture the nucleotide context dependencies in the human genome. This model achieved an average prediction accuracy exceeding 50% for nucleotide

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variations and demonstrated better adaptability than simpler models in somatic cell mutation analysis. Markov models are also widely applied in thermodynamics and statistical mechanics, commonly employed to describe energy state transitions in microscopic particle systems. For example, Nishiyama et al.[4] constructed a Markov model that introduced activity dynamics and maximum transition rates, deriving a new upper bound for entropy production in stochastic thermodynamics, thereby providing a more rigorous statistical description of the second law of thermodynamics.

The dynamic changes in traffic queuing systems are often accompanied by uncertainty and randomness, and Markov processes can help analyze and optimize system performance by modeling state transitions. Barnes and Disney[5] conducted in-depth research on the stochastic structure of Markov processes in finite-state queuing systems, integrating previous findings in this field. Xu et al.[6] proposed a Markov state transition model for single intersections in urban traffic, transforming traffic signal control into a Markov decision-making process. Furthermore, Xia et al.[7] integrated reinforcement learning to construct a short-term traffic flow prediction model based on Markov processes, optimizing adaptive traffic signal timing at urban intersections, and effectively reducing average vehicle delays. This paper will follow this line of thought and introduce the practical application of Markov processes in traffic signal control.

2. Theoretical Foundation

In a discrete-time Markov process, the discrete variable n is used to represent any point in time. X_n describes the state of the system at time n , and the set of all possible states is called the state space, denoted by Ω. For any states $i, j \in \Omega$, p_{ij} represents the transition probability of the system moving from state i to state j .

Definition 1. (Markov Property) A Markov process is a stochastic process that satisfies the Markov property, meaning that the future state depends solely on the current state and is independent of past states. A system is said to have the Markov property if it meet the following three conditions:

1) Memorylessness: The transition probability depends only on the current state i, i.e.:

$p_{ij} = P(X_{n+1} = j|X_n = i) = P(X_{n+1} = j|X_n = i, X_{n-1} = i-1, \dots, X_0 = 0)$;

2) Non-negativity: The transition probability is non-negative, i.e.: $p_{ij} \geq 0$;

3) Normalization: The sum of the transition probabilities from state i to all possible states j equals 1, i.e.: $\sum_{j=1}^{n} p_{ij} = 1$.

Definition 2. (Markov Chain Model) A Markov chain model has the following three properties:

1) State space $\Omega = \{1,2, \dots, \mathrm{n}\}$, where n represents the number of states;

2) Possible state transition pairs (i, j), meaning the system can transition from state i to state j;

3) State transition probability $\mathrm{p_{ij}}$ represents the probability of transitioning from state i to state j.

The Markov chain model describes a sequence of random variables $X_1, X_2, \dots$, which take values in the state space Ω and satisfy the Markov property (Definition 1). The transition probabilities of a Markov chain can be represented by a transition probability matrix, as follows:

$$\begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}$$

Additionally, the transition probability matrix can be represented using a transition probability graph. In the graph, nodes represent states, and directed edges between nodes indicate possible state transitions, with the edge labels representing the transition probabilities p_{ij}.

Definition 3. (Chapman-Kolmogorov Equation)[8] The n-step transition probability $r_{ij}(n) = P(X_n = j|X_0 = i)$ represents the probability that the system transitions from the initial state i to state j after n time steps. $r_{ij}(n)$ can be calculated using the Chapman-Kolmogorov equation (abbreviated as the C-K equation).

Theorem 1. (Properties of the C-K Equation) In the C-K equation, given that $r_{ij}(1) = p_{ij}$, the equation holds for all $n > 1, i, j \geq 1$. The states in a Markov chain can be classified based on the number of times they are visited into those that can be visited multiple times and those that can only be visited a finite number of times. Based on this classification, the long-term visit frequencies of each state can be further analyzed.

Definition 4. (Reachability of Recurrent States) If the n-step transition probability $r_{ij}(n)$ is positive, then state j is said to be reachable from state i. In other words, state i can reach state j within a certain time.

Let $A(i)$ be the set of states reachable from state i. If, for every state j reachable from i, state j can also return to state i, then state i is called a recurrent state. Otherwise, state i is classified as a transient state.

If state i is recurrent, then the set of states $A(i)$ forms a recurrent class. All states in $A(i)$ are mutually reachable, and no state outside $A(i)$ can be reached from these states. Mathematically, for a recurrent state i, if any state $j \in A(i)$, then $A(i) = A(j)$.

Based on the classification of recurrent and transient states, a Markov chain can be decomposed as follows:

1) One or more recurrent classes, along with some possible transient states;

2) States within each recurrent class are mutually reachable, while states in different recurrent classes cannot reach each other;

3)No transient state can reach another transient state;

4) Any transient state may reach one or more recurrent classes.

Definition 5. (Periodicity of Recurrent States) If the states in a recurrent class can be divided into d disjoint sets $S_1, S_2, \dots, S_d$, and all state transitions occur from one set to the next, then the recurrent class is said to be periodic. More specifically, if $i \in S_k$ and $P_{ij} > 0$, then $\begin{cases} j \in S_{k+1}, & k = 1,2, \dots, d-1, \\ j \in S_1, & k = d. \end{cases}$.

If a recurrent class does not have periodicity, it is called aperiodic. For an aperiodic Markov chain, at some point n, for any states $i, j \in R$, $r_{ij}(n) > 0$, indicating that the transition probability stabilizes as time progresses.

In a Markov chain, studying the long-term behavior of states is crucial. When time n becomes large enough, the transition probability $r_{ij}(n)$ converges to a steady-state probability, and the system eventually enters and remains in one of the recurrent classes. If there are multiple recurrent classes, the long-term behavior of the system depends on the initial state. Therefore, we typically analyze the asymptotic properties of individual recurrent classes to deduce the overall long-term behavior of the Markov chain.

Definition 6. (Steady-State Probability) Consider an aperiodic Markov chain with a single recurrent class. For each state j, if the limit $\lim_{n\to\infty} r_{ij}(n) = \pi_j$ exists, then π_j is referred to as the steady-state probability of state j.

Theorem 2. (Steady-State Convergence) For an aperiodic Markov chain with a single recurrent class, the state j and its steady-state probability π_j have the following properties:

1) For every j, $\lim_{n\to\infty} r_{ij}(n) = \pi_j$ exists and holds for all i;

2) π_j satisfies the balance equations $\pi_j = \sum_{k=1}^{m} \pi_k p_{kj}$, with $\sum_{k=1}^{m} \pi_k = 1$;

3) For all transient states j, $\pi_j = 0$; for recurrent states j, $\pi_j > 0$.

This theorem shows that the sum of steady-state probabilities equals 1, indicating that steady-state probabilities form a probability distribution, commonly referred to as the stationary distribution of the chain.

3. Application

Based on the definitions and theorems of Markov processes discussed in Section 2, this section will explore the practical application of Markov processes in traffic signal control, with a focus on queueing theory.

3.1 Birth-Death Process

When the state space of a Markov process is finite, and transitions can only occur between neighboring states or remain unchanged, it is referred to as a birth-death process. In queueing theory, the birth-death process is useful for analyzing key performance metrics such as steady-state distributions, average queue length, and server utilization[9].

For a birth-death process, the balance equations derived from Theorem 2 can be simplified. To aid understanding, we will demonstrate how to derive the balance equations for the birth-death process: Let $b_i = P(X_{n+1} = i+1 \mid X_n = i)$ represent the "birth" process of state i, and $d_i = P(X_{n+1} = i-1 | X_n = i)$ represent the "death" process of state i. The details are illustrated in Fig. 1.

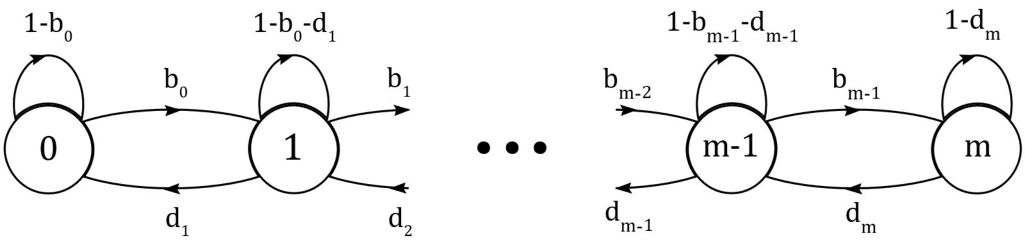

Fig. 1 Illustration of State Transitions in the Birth-Death Process

In a birth-death process, the transition from state i to state $i+1$ is always accompanied by the reverse transition from state $i+1$ to state i (though the reverse transition may not occur immediately). Furthermore, the next transition from $i+1$ back to i occurs before another transition from i to $i+1$ takes place. In other words, transitions from state i to $i+1$ alternate with transitions from state $i+1$ to i. Thus, the probability of transitioning from state i to $i+1, \pi_i b_i$, is equal to the probability of transitioning from state $i+1$ to i, $pi_{i+1} d_{i+1}$, which gives the equation $\pi_i b_i = \pi_{i+1} d_{i+1}$. This equation is known as the local balance equation. From this, we can derive $\pi_i = \pi_0 \frac{b_0 b_1 \cdots b_{i-1}}{d_1 d_2 \cdots d_i}$, $\quad i = 1, \dots, m$, and using the normalization equation $\sum_{k=1}^{m} \pi_k = 1$, we can compute the steady-state probability distribution of the system.

3.2 Application in Traffic Signal Control

In the context of traffic signal control for road queueing systems, suppose the queueing area before an intersection has a fixed capacity, accommodating up to m vehicles. When vehicles arrive at the intersection, they temporarily halt within the queueing area, waiting for the traffic signal to turn green. If the queueing area is already filled with m vehicles, newly arriving vehicles will be unable to enter, potentially leading to traffic congestion and disorder. To efficiently and intelligently manage road queueing systems, it is necessary to analyze how to optimize traffic signal control.

First, consider a short time interval. During this period, at most one vehicle can either arrive at or pass through the intersection, and the number of vehicles in the queue can change by at most one unit. Based on this, the following assumptions are made:

H_1: A new vehicle enters the queueing area with probability $b > 0$;

H_2: If there is at least one vehicle in the queue, a vehicle passes through the intersection during a green light with probability $d > 0$;

H_3: If there is at least one vehicle in the queue, no new vehicles arrive, and no vehicles pass through the intersection with probability $1 - b - d$. If the queue is empty, the probability of the aforementioned events is $1 - b$.

As shown in Fig. 2, the process of vehicle arrivals and passage through the intersection can be described as a birth-death process, with vehicle increases and decreases following the assumptions outlined above.

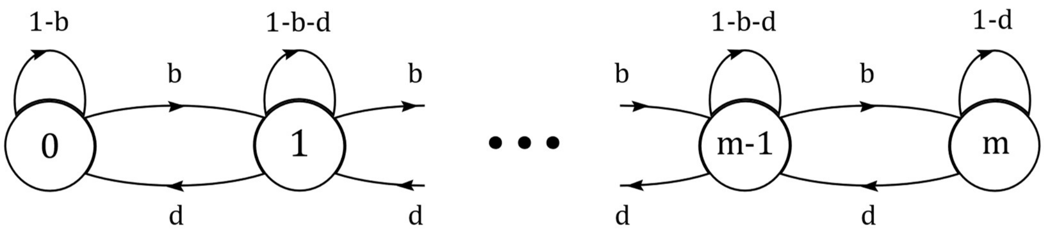

Fig. 2 State Transition Diagram of the Birth-Death Process in the Vehicle Queueing System

According to the local balance equation $\pi_i b_i = \pi_{i+1} d_{i+1}$, we define $q = \frac{b}{d}$, which gives $\pi_{i+1} = q\pi_i$. By recursive derivation, we obtain the general formula $\pi_i = \pi_0 q^i$. Using the normalization equation $\sum_{k=1}^{m} \pi_k = 1$, the steady-state probability of the initial state π_0 and the steady-state probability of state i, π_i, can be determined as below.

$$\pi_0 = \begin{cases} \dfrac{1-q}{1-q^{m+1}}, & q \neq 1 \\ \dfrac{1}{m+1}, & q = 1 \end{cases} \quad ; \quad \pi_i = \begin{cases} \dfrac{1-q}{1-q^{m+1}} q^i, & q \neq 1 \\ \dfrac{1}{m+1}, & q = 1 \end{cases}$$

This indicates that in a long-term stable traffic signal control system, the steady-state probability of having i vehicles in the queueing area is π_i. With these calculations, we can determine the steady-state probabilities of the number of vehicles waiting in the queue under different conditions. This is of great significance in traffic management and planning, as it helps to rationally schedule signal timing adjustments based on traffic flow variations, thereby optimizing traffic movement.

Additionally, we can use the Markov process to determine the probability distribution of each vehicle's position in the queue at different time points. Based on the above traffic signal control system, we reintroduce the following assumptions:

H_1: At each time point, a vehicle moves forward by one position with probability b, or remains at its current position with probability $1 - b$.

H_2: The vehicle's initial position is one of the spaces in the queue, ranging from 1 to m. When a vehicle reaches position 0, it is considered to have exited the queue and is no longer waiting; when it

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reaches position $m+1$, it is considered to have entered the intersection and merged into the traffic flow.

Based on these assumptions, we constructed a model of the queueing system, which describes the state transitions of vehicles between different positions in the queue. The transition probabilities are shown in Fig. 3.

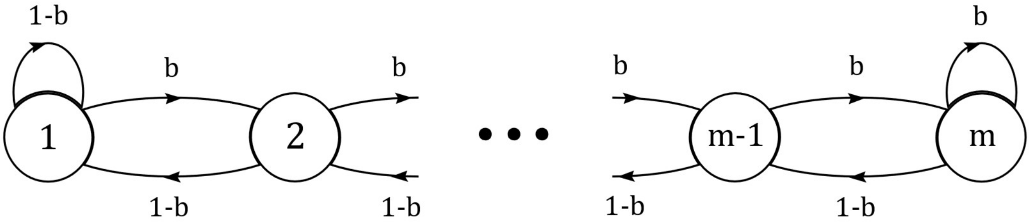

Fig. 3 State Transition Diagram of Vehicle Positions in the Queueing System

Based on the local balance equation $\pi_i b = \pi_{i+1}(1-b)$, we define $q = \frac{b}{1-b}$, leading to the steady-state probability $\pi_i = \frac{q^i}{1+q+q^2+\cdots+q^{m-1}}$. The value of π_i represents the long-term probability of a vehicle being in a specific position in the queueing system at any given time. With these probability distributions, we can evaluate the state distribution of each vehicle from entering the queue to proceeding through the intersection.

3.3 Numerical Experiments

Steady-state analysis provides long-term operational status information of traffic signal control systems, which can assist in optimizing signal timing adjustments. For different traffic volumes, we can flexibly adjust signal timings based on steady-state probabilities to improve intersection efficiency. In the following numerical experiments, we compare the traffic efficiency of dynamic timing strategies and fixed timing strategies under varying traffic conditions, measuring system performance using the average queue length.

The experimental parameters are based on general urban roads, with a length of 40-50 meters, and the queue capacity is set to $m = 10$. The vehicle arrival rate λ starts from 0 and increments by 0.01 until it reaches the passing rate $\mu = 0.8$. System stability is ensured by the condition $\lambda < \mu$.

For the traditional fixed-time strategy, the signal timing is fixed and does not change with the number of vehicles, modeled using the M/M/1 queue system. In this model, the average queue length L_q can be calculated by $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$. For the dynamic signal adjustment strategy based on the Markov process, the green light time will increase with the number of vehicles in the queue. The system state changes are described by the steady-state probability π_i, and the average queue length L_m under this strategy can be calculated using $L_m = \sum_{i=0}^{m} i \cdot \pi_i$.

Fig. 4 illustrates the steady-state probability distribution of the number of vehicles in the queue area at different arrival rates λ. When the arrival rate is low ($\lambda < 0.5$), the system operates under off-peak traffic conditions, resulting in lower traffic volume and shorter queue lengths. As λ increases, the system gradually transitions to peak traffic conditions, especially when $\lambda > 0.6$, where the system approaches a fully loaded state.

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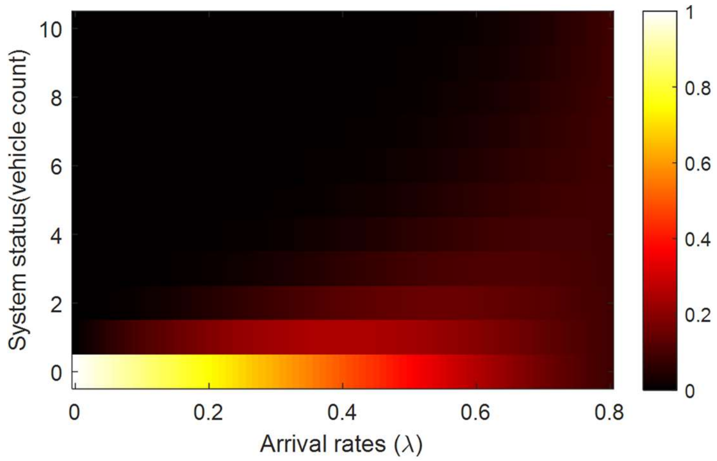

Fig. 4 Steady-State Probability of Vehicle Count as a Function of Arrival Rate

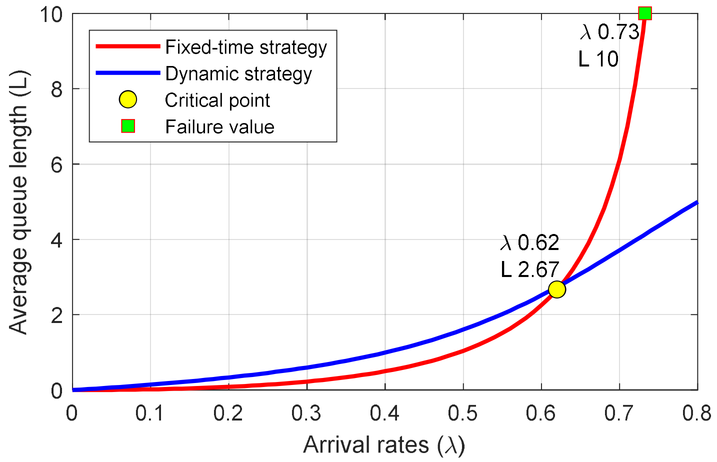

Fig. 5 Comparison of Average Queue Length Under Two Strategies

Fig. 5 presents a comparison of the average queue lengths between the traditional fixed-time traffic signal control strategy and the Markov process-based dynamic traffic signal control strategy across different arrival rates. The following patterns are revealed:

(1) Off-peak period ($\lambda \leq 0.5$): The traditional fixed-time signal control strategy performs efficiently, effectively handling lower traffic volumes and maintaining shorter queue lengths.

(2) Critical point ($\lambda = 0.62$): When the arrival rate reaches 0.62, both strategies show similar performance. At this point, the rate of traffic volume increase accelerates, and the system approaches an unstable state.

(3) Peak period ($0.6 < \lambda \leq 0.8$): The dynamic signal control strategy exhibits a significant advantage. When $\lambda = 0.73$, the traditional fixed-time signal control strategy fails, and the queue length reaches the queue capacity limit (10 vehicles), showing exponential growth.

Numerical experiments indicate that the fixed-time signal control strategy performs well during off-peak periods, but as the arrival rate increases, it gradually becomes unstable, often leading to traffic congestion. In contrast, the Markov process-based dynamic traffic signal control strategy performs significantly better during peak periods, effectively reducing average queue lengths and improving intersection traffic efficiency.

4. Conclusion

This paper systematically investigates the theoretical framework of the Markov process and its practical application in traffic signal control. By modeling the vehicle queuing system as a Markov

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chain based on the birth-death process, the study elucidates the dynamic behavior of vehicles in the queue and the corresponding state transition probabilities. The findings indicate that the model is capable of accurately predicting changes in the number of vehicles within the queue under varying traffic conditions. Furthermore, it enables the dynamic adjustment of traffic signal timings based on steady-state probabilities, thus significantly enhancing intersection traffic flow efficiency.

In the numerical experiments conducted, a comparison was made between dynamic traffic signal control strategies based on Markov processes and conventional fixed-duration signal strategies. The findings imply that dynamic strategies outperform fixed-duration strategies during peak periods, while the latter performs better under low-traffic conditions but struggles in high-traffic scenarios, often leading to congestion. This suggests that Markov process-based dynamic traffic signal control strategy have significant potential in traffic scenarios characterized by large fluctuations in flow rates.

This study provides theoretical support for optimizing traffic signal control, enabling traffic management authorities to dynamically adjust signal timings in response to real-time traffic flow, thus enhancing overall efficiency. Future work will focus on refining model parameters by integrating real-world traffic data and exploring control strategies for complex traffic scenarios, thereby improving the model's adaptability to actual traffic systems and offering more precise optimization tools for traffic management.

References

[1] Bovier A. Markov processes[J]. Lecture Notes Summer, 2012.

[2] Ibe O. Markov processes for stochastic modeling[M]. Newnes, 2013.

[3] Liang Y, Grønbæk C, Fariselli P, et al. Context dependency of nucleotide probabilities and variants in human DNA[J]. BMC genomics, 2022, 23(1): 87.

[4] Nishiyama T, Hasegawa Y. Upper bound for entropy production in Markov processes[J]. Physical Review E, 2023, 108(4): 044139.

[5] Barnes J A, Disney R L. Traffic processes in a class of finite Markovian queues[J]. Queueing Systems, 1990, 6: 311-326.

[6] Xu Y, Xi Y, Li D, et al. Traffic signal control based on Markov decision process[J]. IFAC-PapersOnLine, 2016, 49(3): 67-72.

[7] Xia Xinhai, Xu Lunhui. Urban Intersection Adaptive Traffic Signal Timing Decision with Markov Process Model Using Reinforcement Learning[J]. Highway Engineering, 2018, 43(01): 149-153+239.

[8] Karush J. On the chapman-kolmogorov equation[J]. The Annals of Mathematical Statistics, 1961, 32(4): 1333-1337.

[9] Bhaskaran B G. Almost sure comparison of birth and death processes with application to M/M/s queueing systems[J]. Queueing Systems, 1986, 1: 103-127.