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Three-Component Model-Based Stress Relaxation Calculation Model for PVC Waterproofing Membranes

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Abstract

PVC waterproofing membranes are viscoelastic materials, and a fixed strain is imparted to them by applying initial tension. The stress relaxation characteristics of PVC waterproofing membranes were investigated. The Kelvin model and the three-component model composed of springs in series can express the transient response, creep, and relaxation of viscoelastic solids. Based on the analysis of the experimental data of stress relaxation, the parameters of the three-component model were determined, and the stress relaxation calculation formula of PVC waterproofing membranes was established. This calculation formula can better satisfy the later stage of stress relaxation, but it is not so accurate for the early stage, and there are still shortcomings.

Keywords

Stress Relaxation; Three-Component Model; PVC Waterproofing Membrane; Kelvin Model; Stress.

1. The Significance and Methods Proposed by the Prediction Model

When the strain is constant, the phenomenon of stress decreasing over time is called stress relaxation. From the rheological mechanism perspective, the viscous flow will cause the stress to decay to zero after a long enough time. Therefore, it can be said that under certain strain conditions, a material that quickly tends to zero stress is a fluid, while a material that has stress decaying to a certain value after a considerable amount of time is a solid[1].

PVC waterproofing membranes are viscoelastic materials. A fixed strain is imparted to PVC waterproofing membranes by applying initial tension, allowing for examination of the stress relaxation characteristics of PVC waterproofing membranes as time increases. Additionally, considering that the Kelvin model and the three-component model composed of springs in series can express the transient response, creep, and relaxation of viscoelastic solids, based on the analysis of the experimental data of stress relaxation, the parameters of the three-component model were determined to establish the stress relaxation calculation formula for PVC waterproofing membranes. The appropriateness of the above formula was simultaneously examined through experimentation.

2. The Three-Component Model

Kelvin is a solid viscoelastic body. That is, it consists of two components: an elastic body and a viscous body, where the elastic component forms the skeleton and the viscous component fills the voids in the skeleton. When subjected to an external force, the skeleton and the viscous body are both stressed. As the skeleton deforms, the viscous body also flows, consuming energy on one hand and delaying the deformation of the skeleton on the other. This can be represented by an ideal elastic element and an ideal viscous element in parallel (as shown in Figure 1). The total strain of the two elements equals the total strain of the model, and the stress of the model is the sum of the stresses of the two elements. Under the action of constant stress, the relationship between deformation and time is shown in Equation 1, and the deformation response under load is shown in Figure 2[2].

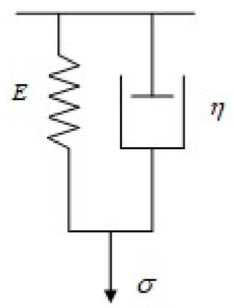

Figure 1. Kelvin Model

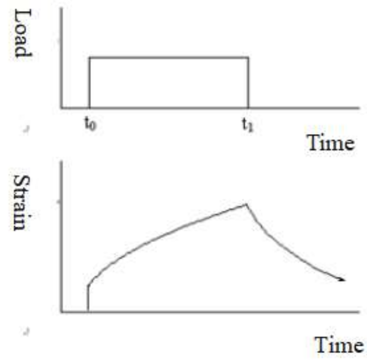

Figure 2. Deformation Response of the Kelvin Model

The Kelvin body does not have instantaneous elastic strain. Strain increases with time. When $t \to \infty$, $\varepsilon \to \sigma/E$, it acts like an elastic solid. When the load is removed at time t1, deformation slowly returns, and when $t \to \infty, \varepsilon \to 0$.

$$\varepsilon = \frac{\sigma}{\eta}\left(1 - e^{-\frac{E}{\eta}t}\right) \tag{1}$$

Because the Kelvin model cannot represent instantaneous elasticity, combining it with a spring in series can form a three-component model, also known as a standard linear solid model. The three-component model can express the instantaneous response, creep, and relaxation of viscoelastic solids. As shown in Figure 3.

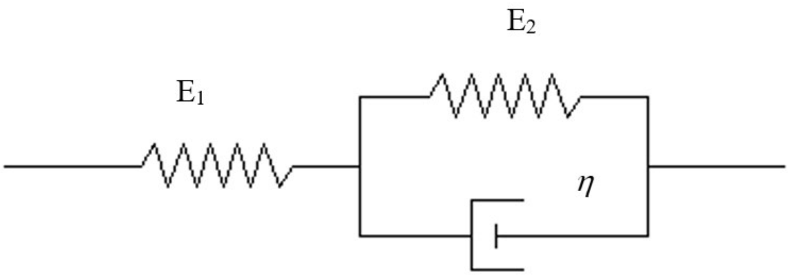

Figure 3. Three-Component Model

Each element's parameters are as shown in the figure. If ε_1 represents the strain of the spring connected in series with the Kelvin model, ε_2 represents the strain of the Kelvin model, and σ and ε represent the stress and strain of the three-component model, the following relationships hold[3]:

$$\varepsilon = \varepsilon_1 + \varepsilon_2 \tag{2}$$

$$\sigma = E_2\varepsilon_2 + \eta\, d\varepsilon_2 / dt \tag{3}$$

$$\sigma = E_1\varepsilon_1 \tag{4}$$

In the equation, $E_1$is the elastic modulus of the spring connected in series with the Kelvin model, $E_2$and η respectively represent the elastic modulus of the spring and the viscosity coefficient of the Kelvin model.After applying the Laplace transform, the relationship between stress and time is obtained as follows:

$$\sigma_t = \frac{E_1E_2}{E_1+E_2}\varepsilon_0 + \frac{E_1{}^2}{E_1+E_2}\varepsilon_0 e^{-\frac{E_1+E_2}{\eta}t} \tag{5}$$

Where:

$$E_1 = \frac{\sigma_0}{\varepsilon_0} \tag{6}$$

$$E_2 = \frac{\sigma_\infty E_1}{\varepsilon_0 E_1 - \sigma_\infty} \tag{7}$$

In the equation:

E_1- the elastic modulus at the beginning of the experiment;

σ_0, ε_0- the stress and strain of the PVC waterproofing membrane at the beginning of the experiment;

E_2- the elastic modulus at the end of the experiment;

σ_∞- the stress of the PVC waterproofing membrane at the end of the experiment.

3. The Stress Relaxation Formula

3.1 Stress Relaxation Calculation Formula at 20°C

To derive the stress relaxation calculation formula, it is first necessary to determine the viscoelastic coefficients. In this paper, the experimental data with an initial tensile rate of 30% are used in combination with Equation 5 to derive the following calculation process.

$$E_1 = 141.85\text{MPa},\ \sigma_0 = 7.79\text{MPa},\ \sigma_\infty = 2.72\text{MPa},\ \varepsilon_0 = 0.055$$

$$E_2 = \frac{\sigma_\infty E_1}{\varepsilon_0 E_1 - \sigma_\infty} = \frac{2.72\times 141.85}{7.79-2.72} = 76.10\,\text{MPa}$$

Selecting an experimental data point, and substituting the values of E_1, E_2, $\varepsilon_0$into Equation 5, we find that $\eta = 50.1MPa\cdot h$.

The stress relaxation calculation formula is as follows:

$$\sigma_t = \frac{E_1 E_2}{E_1 + E_2}\varepsilon_0 + \frac{E_1{}^2}{E_1 + E_2}\varepsilon_0 e^{-\frac{E_1+E_2}{50.1}t} \tag{8}$$

3.2 Comparison between the Stress Relaxation Calculation Value and the Experimental Value at 20°C

Using Equation 8, the calculated values for initial tensile rates of 5%, 20%, 50%, and 60% can be obtained. The calculated values are compared and analyzed with the experimental values, as shown in Figures 4-7.

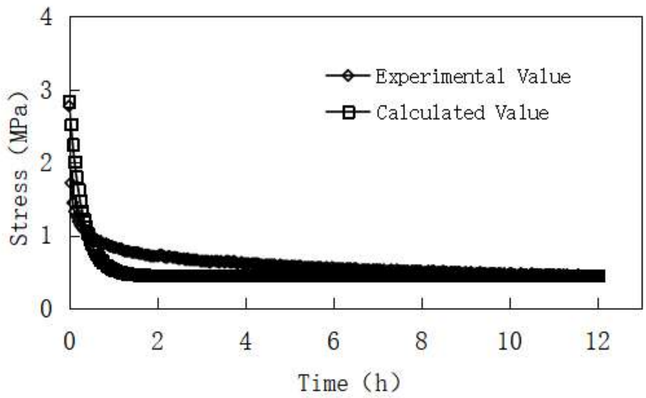

Figure 4. The Relationship between Stress and Time for an Initial Tensile Rate of 5%

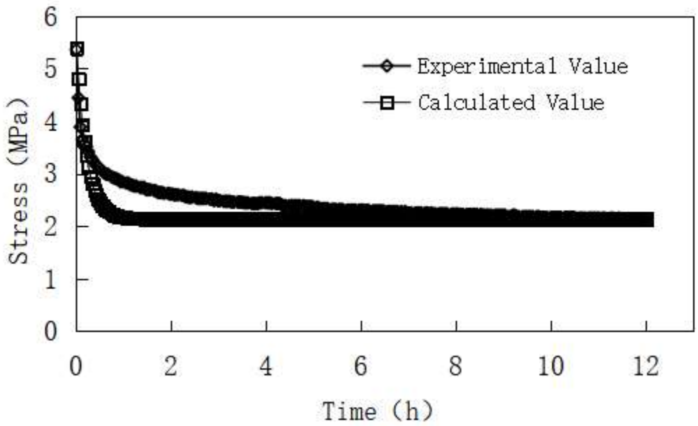

Figure 5. The Relationship between Stress and Time for an Initial Tensile Rate of 20%

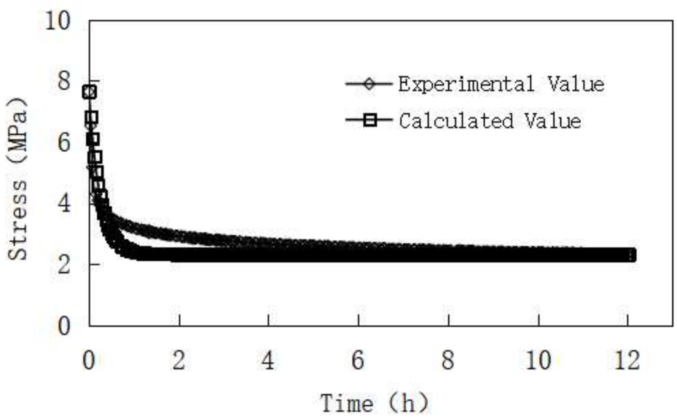

Figure 6. The Relationship between Stress and Time for an Initial Tensile Rate of 50%

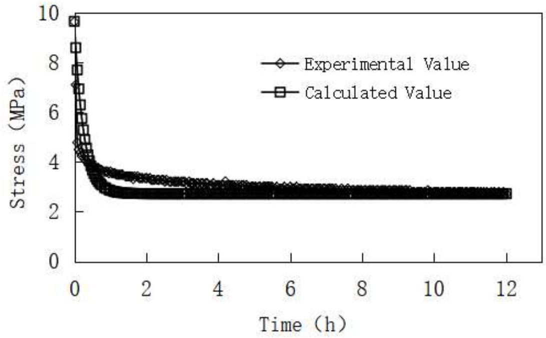

Figure 7. The Relationship between Stress and Time for an Initial Tensile Rate of 60%

From Figures 7, it is evident that the calculated values and the experimental values are quite consistent in the later stages. However, in the early stages, the experimental values appear to have a turning point earlier than the calculated values. This could be due to the fact that the viscous component of the Kelvin model in the chosen three-component model cannot generate instantaneous strain under the influence of external forces. Therefore, the strain of the Kelvin model gradually increases. Because the total strain is constant, this leads to a decrease in the strain of the spring, indirectly resulting in a decrease in stress and causing stress relaxation. Since the viscous component is in a steady-state flow, whereas the relaxation of stress in actual experiments does not show a steady change - faster in the early stages and slower in the later stages, this ultimately results in a less consistent fit for the early-stage data but a better fit for the later-stage data.

3.3 Stress Relaxation Calculation Formula at 0°C

Same as at 20°C, to obtain the calculation formula for stress relaxation at 0°C, it is necessary to first determine the viscoelastic coefficient η. Utilizing the experimental data with an initial tensile rate of 30% and combining it with Equation 5, we can calculate the value as follows: $\eta = 58.2 MPa \cdot h$

The stress relaxation calculation formula is as follows:

$$\sigma_t = \frac{E_1 E_2}{E_1 + E_2}\varepsilon_0 + \frac{E_1^{\ 2}}{E_1 + E_2}\varepsilon_0 e^{-\frac{E_1 + E_2}{58.2}t} \tag{9}$$

Comparison between Calculated and Experimental Values:

Using Equation 9, we can calculate the values for initial tensile rates of 5%, 20%, 50%, and 60%. Then, we can compare and analyze these calculated values with the experimental values.

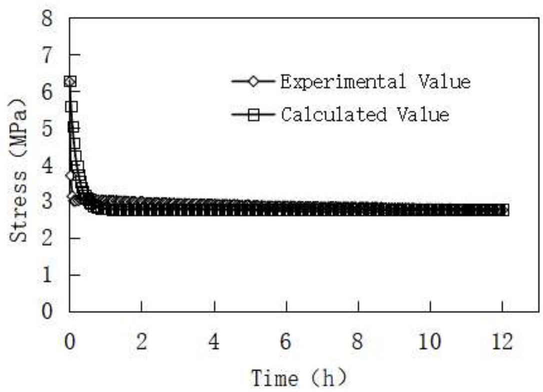

Figure 8. The Relationship between Stress and Time for an Initial Tensile Rate of 5%

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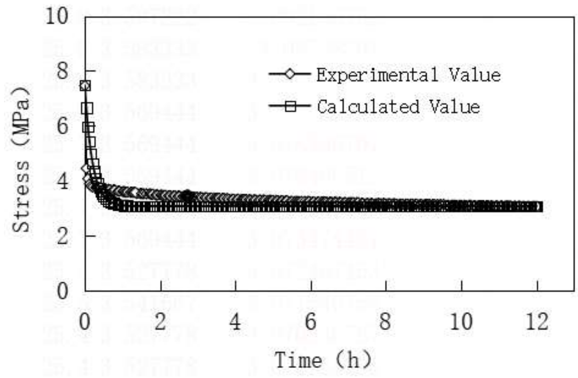

Figure 9. The Relationship between Stress and Time for an Initial Tensile Rate of 20%

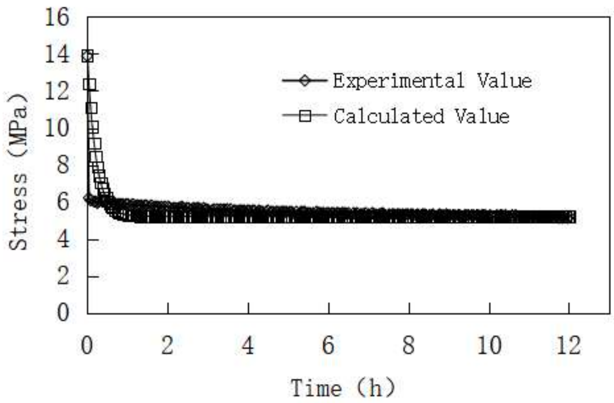

Figure 10. The Relationship between Stress and Time for an Initial Tensile Rate of 50%

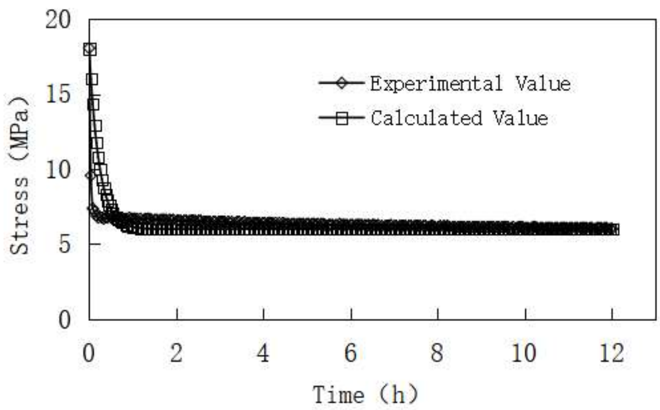

Figure 11. The Relationship between Stress and Time for an Initial Tensile Rate of 60%

From Figures 8-11, we can see that the calculated values match the experimental values quite well. The calculated values appear to have a turning point slightly earlier than the theoretical values, but after the turning point, they match very well. This may also be due to the non-steady-state flow of the viscous component, which does not align with the non-steady-state change in stress relaxation in actual experiments.

3.4 Stress Relaxation Calculation Formula at 69°C

Same as at 20°C, to obtain the calculation formula for stress relaxation at 69°C, it is necessary to first determine the viscoelastic coefficient η Utilizing the experimental data with an initial tensile rate of 30% and combining it with Equation 5, we can calculate the value as follows: $\eta = 39.0MPa \cdot h$

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The stress relaxation calculation formula is as follows:

$$\sigma_t = \frac{E_1 E_2}{E_1 + E_2}\varepsilon_0 + \frac{E_1{}^2}{E_1 + E_2}\varepsilon_0 e^{-\frac{E_1+E_2}{39.0}t} \tag{10}$$

Comparison of Calculated and Experimental Values:

Using Equation 10, the calculated values for initial tensile rates of 5%, 20%, 50%, and 60% can be obtained. Then, the calculated values are compared and analyzed against the experimental values.

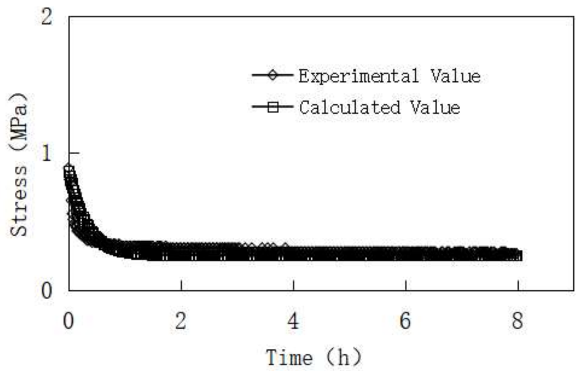

Figure 12. The relationship between stress and time at an initial tensile rate of 5%

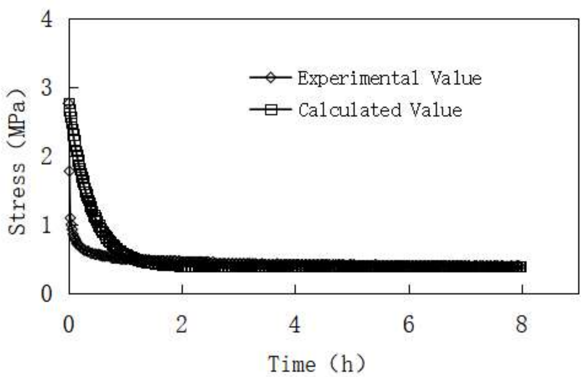

Figure 13. The relationship between stress and time at an initial tensile rate of 20%

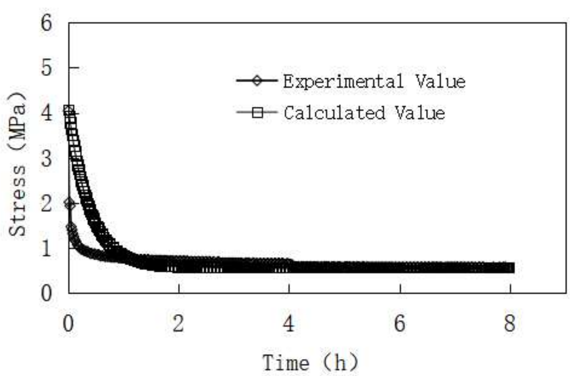

Figure 14. The relationship between stress and time at an initial tensile rate of 50%

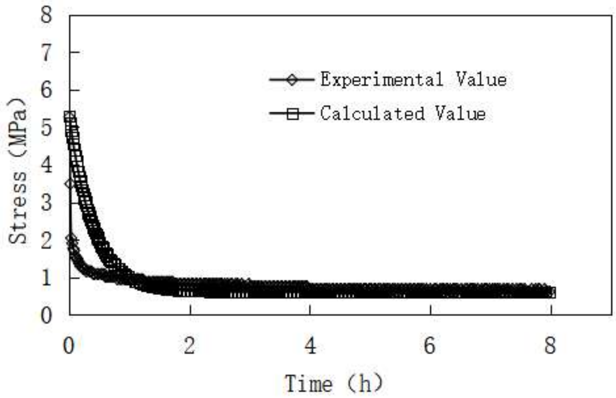

Figure 15. The relationship between stress and time at an initial tensile rate of 60%

From Figures 12-15, it can be seen that the calculated values and the experimental values are in better agreement in the later stages. However, in the early stages, the experimental values appear to have a turning point earlier than the calculated values. This is also due to the influence of the theoretical model's viscous component.

4. Conclusion

The calculation formula derived from the three-component model better fits the later stages of stress relaxation, but it is less accurate for the earlier stages, and there are still some deficiencies. Comparing the curves of stress vs. time at 0°C, 20°C, and 69°C, it can be seen that the initial stress value is highest at 0°C, followed by 20°C, and lowest at 69°C; the time to reach stable stress relaxation is fastest at 0°C, followed by 69°C, and slowest at 20°C; and the degree of fit between the calculated values and the experimental values is best at 0°C, followed by 69°C, and worst at 20°C.

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