Three-Dimensional Wake Model based on Cosine Distribution

Xi Liu[a]

School of Civil Engineering, Central South University of Forestry and Technology, Changsha 410004, China

[a]3232417259@qq.com

Abstract

As the demand for clean energy continues to escalate, the scale of wind power generation has been expanding globally. To comprehensively account for the effects of inflow turbulence and to obtain the three-dimensional distribution of mean wind downstream of wind turbines, this paper proposes a three-dimensional wake model that incorporates shear inflow conditions, based on the cosine function distribution and the Park model. The reliability of using the cosine function as a basis function is validated through wind tunnel experimental data and large eddy simulation data. Furthermore, the accuracy of the proposed model is confirmed by field measurement datas from wind farm.

Keywords

List the; Wind Turbines; Park Model; Cosine Function; Three-Dimensional Wake Model.

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1. Introduction

Compared to other energy sources, wind energy boasts advantages such as being green, clean, and abundant, which has led to its widespread application globally in recent years. For practical wind farms, the arrangement of wind turbines directly influences their power generation efficiency.

The layout of wind turbines is primarily governed by the wake interference effects of the turbines. Currently, the main methods for studying the wake interference effects of wind turbines include wind tunnel experiments, computational fluid dynamics (CFD) simulations, and wake mathematical models [1]. When optimizing the layout, the need to continuously refine the spatial positions of wind turbines results in a multitude of scenarios that need to be analyzed, making wind tunnel experiments and CFD simulations too costly. Therefore, the prevailing practice in layout optimization is to employ simplified wake mathematical models.

As one of the earliest wake models invented, the Park model [2] (also known as the Jensen model) has been validated for its effectiveness and has been widely applied. The Park model is primarily based on the principle of mass conservation and assumes that the downstream wind speed distribution is solely dependent on the axial distance from the rotor. This top-hat form of wind speed deficit has been adopted in many subsequent wake models developed [3, 4]. However, the wind speed distribution downstream of a wind turbine is evidently a complex nonlinear function, and the top-hat form of wind speed deficit has significant shortcomings. In the subsequent development of wake models, some scholars have corrected the wind speed distribution in the wake models to Gaussian distributions [5], polynomial distributions [6], and other functions, proving that these "unimodal" functions are more in line with the actual wind speed distribution downstream of wind turbines. With the continuous expansion of wind turbine sizes, the influence of the incoming wind shear on the downstream wind distribution has gradually increased, and there is a clear correlation between the wind speed at the wind turbine inflow plane and the spatial point position.

To comprehensively account for the effects of shear inflow and to obtain the three-dimensional distribution of average wind downstream of wind turbines, this paper proposes a three-dimensional wake model that considers shear inflow conditions, based on the cosine function distribution and the Park model. The second section of this paper mainly introduces the derivation process of the model, the third section is dedicated to the validation of the model, and the fourth section presents the conclusions.

2. Three-Dimensional Wake Model based on Cosine Distribution

2.1 Park Model

The Park model, one of the earliest wake models to be implemented in engineering practice and still a core wake model in the widely-used wind resource analysis software Wasp, was proposed by Jensen based on the principle of mass conservation [2]. The expression for this model is given by:

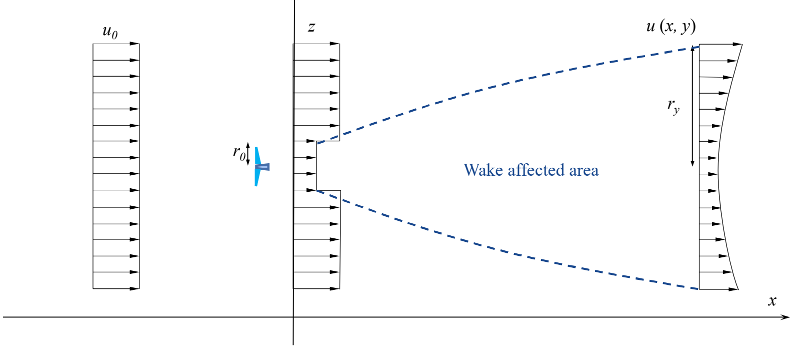
$$u_P = u_0(1-2a/(1+kx/r_0)^2) \tag{1}$$

In the equation, u_P represents the wind speed at the coordinate (x, z) calculated by the Jensen model; u_0 is the wind speed at the inlet of the wind turbine; k is the rate of increase of the wake radius, which can be calculated by $k = 0.5/\ln(z/z_0)$ or determined directly using empirical parameters; r0 is the initial radius of the wind turbine wake; a is the axial induction factor of the wind turbine; x is the relative coordinate of the calculation position.

2.2 Three-Dimensional Wake Model based on Cosine Distribution

In this paper, a three-dimensional wake model for wind turbines is established based on the cosine function distribution assumption, utilizing the Park model. The transverse and vertical coordinate systems are illustrated in Figure 1.

(a) Horizontal coordinate



(b) Vertical coordinate

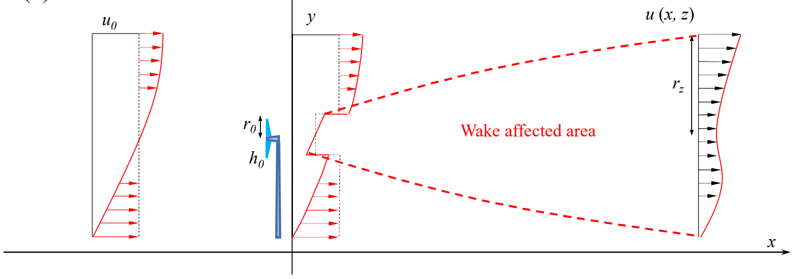


Figure 1. The coordinate system

Firstly, it is assumed that the wake wind speed deficit in the two-dimensional plane where the wind turbine rotor is located conforms to a two-dimensional cosine distribution pattern.

$$u_2 = u_0[1-(A\cos Cr + B)] \tag{2}$$

In the equation, A, B, C are parameters to be determined, and r is the distance from the center of the wake. Based on the periodicity and continuity assumptions of the cosine function, we have:

$$2\pi / C = 2r_w \tag{3}$$

$$u_0[1-(A\cos Cr_w / 2 + B)] = u_0 \tag{4}$$

Substituting equation (3) into equation (4), we obtain:

$$C = \frac{\pi}{r_w} \tag{5}$$

$$B = A^{-1} \tag{6}$$

Based on the linear expansion assumption, the wake influence radius rw downstream of the wind turbine is a function related to x, which can be expressed as:

$$r_w(x) = r_0 + k_w x \tag{7}$$

In the equation, kw is the wake radius expansion rate. Relevant studies have indicated that when using the Park model to solve offshore wind turbine wake problems, the wake expansion rate k can be 0.075, and when targeting onshore wind turbines, k can be 0.04 or 0.05. This paper, based on the modified formula proposed by Sun [7] that is based on changes in turbulence, obtains the expansion rate:

$$k_w = k\frac{I_w}{I_0} \tag{8}$$

$$I_w = \sqrt{I_0^2 + I_+^2} \tag{9}$$

$$I_+ = 0.73a^{0.8325} I_0^{0.0325} (x / D)^{-0.32} \tag{10}$$

Substituting equations (5) and (6) into the two-dimensional cosine distribution form of equation (2):

$$u_2 = u_0[1 - A(\cos\frac{\pi}{r_w}r + 1)] \tag{11}$$

Assuming that this wake model has the same mass flux as the Park model:

$$\int_{-r_w}^{r_w} u_0[1-A(\cos\frac{\pi}{r_w}r+1)]dr = r_w u_P \tag{12}$$

Solving for A yields the following expression:

$$A = 2a/(1+kx/r_0)^2 \tag{13}$$

In Jensen's original conception, the initial wake radius of the Park model was directly equal to the wind turbine rotor radius. However, in the actual development of wind turbine wakes, due to the presence of tip vortices and other vortex phenomena, the real initial wake influence radius is greater than the wind turbine rotor radius. A correction formula based on actuator disk theory for the initial wake radius is used:

$$r_0 = r_d\sqrt{{}^{1-a}\!/_{1-2a}} \tag{14}$$

In the equation, rd represents the radius of the wind turbine rotor.

At this point, the two-dimensional cosine wake model based on the Jensen model can be expressed as:

$$u_2 = u_0[1-(2a/(1+kx/r_0)^2)(\cos\frac{\pi}{r_w}r+1)] \tag{15}$$

Assuming that the incoming wind profile follows an exponential distribution form:

$$U(z) = u_0(\frac{z}{z_h})^a \tag{16}$$

The change in mass due to the incoming wind shear can be expressed as:

$$\vartriangle m = \iint_{S_{r_w}-S_{r_0}} \vartriangle u dA + \iint_{S_{r_0}} \vartriangle u(1-2a)dA \tag{17}$$

$$\vartriangle u = U(z) - u_0 \tag{18}$$

According to the principle of mass conservation, we can derive the following equation:

$$\iint_{S_{r_w}} u_3 dA = \iint_{S_{r_w}} u_2 dA + \vartriangle m \tag{19}$$

By combining equations (15) through (19), we can solve for the three-dimensional wake model based on the cosine distribution proposed in this paper.

$$u_3(x,y,z)=u_0\left[\left(\frac{z}{z_h}\right)^a-\left(\frac{2a}{(1+kx/r_0)^2}\right)\left(\cos\frac{\pi}{r_w}r+1\right)\right]-\frac{2a\iint_{S_{r_0}}\triangle u dA}{\pi r_w{}^2} \tag{20}$$

3. Verification

To validate the accuracy and superiority of the three-dimensional wake model based on the cosine distribution proposed in this paper, we will compare it with published wind tunnel experimental wake measurement data and large eddy simulation (LES) data. This section is divided into three parts: dimensionless cosine distribution comparison, transverse section comparison, and vertical section comparison.

3.1 Comparison of Dimensionless Cosine Distribution

To validate the accuracy of the wind turbine wake model based on the cosine distribution, the average wind speed distributions obtained from the tests in references [8] and [9] are extracted, focusing on the wind speed deficit portion. Using the incoming wind speed and the wind turbine rotor radius, the wind speed deficit is then normalized. The basic parameters corresponding to the extracted data are presented in Table 1.

Table 1. Validation Test Scenario Basic Parameters

Case	D(m)	z_h(m)	U_0(m/s)	C	z_0(m)	I_0
1	0.15	0.125	2.2	0.42	0.00003	0.070
2	80	70	9	0.8	0.5	0.134
3	80	70	9	0.8	0.05	0.094
4	80	70	9	0.8	0.00005	0.048

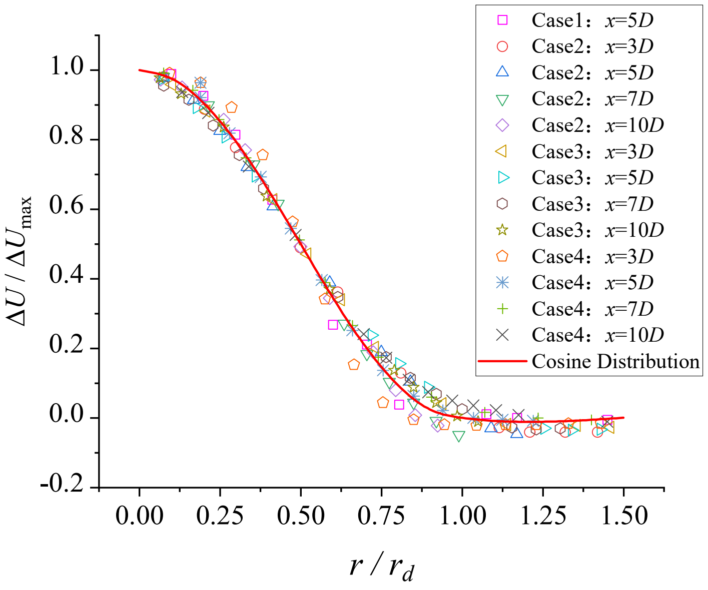


Figure 2. Comparison of Dimensionless Cosine Distribution

Figure 2 presents a comparison between the dimensionless wind speed deficit measured from wind tunnel experiments and large eddy simulations and the cosine function distribution. It is assumed that the wind speed deficit is zero when the dimensionless distance exceeds 1, implying that outside the influence radius of the wind turbine wake, the wind speed is consistent with the incoming wind speed. From the figure, it can be observed that the cosine function closely matches the distribution of dimensionless wind speed deficit across various scenarios and cross-sections, being entirely enveloped by the measured data with a very small enveloping interval. This indicates that when other parameters are appropriately controlled, the cosine function distribution has the potential to accurately describe the wind speed deficit downstream of the wind turbine. The wake model proposed in this paper, which employs the cosine function as the basis function for wind speed deficit distribution, is therefore accurate.

3.2 Transverse Cross-Section Validation

Although the consistency of dimensionless distributions is indicative, it does not necessarily ensure equivalence in downstream wind speed deficits. To further substantiate the accuracy of the proposed wake model, a comparative analysis is conducted using the model against empirical wake data from a wind farm. The transverse cross-sectional validation data employed in this study is extracted from Reference [10]. Li et al. [10] carried out a lidar wind measurement experiment in a wind farm in northwest China, providing comprehensive measurement data. The wind farm is equipped with 200 units of Goldwind GW82/1500 turbines, with a total installed capacity of 300 megawatts. The rotor blade diameter is 82 meters, and the hub height of the wind turbines is 70 meters. LiDAR measurements were utilized to ascertain the wake distribution downstream of a single wind turbine at positions x/D of 2, 3, 4, and 5. To validate the precision of the model, this study selected a dataset of high-quality data, characterized by an incoming wind speed at hub height of 5.4 m/s, a thrust coefficient of 0.79, surface roughness of 0.005 m, and a turbulence intensity at the hub of 10%.

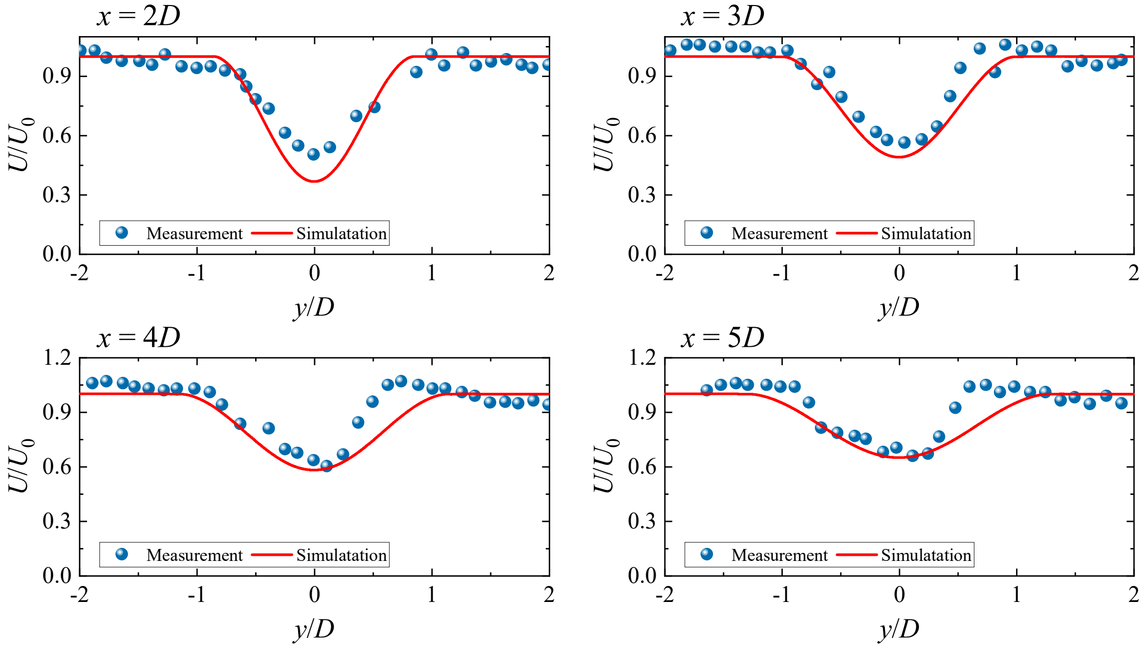


Figure 3. Transverse Cross-Section Validation

Figure 3 illustrates the comparative analysis between the proposed wake model and field-measured data from a wind farm. The results depicted in the figure indicate that the proposed model effectively captures the trend of the downstream average wind speed variation with respect to the coordinate yy, and the simulated wind speed deficit extremum closely matches the actual measurements. This demonstrates the efficacy of the proposed wake model. Additionally, upon examining the field data, it is evident that there is a pronounced asymmetry in the measured values, with the right-hand side of the measured curve deviating significantly from the cosine function. This discrepancy may arise from the fact that the wind turbine in question is situated within a real wind farm, where it is inevitably subjected to interference from other wind turbines. However, the wake model does not account for such interferences, leading to substantial errors on the left side of each transverse cross-section. Consequently, the comparative results are notably less favorable than those of the dimensionless wind speed deficit comparison presented in Section 3.1.

3.3 Vertical Cross-Section Validation

Gao et al. [11] conducted a wake velocity measurement experiment using lidar technology at a wind farm in Zhangjiakou. The wind farm is equipped with 50 wind turbines, each with a capacity of 1.5MW, resulting in a total installed capacity of 75MW. The rotor radius of each wind turbine is 38.5 meters, and the hub height is 65 meters. To acquire high-precision wake distribution data, two distinct types of lidar systems were utilized in the experiment: one for measuring the incoming wind profile and the other for measuring the wake velocity distribution. The vertical wind speed distribution data obtained on January 6, 2019, was employed to verify the accuracy of the wake model. By fitting the wind profile, the wind shear parameter α was calculated to be 0.14. The incoming flow conditions were characterized by a wind speed of 9.2 m/s, an incoming flow turbulence intensity of 11%, and a thrust coefficient of 0.72. Additionally, due to the presence of significant wake disturbances downstream at $x > 5D$, this study employs the wind speed distribution at $x = 3D$ and $x = 4D$ for validation purposes.

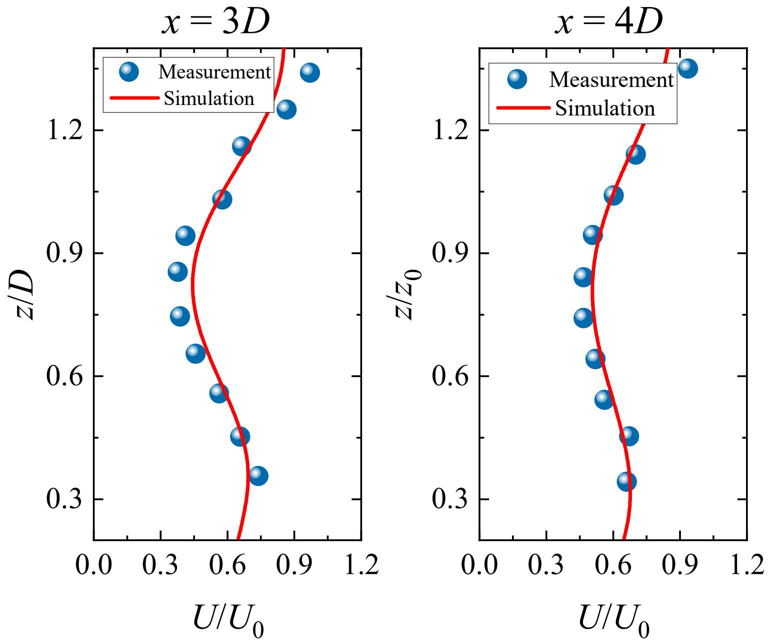


Figure 4. Vertical Cross-Section Validation

Figure 4 presents a comparison of wind speed profiles obtained from the proposed wake model and actual measurement data in the vertical direction. The wind speeds are normalized using the incoming wind speed at hub height, and the z-coordinate is normalized using the wind turbine rotor diameter. Similar to the transverse results, the proposed wake model closely matches the pattern of wind speed deficits downstream of the wind turbine, and the actual values are also very close. This further confirms the accuracy of the proposed wake model.

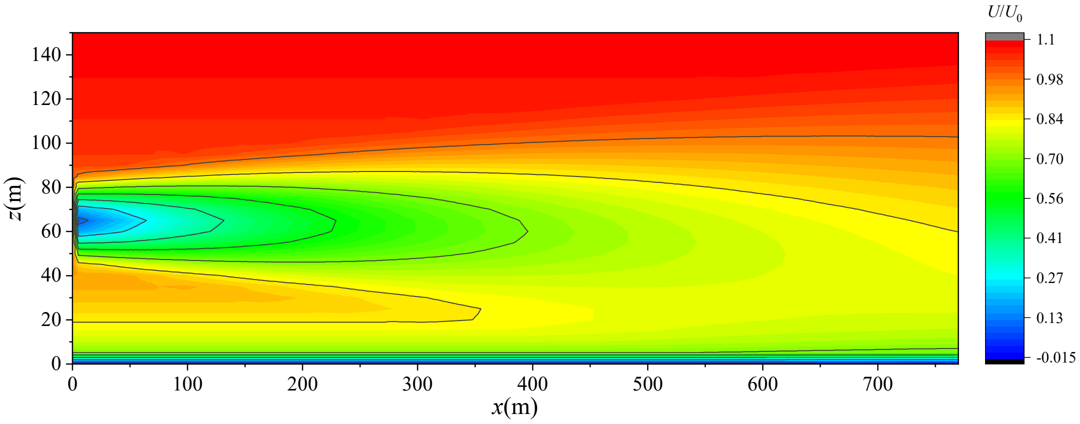


Figure 5. Velocity contours of *x*-*z* plane

Building upon the accurate validation of the wake velocity profiles at 3D and 4D downstream positions, a comprehensive vertical dimensionless velocity contour plot for the entire x-z plane has been further constructed. The results are depicted in Figure X, which illustrates the dimensionless velocity distribution over a downstream area spanning ten times the diameter (770 meters) in length and two times the diameter (154 meters) in width, essentially encompassing the primary influence range downstream of the wind turbine. From Figure 5, it is evident that the vertical wind profile downstream of the wind turbine fundamentally differs from the "top-hat" wind speed distribution assumed by one-dimensional wake models. Additionally, there is a significant discrepancy near the ground with the single-peak wind speed distribution assumed by two-dimensional wake models. This observation underscores the substantial limitations inherent in using one-dimensional or two-dimensional wake models for predicting wake speed distributions in actual wind farms. The three-dimensional wake model proposed in this paper, which incorporates the cosine distribution and accounts for wind shear, offers a more advantageous approach.

4. Conclusion

(1) When employing the cosine function as the basis function for the wake model downstream of wind turbines, it can be entirely enveloped by the wind speed deficit data extracted from actual measurement data, with a very small enveloping interval. Provided that other parameters are appropriately controlled, the cosine function distribution has the full potential to accurately describe the wind speed deficit downstream of wind turbines.

(2) The three-dimensional wind turbine wake model proposed in this paper, based on the cosine distribution, is capable of accurately depicting the transverse and vertical wake velocity distributions downstream of wind turbines. Moreover, influenced by shear inflow, the wind profile downstream of wind turbines significantly deviates from the wind speed distributions assumed by one-dimensional and two-dimensional wake models. The three-dimensional wake model proposed in this paper, which takes into account wind shear, holds a distinct advantage.

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