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The Almost Sure Exponential Stability of Discrete-time Impulsive Stochastic Cohen-Grossberg Neural Network Systems

Xiao Gao[a], Dianqiang Li[b], Yan Lu[c], and Qiaoyu Ma[d]

School of Science and Mathematics, Suqian University, Suqian 223800, China

[a]19549383236@163.com, [b]ldq2015ahdax@126.com, [c]2890146536@qq.com,

[d]1310073811@qq.com

Abstract

This paper investigates the almost sure exponential stability of discrete-time impulsive stochastic Cohen-Grossberg neural network systems (CGNNs). Firstly, for the linear scalar system, sufficient conditions for the almost sure exponential stability of the neural network system are presented. Secondly, by using the average impulsive interval method and the strong law of large numbers, sufficient conditions for the almost sure exponential stability of the general discrete-time impulsive neural network system are given. Finally, the validity of the conclusions is verified through the numerical simulations.

Keywords

Cohen-Grossberg Neural Network; Discrete-time; Impulsive System; Lyapunov Function.

1. Introduction

In recent years, with the development of multidisciplinary integration, many control engineering problems often involve some hybrid and complex systems formed by continuous and discrete dynamics. Among these numerous hybrid systems, the research on impulsive neural network systems has drawn particular attention. In 1983, Cohen and Grossberg proposed the Cohen-Grossberg neural network ([1]). The key characteristics of the Cohen-Grossberg neural network lies in its inherent stability and adaptability, which are attributed to the amplification function and the balance function within the network. The amplification function determines the gain of neurons, while the balance function ensures the self -organizing ability of the network. These features enable this neural network to perform outstandingly in areas such as pattern recognition, associative memory, and optimization problem- solving. In recent years, there has been an abundance of research findings regarding the behavioral aspects of this neural network ([2, 3]). Chaouki ([4]) introduced a generalized approach during the research and defined an interval range to study the convergence of this neural network. Hu et al. ([5]) defined a new class of periodic functions and established sufficient conditions for the existence and stability of anti-periodic solutions of fuzzy CGNNs on time scales. Li et al.([6]) investigated the exponential stability of a neural network system with a Markov switching system and time-delay terms under the mean-square of the mathematical expectation, breaking through the traditional limitation of only studying stochastic CGNNs without time - delay. Zheng ([7]) studied the robust stability of the Cohen - Grossberg neural network with piece-wise constants, using the small-gain theorem as a criterion to verify the system's robust stability. Jiang et al.([8]) studied the dynamic behavior of the fractional-order inertial CGNNs with time-varying delays and provided sufficient conditions for determining the global Mittag-Leffler stability and periodicity of the system's solutions. However, to date, there are still relatively few research achievements on the almost-sure exponential stability of the impulsive stochastic Cohen-Grossberg neural network in discrete-time.

This paper will study the almost-sure exponential stability of the impulsive stochastic Cohen-Grossberg neural network in discrete-time.

2. Preliminary Knowledge

This paper will adopt the following notations: Denote R as the set of real numbers, define $|x|$ as Euclidean norm, and the symbol T represents the transpose of a matrix or a vector in algebra. For a matrix A, $|A|=\sqrt{trace(A^T A)}$ represents the trace norm, and I represents the identity matrix. Denote $(\Omega,\{F_t\}_{t\geq 0},P)$ as a complete probability space with a σ-algebra flow $\{F_t\}_{t\geq 0}$, so that we can handle random variables and events more flexibly and ensure the validity of the conclusions of the stability proof. Denote $\omega(n)=(\omega_1(n),\omega_2(n)),\ldots,\omega_m(n))^T$ as an m-dimensional standard Brownian motion in this probability space, and each of its components strictly satisfies the properties of independent increments, stationary increments, normalization, and continuity. For $E(\zeta)=\sum_{i=1}^{n} x_i p_i$ it represents the mathematical expectation of ζ where x_i represents the value taken by the random variable ξ and p_i is the probability that ξ takes the value x_i. Based on the impulsive stochastic CGNNs in the continuous-time state, the stochastic CGNN in the discrete - time state is given as follows:

$$\begin{cases} x(n+1)=-a(x(n))[b(x(n))-Ag(x(n))]+\sigma(x(n)\omega(n), n\neq n_k, \\ x(n_k)=C_k x(n_k-1), k\in N. \end{cases} \tag{1}$$

Among the above-mentioned symbols, N is a positive integer. For $x(n)$, which is an n-dimensional state vector, and $x(0)=x_0\neq 0$, The matrix $a(x(n))$ is an n-dimensional amplification function, $b(x(n))$ is an n-dimensional neuron behavior function, $A\in R^{n\times n}$ is the connection-weight matrix, $g(x(n))$ is specially a n-dimensional neuron activation function ,and $\sigma(x(n))\in R^{n\times n}$ is specially a noise-intensity matrix function and the equation $x(n+1)=-a((x(n)))[b(x(n))-Ag(x(n))]+\sigma((x(n))\omega(n), n\neq n_k$ represents the general function of neurons in the discrete-time state. $x(n_k)=C_k x(n_k-1)$ represents the state change of the system at the impulse point $x=n_k$, where C_k represents the impulse-intensity matrix.

For each neuron function in the system equation (1), the following assumptions are given.

Assumption1: There exist positive constants $\underline{\alpha_i}$ and $\overline{\alpha_i}$ and matrices $\underline{u_i}$, $\overline{u_j}$,where $i\neq j$, and $i,j=1,2,\ldots n$, such that

$$\underline{u_i}\leq \frac{g(x(n_i))-g(x(n_j))}{x(n_i)-x(n_j)}\leq \overline{u_j},$$

$$\frac{b(x(n_i))-b(x(n_j))}{x(n_i)-x(n_j)}\geq \beta_{ij},$$

$$0\leq \underline{\alpha_i}\leq a(x(n_i))\leq \overline{\alpha_i}.$$

$$\text{where } \underline{\alpha_i}=\max_{1\leq l\leq n}\{\underline{\alpha_i}\}, \overline{\alpha_i}=\max_{1\leq l\leq n}\{\overline{\alpha_i}\}.$$

Definition 1([10]). For any impulse sequence $\zeta=(n_k:k\in N)$, if it satisfies

$$\frac{n}{T_a} - N_0 \le N(n,0) \le \frac{n}{T_a} + N_0 . \tag{2}$$

Then $N_0 \ge 0$ and $T_a \ge 0$ are respectively called the elasticity number and the average impulse time - interval, where $N(n,0)$ represents the number of impulses on the interval $(0,n]$.

Definition 2 ([11]).(Strong Law of Large Numbers): For a continuous real - valued local martingale $M = \{M_t\}_{t\ge 0}$ with a value of 0 at $t=0$,we have:

$$\begin{cases} \lim\limits_{t\to\infty} < M,M >_t = \infty \Rightarrow \lim\limits_{t\to\infty} \dfrac{M_t}{< M,M >_t} = 0, \\ \limsup\limits_{t\to\infty} < M,M >_t = \infty \Rightarrow \lim\limits_{t\to\infty} \dfrac{M_t}{t} = 0. \end{cases}$$

where $N_0 \ge 0, T_a \ge 0$ respectively represent the elasticity number and the average time - interval, which are used as measures of the system's uncertainty.

Definition 3([12]). For a simple linear scalar stochastic system:

$$\begin{cases} x(n+1) = ax(n) + bx(n)\omega(n), n \ne 2k-1, n \in N, \\ x(2k) = \rho x(2k-1), k \in N_+. \end{cases}$$

If the initial value $x(0) = x_0 \ne 0, a \ne 0, |\rho| > 0$ and $\omega(n)$ is a standard Brownian motion, then for any real number n, the following inequality holds, and the almost-sure exponential stability of the system is satisfied:

$$\ln|\rho| + E\ln|a + b\omega(n)| < -\varepsilon, \varepsilon \ge 0 . \tag{3}$$

For the above - given inequalities, the Monte Carlo simulation method is adopted, and the boundary curves of the inequalities are given to further illustrate the validity of the conclusions, as shown in Figure 1. Boundary curve and Figure 2:

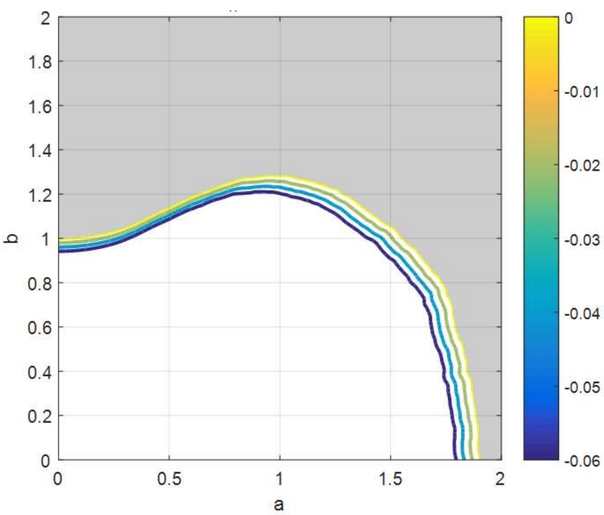

Figure 1. Boundary curve

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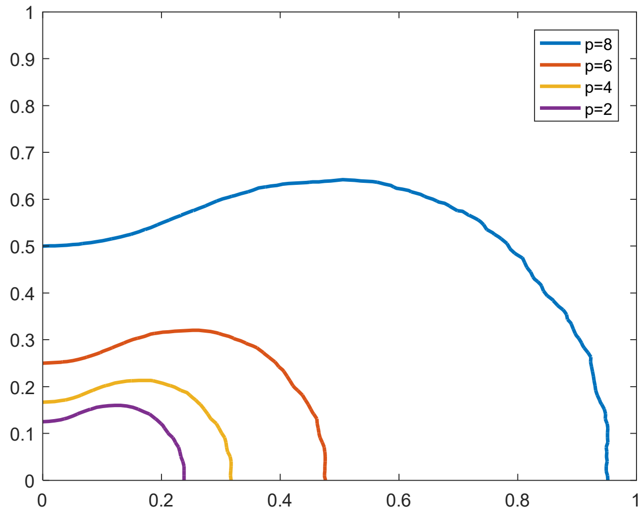

Figure 2. Boundary diagram of p value change

Proof: For any time instant $n \in I[2(k-1),2k), k \neq 0$, we have:

$$|x(n+1)|^2 = (a+b\omega_n)^2 |x(n)|^2 .$$

When $n = 2k$, the following equation can be obtained:

$$|x(2k)|^2 = \rho^2 |x(2k-1)|^2 .$$

Through further direct calculation, it can be shown that for $n \in [0,\infty)$, the following equation holds:

$$\ln|x(n)| = \frac{1}{2}(\ln|x_0| + \sum_{0<2k<n} \ln\rho^2 + \sum_{j=0}^{n} \ln(a+b\omega_j)^2) .$$

At this time, according to the strong law of large numbers in Definition 3, the explicit solution of this linear scalar system is:

$$\ln|x(n)| = \ln|\rho| + E\{\ln|a+b\omega_n|\} = \ln|\rho| + \ln|a| + \gamma(c) \leq 0 .$$

where $\gamma(c) = E[\ln|1+c\omega_n|], c = \frac{1}{2}$, and the graph of the function $\gamma(c)$ is shown in Figure 3 below.

Then this linear scalar system is said to be almost surely exponentially stable.
Generalized to the general case, it is:

$$\ln|\rho| + E\{\ln|a+b\omega_n|\} < -\varepsilon, \varepsilon > 0 .$$

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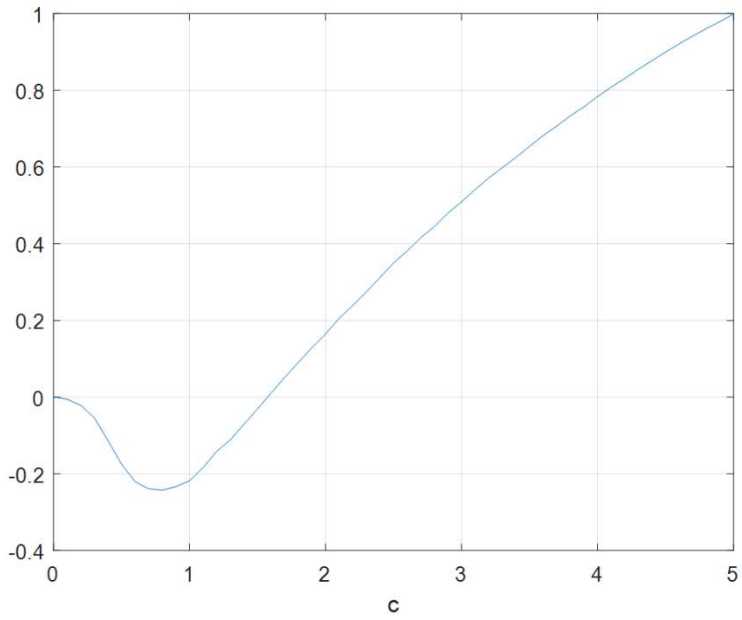

Figure 3. The graph of the function C

Definition4([13]). For the stochastic CGNNs in the discrete-time state, if for any $n \in I[2(k-1),2k), k \neq 0$, the following is satisfied:

$$\lim_{n\to\infty} \sup \frac{1}{n} \log|x(n)| < 0 . \tag{4}$$

then the discrete neural network is said to have almost - sure exponential stability.

For the convenience of calculation below, we denote

$$f(x(n)) = -a(x(n))[b(x(n)) - Ag(x(n))] .$$

3. Main Results

By using the average - impulse method and constructing a suitable Lyapunov function, the following main conclusions can be obtained.

Theorem 1: If Assumption 1 holds, and there exist real numbers $c_1, c_2, c_3, \theta, \varepsilon \in R, c_1 < c_3$ and $Q \in R^{n\times n}$ such that the following conditions are satisfied:

(i) For any time - instant $n \in (n_k - 1, n_k), k \in N$, all satisfy:

$$f^T(x(n))Qf(x(n)) \le c_1 x^T(n)Qx(n) .$$

(ii) For any time - instant $n \in (n_k - 1, n_k), k \in N$, all satisfy:

$$c_2 x^T(n)Qx(n) \le \sigma^T(x(n))Q\sigma(x(n)) \le c_3 x^T(n)Qx(n) .$$

(iii) For any time - instant $n \in (n_k - 1, n_k), k \in N$, all satisfy:

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$$C_k^T x(n_k-1)QC_k x(n_k-1) \le \rho x^T(n_k-1)Qx(n_k-1).$$

(iv) For any time - instant $n \in (n_k-1, n_k), k \in N$, and when $\lambda \ge 2(c_1+c_3) \ge 0$ all satisfy:

$$f^T(x(n))Q\sigma(x(n))y + y^T\sigma^T(x(n))Qf(x(n)) \ge f^T(x(n))Qf(x(n)) + y^T\sigma^T(x(n))Q\sigma(x(n)) - \lambda x^T Qx.$$

Then

$$\limsup_{n\to\infty}\frac{1}{n}\log|x(n)| \le \frac{1}{2}((2c_1)^\theta + \Delta(2c_1,c_2,\theta) - 2\varepsilon) \le 0,$$

Where

$$\begin{cases} 0<\theta<1, \\ \varepsilon > \dfrac{1}{2}[(2c_1)^\theta + \Delta(2c_1,c_2,\theta)], \\ \Delta(2c_1,c_2,\theta) = \displaystyle\int_0^{(2c_1)^{1-\theta}/c_2} [\frac{u}{c_2} - 1 - \log u + \log c_2]\overline{M}(u,1)du. \end{cases}$$

That is, the discrete - time impulsive neural network system is almost surely exponentially stable.

Proof: Select the Lyapunov function $V(x(n+1)) = x^T(n+1)Qx(n+1)$,where Q is a positive - definite diagonal matrix. From the system equation (1) and the Lyapunov function, we can obtain:

$$\begin{aligned} V(x(n+1)) &= [f(x(n)) + \sigma(x(n))\omega(n)]^T Q[f(x(n)) + \sigma(x(n))\omega(n)] \\ &= f^T(x(n))Qf(x(n)) + \omega^T(n)\sigma^T(x(n))Q\sigma(x(n))\omega(n) + 2f^T(x(n))Q\sigma(x(n))\omega(n). \end{aligned}$$

Define

$$\varphi(n) = \frac{V(x(n+1))}{V(x(n))} = \frac{f^T(x(n))Qf(x(n)) + \omega^T(n)\sigma^T(x(n))Q\sigma(x(n))\omega(n) + 2f^T(x(n))Q\sigma(x(n))\omega(n)}{x^T(n)Qx(n)},$$

from the above-mentioned formula, we can get

$$\varphi(n) = q(n) + p(n)|\omega(n)|^2 + \frac{2f(x(n))Q\sigma(x(n))\omega(x(n))}{x^T(n)Qx(n)}, \tag{5}$$

where $q(n) = \dfrac{f^T(x(n))Q\sigma(x(n))}{x^T(n)Qx(n)} \le c_1$, $c_2 I \le p(n) = \dfrac{\sigma^T(x(n))Q\sigma(x(n))}{x^T(n)Qx(n)} \le c_3 I.$

For $2f(x(n))Q\sigma(x(n))\omega(x(n))$, select a pair of appropriate stochastic sequences $\{\overline{q(n)}: -1 \le \overline{q(n)} \le 1, n\in N\}$ and $\{\overline{p(n)}: -1 \le \overline{p(n)} \le 1, n \in N\}$ such that

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$2f(x(n))Q\sigma(x(n))\omega(x(n)) = \overline{q(n)} f^T(x(n))Qf(x(n)) + \overline{p(n)}\omega^T(n)\sigma^T(x(n))Q\sigma(x(n))\omega(n)$ From the definition of $\phi(n)$, we can get

$$\varphi(n) = q(n)(1+\overline{q(n)}) + \omega^T(n)p(n)(1+\overline{p(n)})\omega(n). \tag{6}$$

From (6) and (iv), we can get

$$\varphi(n) = \varepsilon_n + \phi_n^2, \tag{7}$$

Where

$$0 \le \underline{q(n)} \le q(n), \varepsilon_n = q(n)(1+\overline{q(n)}) - \underline{q(n)} - 2\sqrt{\underline{q(n)}[\omega^T(n)p(n)(1+\overline{p(n)})\omega(n)]} \le 2c_1,$$
$$\phi_n = \sqrt{\underline{q(n)}} + \sqrt{\omega^T(n)p(n)(1+\overline{p(n)})\omega(n)}.$$

Taking the logarithm of $\phi(n)$ gives

$$\log V(x(n+1)) = \log V(x(n)) + \log\varphi(n). \tag{8}$$

From equation (8) and mathematical induction, it can be known that for the impulse point j in the system (1) within the interval $[n_k, n+1]$

$$\log V(x(n+1)) = \log V(x(n_k)) + \sum_{j=n_k}^{n}\log\varphi_j. \tag{9}$$

From the definition of the Lyapunov function, it is known that

$$V(x(n_k)) = C_k^T(x(n_k-1))QC_k(x(n_k-1)).$$

Combining (iii) and the above-mentioned formula, we can obtain

$$V(x(n_k)) \le \rho x^T(n_k-1)Px(n_k-1) = \rho V(x(n_k-1)). \tag{10}$$

From equation (2), we can get

$$\rho^{N(n,0)} \le \begin{cases} \rho^{\frac{n}{T_a}+N_0} = \rho^{N_0} e^{\frac{\ln\rho}{T_a}} n, \rho \ge 1, \\ \rho^{\frac{n}{T_a}-N_0} = (\frac{1}{\rho})^{N_0} e^{\frac{\ln\rho}{T_a}} n, 0 < \rho < 1. \end{cases} \tag{11}$$

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From equations (9) , (10) and mathematical induction, we can obtain

$$\log V(x(n)) \leq \log V(x_0) + \log \rho^{N(n,0)} + \sum_{j=0}^{n} \log \varphi_j . \tag{12}$$

Substituting equation (11) into equation (12), we can get

$$\log V(x(n)) \leq \log V(x_0) + \log q + \frac{\ln|\rho|}{T_a} n + \sum_{j=0}^{n} \log \varphi_j , \tag{13}$$

where $q = \min\{\rho^{N_0}, (\frac{1}{\rho})^{N_0}\}$.

Combining formula (7) and the inequality $\ln(1+a) \leq a, a \geq 0$,, we can obtain

$$\log(\varepsilon_n + \phi_n^2) \leq \log(2c_1 + \phi_n^2) \leq \begin{cases} (2c_1)^\theta + \log \phi_n^2, \phi_n^2 \geq \dfrac{2c_1}{e^{(2c_1)^\theta} - 1}, \\ 2c_1 + \phi_n^2 - 1, \phi_n^2 < \dfrac{2c_1}{e^{(2c_1)^\theta} - 1}, \end{cases} \tag{14}$$

where $\theta \in (0,1)$ is a constant.

From the above - mentioned formula, we can get

$$\mathrm{E}\{\log \varphi_n\} \leq (2c_1)^\theta + \mathrm{E}\{\log \phi_n^2\} + \delta(2c_1, \overline{q(n)}, \overline{p(n)}, p(n), \theta), \tag{15}$$

where $\delta(2c_1, \overline{q(n)}, \overline{p(n)}, p(n), \theta) = \int_0^{2c_1/e^{[(2c_1)^\theta - 1]}} (u - 1 - \log u) \overline{M}_{\varphi_n^2}(u) du$, $\overline{M}_{\varphi_n^2}$ is the probability-density function of φ_n^2 , and $\phi_n \sim N(\sqrt{\underline{q(n)}}, |p(n)|(1+\overline{p(n)})|)$.

From condition (iii) and condition (iv), we can get

$$\begin{cases} 0 \leq \underline{q(n)} \leq q(n) \leq c_1 \\ -1 \leq \overline{p(n)} \leq 1 \\ c_2 I \leq p(n) \leq c_3 I \\ \lambda \geq 2(c_1 + c_3) \geq 0 \end{cases}$$

Since the function $h(u) = u - 1 - \log u$ is monotonically decreasing on $(0,1]$, when $\varphi_n = \sqrt{\omega^T(n) c_2 \omega(n)} \sim N(0, c_2)$, for $\delta(2c_1, \overline{q(n)}, \overline{p(n)}, p(n), \theta)$ can reach its maximum value here.Therefore, (φ_n / c_2) is a chi - square random variable with 1 degree of freedom, that is

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$$\delta(2c_1,\overline{q(n)},\overline{p(n)},p(n),\theta)\leq\Delta(2c_1,c_2,\theta)$$

Since $\{\log\phi_n\}$ is an L - mixing process and $\{\log\phi_n\}$ is integrable, by applying Definition3 (the strong law of large numbers) and combining formula (13) and formula (15), we can obtain

$$\limsup_{n\to\infty}\frac{1}{n}\log V(x(n))\leq(2c_1)^{\theta}+\mathrm{E}\{\log\varphi_n^2\}+\Delta(2c_1,c_2,\theta)+\frac{\ln|\rho|}{T_\alpha}. \tag{16}$$

Next, introduce a number $\varepsilon>0$ such that

$$\begin{cases}\varepsilon>\dfrac{1}{2}[(2c_1)^{\theta}+\Delta(2c_1,c_2,\theta)]\\ c_1<\min\limits_{|p(n)|\in[c_2,c_3]}H_\varepsilon(|p(n)|)\end{cases}.$$

At this time, according to formula (3) in Definition 3, the following formula holds:

$$\frac{\ln|\rho|}{T_\alpha}+\mathrm{E}\{\log\varphi_n^2\}=\frac{\ln|\rho|}{T_\alpha}+2E\{\log|\sqrt{\underline{q(n)}}+\sqrt{\omega^T(n)p(n)(1+\overline{p(n)})\omega(n)}|\}<-2\varepsilon,\varepsilon>0.$$

From formula (16), we can get

$$\limsup_{n\to\infty}\frac{1}{n}\log|x(n)|=\limsup_{n\to\infty}\frac{1}{2n}\log V(x(n))\leq\frac{1}{2}((2c_1)^{\theta}+\Delta(2c_1,c_2,\theta)-2\varepsilon)<0. \tag{17}$$

That is, this neural network is almost surely exponentially stable.

4. Numerical Simulation

Consider the following discrete - time impulsive stochastic Cohen - Grossberg neural network

$$\begin{cases}x(n+1)=-a(x(n))[b(x(n))-Ag(x(n))]+\sigma(x(n)\omega(n),n\neq n_k\\ x(n_k)=C_kx(n_k-1),k\in N\end{cases},$$

where

$$a(x(n))=\begin{bmatrix}0.95e^{-0.1\times(x_1^2+x_2^2)[\mathrm{sgn}(x_1)x_1+\mathrm{sgn}(x_2)x_2]} & 0\\ 0 & 0.95e^{-0.1\times(x_1^2+x_2^2)[\mathrm{sgn}(x_1)x_1+\mathrm{sgn}(x_2)x_2]}\end{bmatrix}$$

$$g(x(n))=\begin{bmatrix}x_1e^{-0.65\times(x_1^2+x_2^2)} & 0\\ 0 & x_1e^{-0.65\times(x_1^2+x_2^2)}\end{bmatrix}\ b(x(n))=\begin{bmatrix}0.95x_1(1-0.65\times(x_1^2+x_2^2))\\ 0.95x_2(1-0.65\times(x_1^2+x_2^2))\end{bmatrix}$$

$$\sigma(x(n))=\begin{bmatrix}0.0001 & 0\\ 0 & 0.0001\end{bmatrix}.$$

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From (i) - (iv) in Theorem 1, we can take:

$$c_1 = 0.8096, c_2 = 10^{-10}, c_3 = 100, \theta = \frac{1}{2}, \varepsilon = 0.001, C_k = 1.2I$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} -2.5 & 0 \\ 0 & 2.5 \end{bmatrix}$$

At this time, combined with Matlab, numerical simulation is carried out on this system. For the impulse interval T_α, the average - time impulse interval is used. The change of the Lyapunov function of this system over time in the discrete state is represented by scaling the increment of Brownian motion.

At this time, according to Theorem 1, this neural network is almost surely exponentially stable. Select the initial value $x_0 = [0.1, 0.1]^T$. The single - sample state trajectory as time changes is shown above, see Figure 4 and Figure 5.

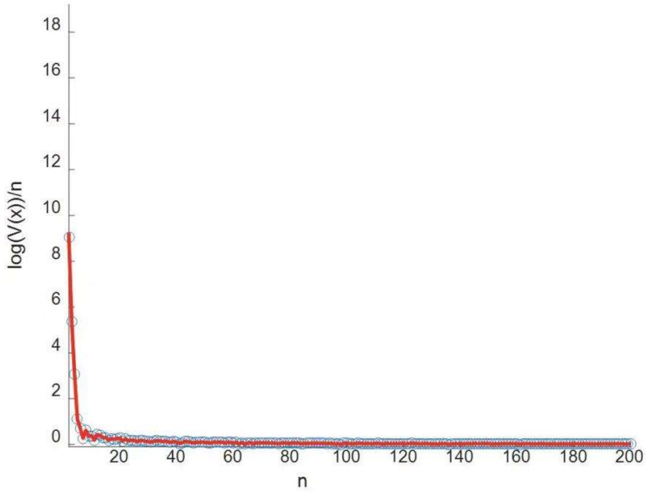

Figure 4. The graph of the value of the limit - superior of log(V(x))/n changing over time

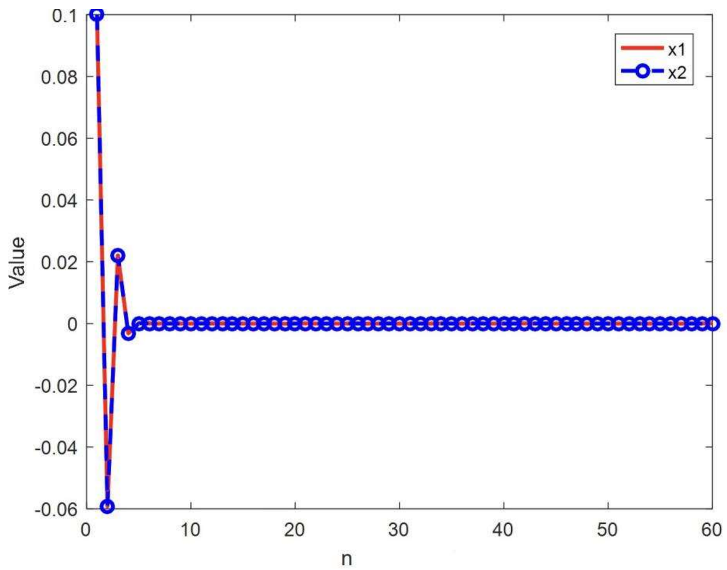

Figure 5. The sample trajectory point plot of x1 and x2

5. Conclusion

This paper studies the almost sure exponential stability of discrete - time impulsive stochastic Cohen - Grossberg neural networks. By choosing appropriate Lyapunov functions and average impulsive methods, sufficient conditions for the almost - sure exponential stability of discrete - time impulsive stochastic Cohen - Grossberg neural networks are given, and the effectiveness of the conclusions is verified by numerical simulation examples.

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