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Empirical Analysis on Different Portfolio Management Strategies

Xiaoxi Li*

School of Economics, Guangzhou College of Commerce, Guangzhou 511363, China

*20180132@gcc.edu.cn

Abstract

This thesis examines portfolio management strategies, comparing optimization models with the equal-weight (1/N) strategy. A literature review highlights 1/N's robustness due to simplicity and reduced estimation errors, often outperforming complex models like Black-Litterman. The empirical analysis replicates the Black-Litterman 1/N model using 25-FF data, evaluating performance metrics such as Sharpe ratio and Omega measure. While Black-Litterman achieves lower volatility, 1/N outperforms in net returns due to lower transaction costs. The study underscores the importance of mitigating estimation risks and incorporating diverse asset classes to enhance advanced strategies, while highlighting 1/N's practical appeal in real-world applications.

Keywords

Portoflio Management; Black-Litterman; Equal-weight Strategy.

1. Introduction

The thesis is divided into three sections, the first section is the literature review of Demiguel(et.al,2007)[1], Bessler(et.al, 2014)[2] and Kirby and Ostdiek (2012)'[4]s research, demonstrating comparison between different optimizing portfolios and equal weight portfolios. I also discuss the comparison between different papers in the main findings section. The second section is the empirical analysis of Black-Litterman-1/N model in Bessler et.al (2014) [2]'s paper. The third section concludes the whole essay combining my empirical analysis and the literature review.

2. Literature Review

2.1. First Portfolio Strategy

Table 1. 14 Models

Numble	Scheme 1
1	MSCI World index(developed country)
2	MSCI Emerging markets stock
3	US government bond
4	Merill Lynch US High Yield 100 Bond index
5	S&P GSCI light Energy Index
6	Value-weighted market portfolio
7	Mackinlay and Pastor's missing-factor model
8	Sample-baesd mean-variance with short-sale constraints
9	Bayes-Stein with short-sale constrains
10	Minimum-variance with short-sale constraints
11	Minimum-variance with generalized constraints
12	Kan and Zhou's(2007) "three-fund" model
13	Mixture of minimum-variance and 1/N
14	Garlappi, Uppal, and Wang's (2007) multi-prior model

(Demigurl et. al, 2007,p.1917)[1].

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Demiguel (et.al,2007)[1] compares 14 models to the equal weight portfolio strategy. The 14 models are listed in the table.

2.1.1. Datasets

Seven excess return (over 90 US T-bill) empirical datasets are used. The "S&P Sectors" index includes 10 industry portfolios in US of consumer goods, manufacturing goods and so on (Demiguel et.al, 2007)[1]. International equity Index mainly includes the monthly returns of Canada, France, Germany, Italy, Japan, Switzerland, UK and US. (Demiguel et.al, 2007)[1].

Table 2. Index

Number	Data	Time period
1	S&P and US equity market portfolio	01/1981-12/2002
2	10 industry portfolio	07/1963-11/2004
3	Eight country indexes and the world index	01/1970-07/2001
4	SMB and HML portfolios and the US equity market portfolio	07/1963-11/2004
5	Twenty- size-and book-to-market portfolios and the MKT,SMB and HML portfolios	07/1963-11/2014
6	Twenty size-and book-tomarket portfolios and the MKT, SMB	07/1963-11/2014
7	Twenty size- and book-to-market portfolios and the Mkt, SMB, HML and UMD portfolios	07/1963-11/2014
8	Simulated data	2000 years

(Demiguel, 2007, p.1968-p.1949)[1]

2.1.2. Evaluation Method

For each dataset we calculate returns for different optimized portfolios. The paper uses a rolling window approach; and the window size is M=120 months. For period 0-t month, we calculate the input parameter sample return and covariance matrix of the sample portfolios, and use these two parameters to decide the weights of the t+1 (out-of-sample) portfolios, and therefore the t+1 expected returns for different optimized portfolio are calculated accordingly (Demiguel, et.al, 2007) [1]. By rolling the window each time by one month, we drop the t=0 returns of the portfolios and add the t+1 returns. We do it recursively until the end of the out-of-sample period (Demiguel et. al, 2007, pp 1928) [1]. After getting T-M optimized portfolio returns, we can then calculate the expected return and variance. Sharpe ratio and other performance measures can be further calculated.

The performance measures used in this journal are in-sample Sharpe ratio(μp/σp), out-of-sample Sharpe ratio, certainty-equivalent return and average sum of the absolute value of the trades for N assets (Demiguel et.al, 2007, p.1928) [1].

2.1.3. Main Findings

The difference in in-sample and out-of-sample Sharpe ratio explains that the estimation error is significant. As out-of-sample Sharpe ratio always underperforms in-sample Sharpe ratio (Demiguel et.al,2007) [1].

1/N portfolio strategy dominates all the other portfolio strategies in terms of Sharpe ratio ,CEQ and trading volumes(Demiguel et.al,2007) [1].

Value–weighted market portfolio performs better than 1/N strategy in trading volumes by passive managing when no trading is conducted (Demiguel et.al, 2007, p1956) [1].

The empirical result is quite influenced by the estimation window size, because it will take 6000 months as the estimation window size for minimum variance portfolio strategy to outperform 1/N strategy (Demiguel et.al,2007) [1].

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2.2. Second Portfolio Strategy

Bessler et. al (2014) [2] compares Black-Litterman (posterior return), the mean-variance , Bayes-Stein(shrinkage return) and minimum variance model to 1/N portfolio strategy (both the strategic weight and equal weight).

2.2.1. Datasets

The article includes different sets of assets class index internationally. The data period is from January 1988 to December 2011(Bessler et. al, 2014) [2]. The data categories are shown in the table:

Table 3. Data Categories

Numble	Scheme 1
1	MSCI World index(developed country)
2	MSCI Emerging markets stock
3	US government bond
4	Merill Lynch US High Yield 100 Bond index
5	S&P GSCI light Energy Index

In further analysis with regards to the business cycle, the period is divided to three sub-periods, the break point being normally the year ofeconomic crisis like Asia crisis or the new economy emerging year (Bessler et.al, 2014, p.16) [2].

2.2.2. Evaluation Method

Similarly to Dimeguel et.al's(2007) evaluation, the rolling window method is applied in Besseler et.al's(2016). Window size for estimated returns is 36 months while for the estimated covariance matrix is 12 months. By using the sample estimated returns and covariance matrix as inputs, then we can get the weights and calculate the returns for different optimized portfolios.

Different from Demiguel et.al(2007) [1], all five datasets are combined to calculate the performances of the optimizing portfolios, while Demiguel et.al(2007) [1] calculate each dataset's performance for different optimizing portfolios.

As for strategic naïve approach, the weights are assigned based on investors' attitude towards risks. Investors are divided into conservative, moderate and aggressive type with different maximum expected volatility (5%,10% and 15%) (Bessler et.al,2014) [2].

Performance methods used in Bessler et.al's(2014) [2] paper are Net Sharpe Ratios, Maximum drawdown and portfolio turnover.

With different constraints in our model, Bessler et.al(2014) also test the sensitivity of performance measures to the different assigned weights, volatility, variation of input parameter tau and different estimation windows size.

2.2.3. Main Findings

According to Besseler et. al(2014), the BL portfolios dominates all the other portfolio strategies on an ongoing basis with regards to the net Sharpe ratio and the omega measures (which is better). Furthermore, BL models have lower volatility based on better confidence in forecasting returns, which remedies for the estimation error in all the other portfolios strategies.

The robustness analysis shows that the performance of different optimizing portifolio strategy is not sensitive to risk aversion parameter, short-selling constraints, and volatility (Besseler et.al, 2014).

In the case of stock-only market, BL models are not able to outperform the 1/N model because of the effect of the transaction costs and estimation error (Bessler et.al,2014) [2]. Moreover, as

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other assets, especially commodities, provide the hedging advantage, the allocation effect is weakened (Bessler et.al,2014) [2].

2.3. Third Portfolio Strategy

Kirby and Ostdiek (2012)[4] illustrates the reason why MV cannot outperform 1/N strategy as a result of high assumed targeted expected return in Demiguel et,al's(2014) [1]'s case.

The paper mainly compares the volatility timing and reward-to-risk portfolio model to simple naïve portfolio strategy. The two models incorporate a new parameter, η, which adjusts the weights' reaction to volatility changes. The reward-to-risk portfolio method further incorporates expected conditional returns when calculating weights.

2.3.1. Datasets

The sample period is from July 1963–December 2008(Kirby and Ostdiek, 2012, p.563) [4] , and the estimation window size is 120. Totally, there are 546 monthly observations. Four datasets are used in the paper, US T-bills, US equity index 10 portfolio, 25 Size/BTM dataset and 10 Momentum dataset.

2.3.2. Evaluation

In accordance to the Demiguel(2007) [1] model, the rolling window method is also used to compare with the Demiguel el.al (2007) [1] and Bessler et.al (2014)' [2]s model. The calculation of weights is not related to utility optimization, but straightforwardly calculate from the formula.

The weights of the VT strategy are:

$\omega_{it} = (1/\hat{\sigma}^2_{it})^{\eta} / \sum_{i=1}^{N} (1/\hat{\sigma}^2_{it})^{\eta}$ $\quad i = 1, 2, \ldots, N,$

(Kirby and Ostdiek, 2012, p.448) [4]

The weights of the return-to-risk strategy:

$\omega_{it} = (\hat{\mu}_{it}/\hat{\sigma}^2_{it})^{\eta} / \sum_{i=1}^{N} (\hat{\mu}_{it}/\hat{\sigma}^2_{it})^{\eta}$ $\quad i = 1, 2, \ldots, N,$

(Kirby and Ostdiek, 2012,p.449) [4]

The weight of return-to-risk strategy using betas

$\omega_{it} = (\beta_{it}/\sigma^2_{it})^{\eta} / \sum_{i=1}^{N} (\beta_{it}/\sigma^2_{it})^{\eta}$ $\quad i = 1, 2, \ldots, N,$

(Kirby and Ostdiek, 2012, p.450) [4]

σ^2_{it} is the sample variance of portfolio i at time t and μ_{it} is the sample average return of portfolio i at time t.

Performance measures used in the paper are Sharpe ratio and quadratic function:

$U(R_{p,t+1}) = W_t(1 + R_{p,t+1}) - 1/2\alpha W^2_t (1 + R_{p,t+1})^2$ (18)

(Kirby and Ostdiek, 2012, p.451) [4]

The essay calculates the Sharpe ratio and quadratic utility function of Models with different η ,(η=1,2,4), and betas.

2.3.3. Main Findings

Minimum variance model performs worse because of high-targeted return, and high transaction costs. But by introducing the tuning parameter η, we can control the turnover and achieve higher performance (Kirby and Ostdiek, 2012). [4]

According to (Kirby and Ostdiek, 2012) [4], when η is higher, both VT strategy and return-to risk strategy have a high Sharpe ratio. Besides, VT strategy and return-to–risk strategy both outperforms 1/N strategy with or without transaction costs.

Compared with Demiguel et.al's (2007) [1] paper, the reason why MV underperforms the 1/N strategy, except for the estimation window size, is that the high expected targeted return(Kirby and Ostdiek, 2012) [4]. If the tuning parameter is introduced, the turnover of our portfoilios

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can be controlled. In this way, VT strategy and return-to-risk strategy are able to outperform the 1/N portfolio strategy.

3. Empirical Analysis of Black-Litterman 1/N

3.1. Implementation Details

For Black-Litterman's model, the main goal is to find the posterior estimated return and covariance matrix based on proper priors, and to use the new parameter inputs to calculate the weights and the return of the optimizing portfolio.

In my analysis, I use 25-FF monthly data from Jan 1976 to Dec 2015 to replicate BL-1/N model in Bessler et al.'s (2014) [2] paper.

Firstly, the implied return π=λ∑w* is calculated (Bessler et al, 2014) [2].

λ is the risk aversion parameter and it is calculated using the 25-FF dataset (market return-risk free rate).In dataset 25-FF, the historic average market return is 0.00647361 and the variance is 0.00289563. As λ =μ/variance (Han,2017), the risk aversion parameter is 0.00647361/0.00289563= 2.24.

With 25 portfolios in our analysis, the weight variable should be a 25X1 vector. In the BL-1/N model, equal-weight reference portfolio is used (1/25 as the element).

3.1.1. Posterior Expected Returns and Variances of BL-1/N Model

Vector Q and matrix Ω represent the distribution of investor posterior view. Vector Q represents the expected return and matrix Ω is the expected variance ("reliability of investor view") (Besseler 2014).

It is assumed that we have 25 absolute investor views for asset 1, 2,...25. We assume that the expected returns will be 0.008307, 0.009677, 0.01279 and so on, calculated from historical average returns from time-period Jan 1976 to June 2016). Therefore, a 25X25 binary matrix P ("Viewer portfolio" Meucci(2001) [5]) and 25x1 Vector Q can be set up as variables in Matlab.

According to Meucci(2001) [5], we can calculate Ω=(P(1/c ∑)P')), and c is the estimates of the variance of historic returns. Therefore, Ω (25x25 matrix) is established.

C matrix is a diagonal matrix with variances on the diagonal. In Matlab, we can first calculate the covariance matrix in t=1, and use Matlab order c=diag(diag(covariance)) to create the variable. By rebalancing our portfolios every time, c changes with the covariance matrix.

As for tau, in accordance to the Bessler et.al(2014) [2]'s paper, we assume it to be 0.1 at first and conduct sensitivity analysis later. Posterior BL estimated return and volatility used the equations in the paper. (Bessler et.al, 2014) [2]

μ_bl=[(ꚍ ∑)(-1)+P` Ω-1P]-1[(ꚍ∑)(-1) π+P` Ω-1Q] (Thiel, 1971, quoted in Bessler et.al, 2014) [2]

∑_bl=∑+[(ꚍ ∑)-1+P` Ω-1P]-1 (Satchell and Scoscroft, 2000, quoted in Bessler et. al, 2014) [6]

When t=1,2,...480, the input parameter is described in Matlab as:

Er_t=((0.01*Cr(:,:,t))^(-1)+p'*omega^(-1)*p)^(-1)*((0.025*Cr(:,:,t))^(-1)*pie+p'*omega^(-1)*q)

Cr_t=Cr(:,:,t)+((0.01*Cr(:,:,t))^(-1)+p'*omega^(-1)*p)^(-1)

*P stands for P binary matrix and pie is the implied return term.

Finally, we can calculate the weights of our optimized portfolio from t (starting point 197601 to 201512) by maximizing the unconstrained utility maximization function using the Er_t and Cr_t inputs in the Matlab:

w'μ -1/2 λ w'∑w(Han,2017)[3]

There is a "no short-sale" constraint in my analysis and I compute the Sharpe ratio before and after transaction costs.

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3.1.2. Equal Weight Portfolio

The matrix of the weight of equal-weight portfolio is a 480*25 matrix and all the elements are 0.04.

3.1.3. Evaluation

A rolling window method is used and the estimated window size is 120.

For each time point starting from t=197512, we can calculate the weights of each portfolio and rebalance it every time we move forward to next month. Therefore, we have a 480x25 matrix to describe the weights of each of the 25 portfolios.

Finally,I use the 480 monthly out-of-sample returns of our optimized portfolio strategies and calculate the Sharpe ratio and the Omega performance measure.

Omega measure is calculated as:

omegai= $\sum_{n=1}^{T} \mathrm{Max}(0, \mathrm{rt,i} - rf) / \sum_{n=1}^{T} \mathrm{Max}(0, \mathrm{rf} - rt, i)$

(Bessler et.al, 2014)

4. Empirical results for the full sample 25-FF factor from December 1976 to June 2016

4.1. Performance Measure

According to Table 1(transaction cost 30 bp), the Sharpe Ratio and Net Sharpe ratio (after transaction cost) BL-1/N are both lower than the 1/N strategy. While in the Bessler(2014)' [2]s paper, the BL-1/N strategy consistently outperforms the 1/N model.

As for the volatility, in accordance with the essay, the BL-1/N always has lower volatility compared to 1/N strategy.

For the Omega measure, namely the average gains relative to losses (Besseler,2014). 1/N portfolio also outperforms BL models.

Table 4. Empirical Results

Investor volatility targeted=0.05	**1/N**	**BL-1/N**	**BL-1/N without investor view**
Net Mean Return	0.0083	0.0074	0.0079
Volatility	0.0493	0.0475	0.0505
Net Sharpe Ratio	0.1694	0.1568	0.1555
Sharpe Ratio without transaction costs 30 bp	0.1694	0.1573	0.1785
Omega Measure	0.5621	0.5221	0.4902

4.1.1. Sensitivity Analysis of Transaction Cost when tau=0.1.

In Table 4, the net Sharpe ratio is not so sensitive when transaction cost changes. In Bessler et.al' [2]s case, with transaction costs changing from 30-50 bp, the Net Sharpe Ratio does not change a lot (0.64-0.72) as well.

Table 5. Robust Analysis Results

Different Transanction cost Conservative	**BL-1/N after transaction**	**1/N**
30	0.1568	0.1694
40	0.1567	0.1694
50	0.1565	0.1694

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4.1.2. Sensitivity Analysis for Variation of Implied Returns (τ)

Transaction cost is kept at 30 b.p. Similarly in BL-1/N's case, net Sharpe ratio does not necessarily change significantly when τ changes. When t=0.1, BL-1/N model performs the best.

Table 6. Robust Analysis Results

Different τ	BL-1/N	1/N
0.3	0.1575	0.1694
0.15	0.1570	0.1694
0.1	0.1568	0.1694
0.05	0.1567	0.1694
0.025	0.1566	0.1694

4.1.3. Compare the Differences

The use of stock-only datasets may explain the differences between Table 1 results the Bessler et.al [2]paper. Since in Bessler et.al (2014) [2], the datasets spread from stocks, government bonds to energy resources index. The wide spreads of assets undoubtedly can provide more diversification and also the hedging effect.

Secondly, Q vector and P matrix are subject to subjective views. Besides, Ω is calculated based on Q, P and historic forecast error (Meucci,2010)[5]. The confidence of our investor view is difficult to be ascertained. The confidence of investor view is plausible, since in our case, the absolute view on the asset expected returns (Q vector) assumption is purely based on totally 1080 returns from period July 1929 to June 2016 for each of the 25 portfolios, the estimation error is unknown.

Furthermore, according to Demiguel(2007) [1], at least when the size of estimation window is 6000 months can a MV portfolio outperforms the 1/N strategy for 25 assets. In our case our estimation window size is only 120, we are not sure whether the result will change significantly with large window size.

Many estimators need to be estimated in BL-1/N case. For example, τ measures the departure of the reference market portfolio (in our case is the equal weight market portfolio), both my and Bessel's robustness check imply that the Sharpe ratio is not sensitive to the changes, even in a broader asset class. But it is uncertain that if t will significantly influence the Sharpe ratio in a large window size.

5. Conclusion

My empirical analysis implies that the BL-1/N strategy slightly underperforms the 1/N strategy. Besides, the performance measurement is not so sensitive to the changes of transaction costs and input parameter t. The overall empirical results are in accordance to the paper Bessler et.al (2014).

The use of 25-FF stock-only weakens the effect of the diversification effect and offsets the allocation effect (Bessler et.al,2014).

The effectiveness of optimizing portfolio strategy of the literature in discussion mainly depends on the mitigation of estimation risk.

Bessler et.al(2014) mainly employs investor view vector and matrix and Kirby and Ostdiek(2012) uses tuning parameter η to mitigate the estimation risk.

If the estimation risk can be correctly mitigated, then BL, VT and return-to-risk strategy is more possible to outperform the 1/N portfolio. Hence, the estimation of investor view and tuning parameter is essential. If investors are confident with our estimation, then using BL, VT and return-to-risk is better off. For BL portfolio, we need to include other assets such as commodities to outperform the 1/N strategy.

As for the equal-weight portfolio strategy, it is only realisable if investors are able to find the true market portfolio. With a number of assets classes in the world, the exact weight is hard to find. Some of the assets might not be tradable on the market. Finally, as price changes through time, the adjustment of weights in order to maintain the equal weight will be tedious.

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