Game-Theoretic Analysis of Grain Sales Strategies: Nash Equilibrium and Cooperative Bargaining

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Abstract

This study uses game-theoretic models, including Nash equilibrium and Rubinstein bargaining theory, to analyze the strategic interactions between grain farmers and enterprises in Heilongjiang, China. A mathematical framework was developed to incorporate moisture content, discount factors, and threat points to derive equilibrium conditions with a focus on profit distribution under price mechanisms. The analysis reveals that farmers face a trade-off between higher profits from enterprise cooperation, which emphasizes grain quality, and lower-risk market sales, while enterprises balance procurement costs against resale premiums. Through Python simulations with real data, the analysis suggests that smallholders tend to prefer direct market sales due to quality-related risks, whereas large-scale farmers benefit more from cooperative agreements. The bargaining model further identifies Pareto-optimal solutions under dynamic negotiations, and highlights how first-mover advantages and discount factors shape outcomes. The findings suggest policy interventions to better align incentives, reduce negative-sum scenarios, and promote sustainable farmer-enterprise partnerships. This study has significant practical and theoretical implications.

Keywords

Farmers and enterprises, bargaining, Nash equilibrium, discount factor, game theory, cooperation, moisture content.

1. Introduction

According to the National Bureau of Statistics, in 2024, the total grain output of Heilongjiang Province reached 160.034 billion jin (equivalent to 80.017 billion kilograms), making it the highest in China for 15 consecutive years. However, with the profits distributed across the industrial chain being highly uneven, the market exhibits a trend of a secular decline in profit margins across successive years. The quality of grain is also valued [1, 2]. For instance, to avoid panic selling and malicious price suppression induced by continuous price level decline, the government adopts the policy of taking over grain from farmers whose grain is unable to meet the quality requirements of enterprises at a low price. However, if farmers want to participate in the program, they must first pass the inspection of quality. The minimum purchase price is set with clear quality standards, ensuring that only the grain that meets the basic quality requirements can be brought into the guarantee scope. Wheat quality is classified into three grades according to the national standards, with the minimum purchase price of 1.19 yuan/jin, second-grade wheat has the minimum purchase price of 1.21 yuan/jin, and first-grade wheat has a minimum purchase price of 1.23 yuan/jin. Thus, domestic policies are prompting enterprises, grain processing centers, and farmers to address challenges in establishing market-oriented joint operation mechanisms from the perspective of improving grain quality. This paper will use the approaches of the bargaining model in game theory, Python modeling operations, and the Rubinstein model to deduce the total profits of farmers selling grain to the market directly and cooperating with enterprises while accounting for factors such as moisture content. Moreover, this article aims to obtain the mixed-strategy Nash equilibrium of the bargaining between farmers and enterprises, which is supported by significant theoretical and experimental foundations. This paper demonstrates the derivation and calculation process, as well as incorporates historical data for analysis. Section 2 will present the profit calculation formula, derivation process, and explanation for multiple sales methodologies, including the impact of moisture content. Section 3 will discuss the derivation process and results of the bargaining model, as well as infer the optimal strategy for both farmers and enterprises. Section 4 provides a summary of the findings and conclusions.

2. Determination of Equilibrium Price

The primary reason why game theory models can be effectively applied to the governance of raw grain supply chains is that farmers and enterprises often have inherent conflicts in resource allocation, as both parties hope to maximize their own interests given the costs they have incurred. This section will systematically discuss and calculate the decisions made by farmers, aiming to derive the choices they should make based on different contexts to maximize their expected payoffs. There are many different ways for grain to be sold in the final market. Two methodologies for grain commercialization will be discussed:

2.1. Cooperation with Grain Processing Enterprises

Generally, enterprises will equip a drying tower to address the issue of excessive grain moisture. Enterprises will give farmers various returns based on the moisture content of the grain they provide. If the moisture content is low, they can obtain high returns. In this way, farmers obtain higher profits from enterprises than from directly selling them to the market [2]. In contrast, if the moisture content is too high, they will be rejected by the enterprise and experience depreciation when sold to the market again. Enterprises will pay for processing fees, and then further process or package the grain before selling it to the market.

2.2. Direct Sales to the Market

The other option is to sell their grain directly to the market for a lower return than selling to businesses, but it can eliminate the risk of being rejected by businesses due to excessive moisture. To model the strategic interaction of farmers and derive a Nash equilibrium to maximize benefits with enterprises, the following variables will be introduced.

- P_m: Current market price.
- γ: Relative premium rate offered by enterprises ($0 < \gamma < 1$).
- P_e: price given by the enterprise, current market price times the relative premium of the enterprise ($0 < \gamma < 1$).
- δ: discount factor, the percentage of the reduction of the profit after each additional selection ($0 < \delta < 1$).
- q: the probability that the product is high quality (low moisture content) ($0 < q < 1$).
- r: the probability that the product is low quality (high moisture content) ($0 < r < 1$).
- η: when the product is of medium quality, the percentage of reduction is ($0 < \eta < 1$).
- θ: the percentage reduction of the price of the product related to the market price after being returned by the enterprise ($0 < \theta < 1$).
- C_f: the farmer's unit production costs
- C_e: the enterprise's handling processing costs

After listing all of the variables, we could figure out the benefit function based on the variables. We could list the total revenue first. There are two choices:

1) Sell to the market: Benefit $= P_m$

2) Sell to the enterprise: $P_e = (1+\gamma) \cdot P_m$

If farmers choose to sell their grain to the enterprise, they will definitely be affected by the discount factor, as the enterprise will spend a lot of time drying the grain compared to directly selling to the market. Next, figure out the benefit of the farmer when the grain has different moisture contents.

Table 1. A farmer's expected benefit from grain quality when selling to the enterprise.

Conditions	Probability	Benefit considered with discount factor
high quality(low moisture content)	q	$\delta \cdot P_e$
relatively low quality(still accepted)	1-q-r	$\delta \cdot (1-\eta) \cdot P_e$
low quality(returned)	r	$\delta^2 \cdot \theta \cdot P_m$

Notice: the product that is returned will experience two processes, it will be returned by the enterprise and then resold to the market, so we would time the discount factor twice, and we should time P_m, not P_e.

Then, we could figure out the expected benefit function when the farmer chooses to sell to the enterprise.

$$\text{Benefit} = q \cdot \delta \cdot P_e + (1-q-r) \cdot \delta \cdot (1-\eta) \cdot P_e + r \cdot \delta^2 \cdot \theta \cdot P_m$$

Then, we exchange the total revenue for the net income.

πm: the net income when the farmer chooses to sell to the market directly

πe: the net income when the farmer chooses to sell to the enterprise

We will make a change to the previous function, minus the cost from the total revenue, to get the net income.

Sell to the market: $\pi m = P_m - C_f$

Sell to the enterprise: $\pi e = q \cdot (\delta \cdot P_e - C_f) + (1-q-r) \cdot (\delta \cdot (1-\eta) \cdot P_e - C_f) + r \cdot (\delta^2 \cdot \theta \cdot P_m - C_f)$

This formulation reflects the net income under each possible outcome, weighted by its respective probabilities.

The only variable that could be controlled by the farmer is δ. Therefore, the two equations can be successively combined to calculate the Nash equilibrium when farmers sell to the market and enterprises as:

$$\pi m = \pi e$$

$$P_m - C_f = q \cdot (\delta \cdot P_e - C_f) + (1-q-r) \cdot (\delta \cdot (1-\eta) \cdot P_e - C_f) + r \cdot (\delta^2 \cdot \theta \cdot P_m - C_f)$$

After collating:

$$(r \cdot \theta \cdot P_m) \cdot \delta^2 + (1 - \eta + q \cdot \eta - r + r \cdot \eta) \cdot P_e \cdot \delta - P_m = 0$$

If the function has a solution, it must obey b^2-4ac≥0 as:

$$(1\text{-}\eta+q\cdot\eta\text{-}r+r\cdot\eta)^2+4(r\cdot\theta\cdot P_m)\cdot P_m\geq 0$$

If the conditions are met, we could figure out the solutions of δ as:

$$\delta=\frac{-b+(b^2\text{-}4ac)^{\frac{1}{2}}}{2a}\text{ or }\frac{-b-(b^2\text{-}4ac)^{\frac{1}{2}}}{2a}$$

$$\delta=\frac{-(1\text{-}\eta+q\cdot\eta\text{-}r+r\cdot\eta)+[(1\text{-}\eta+q\cdot\eta\text{-}r+r\cdot\eta)^2+4(r\cdot\theta\cdot P_m)\cdot P_m]^{\frac{1}{2}}}{2(r\cdot\theta\cdot P_m)}$$

$$\text{or}=\frac{-(1\text{-}\eta+q\cdot\eta\text{-}r+r\cdot\eta)-[(1\text{-}\eta+q\cdot\eta\text{-}r+r\cdot\eta)^2+4(r\cdot\theta\cdot P_m)\cdot P_m]^{\frac{1}{2}}}{2(r\cdot\theta\cdot P_m)}$$

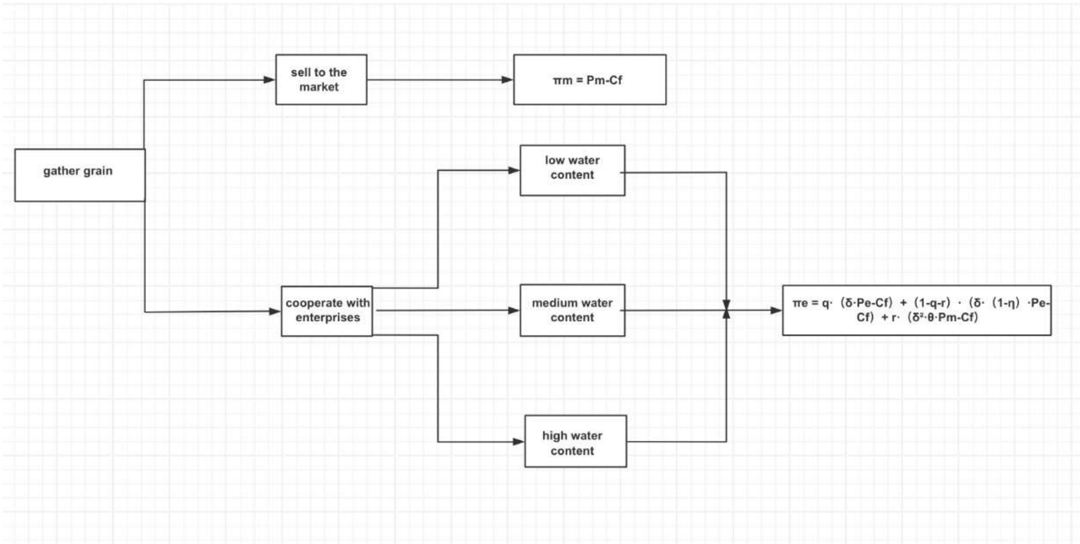

Figure 1. Decision tree of farmer's grain-selling options and expected benefits.

Table 2. Payoff matrix of farmers and grain processing enterprises under different grain quality conditions.

Farmers/Grain Processing Enterprise	High quantity	Low quantity	Reject
Cooperate with Grain Processing Enterprise	$\frac{\delta\cdot P_e}{(d-1)\cdot P_e-C_e}$	$\frac{\delta\cdot(1-\eta)\cdot P_e}{P_e\cdot(1-n)\cdot(d-1)-}C_e$	$\frac{\delta^2\cdot\theta\cdot P_m}{0}$
Cooperate with the Market	$\frac{P_m}{0}$	$\frac{P_m}{0}$	$\frac{P_m}{0}$

The first Python program we developed implements the variables and modeling framework introduced above, which will assist in calculating the equilibrium price and validating the theoretical predictions of the model. By simply inputting the values of each variable, we can separately derive the net profits from sales to the market and the equilibrium net profits from cooperation with the grain processing enterprise. Additionally, we emphasize the comparative functionality of key variables to derive more practical data.

The value of the discount factor will be determined based on the individual circumstances of the farmers. In the most beneficial conditions for farmers, the value of the discount factor is set to 1, representing that the value of the grain will not depreciate over time. There are several possible factors that may affect the discount factor, including:

1) Financial Costs: Farmers may lack sufficient funds, and there is a possibility of relying on loans. The later farmers receive the cost of food, the more interest they will need to pay.

2) Opportunity and Time costs: If farmers waste too much time when trading with businesses, they may lose a lot of the benefits they may have gained during this time.

3) Changes in Quality: If the transaction is delayed for too long, there may be a risk of deterioration and mold growth in the quality of the grain, which will be defined as low-quality grain and result in reduced profits.

4) Farmer's Identity: If a farmer is a large-scale planter, they will experience minimal decline in their profits, and the value of the discount factor will be closer to 1. In contrast, if it is a farmer from a distant area, their income is often unstable, and the delay in time can lead to a significant decline in their income, resulting in a lower discount factor value.

The remaining variables represent uncontrollable factors for farmers, and their possible explanations are as follows:

1) Price Reduction Rate (η): This reflects the low quality of grain. Enterprises have already incurred the drying cost for farmers, but farmers bring low-quality grain. Therefore, enterprises will choose to reduce the benefits to farmers.

2) Relative Enterprise Premium (γ): is due to the fact that enterprises choose to attract farmers to trade with them rather than the market, while also avoiding the possibility of farmers giving up cooperation due to the risk of returns.

3) Resale Price Decline Rate (θ): After grain is returned, its market value decreases due to reduced buyer acceptance, necessitating a price reduction upon resale.

Based on the profit calculation formula derived in the previous paper, the effectiveness of the model will be verified through actual data. Taking corn as an example, the planting cost, market price, the probability of grain farmers being judged as high-quality after inspection, probability of rejection, the discount factor (δ), the relative premium rate of the grain processing enterprises, the relative price reduction rate of the grain processing enterprises, and the price rection rate of the market after being rejected by the grain processing enterprise are introduced as real data, and the Python tool is used to calculate. According to the official statistics of the National Compilation of Agricultural Product Cost and Income Data, the average cost of corn in Heilongjiang is 0.79 yuan/jin, and the market price is about 1.25 yuan/jin. The purchase quality statistics of China Grain Storage Heilongjiang Branch mention that high-quality corn accounts for about 35% of the total, and the probability of being returned by the company is about 10%. For farmers, the discount factor is about 0.83, and for enterprises, the relative premium rate is about 8%, and the price reduction rate is about 10%. The price reduction rate of grain that can only re-enter the market due to excessive moisture is about 25%. According to Python calculations, the profit of grain farmers selling directly to the market is about 0.460 yuan/jin, while the profit of cooperating with enterprises is about 0.221 yuan/jin. In this case, smallholder farmers tend to sell their grain directly to the market.

3. Bargaining

3.1. Model Assumptions and Basic Framework

In the process of cooperation between farmers and enterprises, we assume that both parties are rational. They will maximize their own profits to the greatest extent, so both parties will engage in bargaining to achieve the distribution of benefits. During the negotiations, we believe that the delay caused by the disharmony of each round of bargaining will bring a loss of profits, resulting in a loss of interest for both parties.

This is modeled using the discount factor:

$$\varepsilon\ (0 < \varepsilon < 1)$$

Discount factor: (ε) Time value of money.
The profits of farmers and enterprises are as follows:
Farmer: $P_e - C_f$
Enterprise: $(d - 1) \cdot P_e - C_f$
If the value of ε is higher, it indicates that the time cost is lower, and both the farmers and the enterprises have more patience (patience actually refers to the psychological and economic capacity of the participants) to negotiate.

3.2. Threat Points and Negative-Sum Scenarios

Moreover, in the bargaining process, there are threats to the interests of both farmers and enterprises. If the price agreed is lower than the threat point, it will cause losses to farmers or enterprises and lead to the breakdown of negotiations. The result is that farmers can only return to the market to sell their grain again, while enterprises need to search for high-quality farmers again, resulting in time-induced inefficiencies for both parties' interests due to discount factors. As a result, both sides achieved a negative-sum game (ε) in the game, which is a result that neither side wants to see, even if they lose.
Threat point formula:

- Farmer: $\pi e \geq \pi m$
- Enterprise: $(d - 1) \geq (1 - \eta) \cdot (k - 1)$

As a result, for both farmers and enterprises, the price proposed by either party should exceed the farmer's threat point and be less than the threat point of enterprises. Moreover, the equilibrium price is expected to fluctuate near the boundary point where the threat points of the two parties meet.

3.3. Secondary Options and Adjusted Threat Points

It is worth mentioning that enterprises do not rely exclusively on high-quality agricultural products. Instead, they have a secondary option: purchasing lower-quality crops from alternative suppliers. However, we assume that the procurement capacity is limited and cannot simultaneously purchase high-quality farmers' grain and low-quality crops from other sources, and the quality of raw material affects the pricing of the enterprise's final products directly. Consequently, the profit obtained by the enterprise in the price it sets should be greater than or equal to the profit obtained from purchasing low-quality grain. Therefore, the profit obtained by the enterprise in the price it sets should be greater than or equal to the profit obtained from purchasing low-quality grain. Consequently, the threat point formula of the enterprise should be reconsidered (for enterprises, it is necessary to add the revenue from purchasing low-quality raw materials to the original threat point formula; for farmers, it is necessary to add the revenue from market sales to the original threat point formula).

3.4. Dynamic Bargaining Model and Python Simulation

The second Python program is based on the first, but instead of focusing on computation, it emphasizes a dynamic model of unlimited bargaining between farmers and grain processing enterprises. In this program, users have the option to play as a farmer and engage in a bargaining game with grain processors as they grow high-quality crops. The prerequisite is that the profit obtained by the grain processing enterprise after packaging the farmer's crops (which can be simplified to $(d - 1) \cdot P_e - C_e$, where the packaging premium rate $d > 1$) must be greater than the profit obtained when the grain processing enterprise find secondary crops (assuming the purchase price of secondary crops is the same as that of lower-quality crops, it can sell at $k \cdot P_e \cdot (1 - \eta) - (1 - \eta) \cdot P_e - C_e$, where k is the depreciation rate of secondary crops upon sale, simplified to $P_e \cdot (1-\eta) \cdot (k-1) - C_e$, and $0 < k < 1$. When the farmer puts forth a new price (assumption P_n), if the profit from high-quality crops ($d \cdot P_e - P_n - C_e$) is still greater than that from secondary crops (i.e., $d \cdot P_e - P_n - C_e > P_e \cdot (1-\eta) \cdot (k-1) - C_e$, which simplifies to $P_n < P_e \cdot (d + k - k\eta + \eta - 1)$), the grain processing enterprise will agree to the bargaining; otherwise, it will reject it. When negotiations break down, farmers have no choice but to put their crops on the market, which will cause losses due to the discount factor and reduce profits, while grain processors will switch to a strategy of selling inferior crops.

3.5. Rubinstein Model for Complete Information Dynamic Game

Then, we could introduce the Rubinstein model [3, 4] to analyse the strategy of the complete information dynamic game (assumed) in the bargaining between the farmer and the enterprise. The common knowledge in it is the quality of the grain and its market price. Now, we assume the bargaining goes through n rounds, and in the (n+1)st round, the negotiation failed and the cooperation broke down. Due to the existence of the first-mover's advantage, we would analyse two different situations based on whether the farmer is the first-mover or the enterprise is the first-mover. Furthermore, we consider two situations when n is an odd number and when n is an even number. The Pareto optimality of the balanced shares of farmer and enterprise in the two situations of the final proposal initiated by the farmer and the final proposal initiated by the enterprise is analyzed, respectively (ε Pareto optimality). When considering n is an odd number (During the nth round of bargaining, the benefits will be distributed by the farmers). We assume that in the first round, the proportion of the revenue obtained by the farmer to the total revenue is A_n, and the proportion of the revenue obtained by the enterprise to the total revenue is B_n.

Table 3. An iterative allocation plan across N rounds.

Round	Allocation Plan
1	(A_n, B_n)
2	$(A_n - 1, B_n - 1)$
...	...
N	(A_1, B_1)

3.6. Recursive Allocation and Fixed-Point Analysis

To make sure the cooperation would not break down, the threat point should be considered.

Step 1: n^{th} round allocation by the farmer

In the n^{th} round, the B_1 allocated by the farmer to enterprises should be exactly equal to the threat points of the enterprise to maximize their own interests:

$B_1 =$ Threat Point of Enterprise, $A_1 = 1 - B_1$

Step 2: $(n-1)^{st}$ round allocation

$$A_2 = A_1 \cdot \varepsilon_1, B_2 = 1 - A_2$$

Step 3: $(n-2)^{nd}$ round allocation

$$B_3 = B_2 \cdot \varepsilon_2 = (1 - A_2) \cdot \varepsilon_2 \text{ , } A_3 = 1 - B_3 = 1 - \varepsilon_2 (1 - A_1 \cdot \varepsilon_1)$$

Step 4: Recursive relation for subsequent rounds

$$A_n = 1 - \varepsilon_2 (1 - \varepsilon_1 \cdot A_{n-2})$$

Step 5: Transform to a geometric progression
Let

$$A_n - k_1 = r \cdot (A_{n-2} - k_1)$$

where k_1 is the fixed point.
Step 6: Find the fixed point
Original equation:

$$k_1 = 1 - \varepsilon_2(1 - \varepsilon_1 \cdot k_1)$$
$$k_1 \cdot (1 - \varepsilon_1 \cdot \varepsilon_2) = 1 - \varepsilon_2$$
$$k_1 = \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2}$$

Step 7: Subtract the fixed point and transform

$$A_n - \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2} = 1 - \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2} - \varepsilon_2(1 - \varepsilon_1 \cdot A_{n-2})$$
$$A_n - \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2} = 1 - \varepsilon_2 + \varepsilon_1 \cdot \varepsilon_2 \cdot A_{n-2} - \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2}$$
$$A_n - \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2} = (1 - \varepsilon_2) \cdot (1 - \frac{1}{1\text{-}\varepsilon_1 \cdot \varepsilon_2}) + \varepsilon_1 \cdot \varepsilon_2 \cdot A_{n-2}$$
$$A_n - \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2} = \varepsilon_1 \cdot \varepsilon_2 \cdot (A_n - \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2})$$

At this point, we have transformed the original recursive sequence into a geometric progression.

$$A_{n-2} - \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2} = \varepsilon_1 \cdot \varepsilon_2 \cdot (A_{n-4} - \frac{1\text{-}\varepsilon_2}{1\text{-}\varepsilon_1 \cdot \varepsilon_2})$$
$$A_n\text{-}\frac{(1\text{-}\varepsilon_2)}{(1\text{-}\varepsilon_1 \cdot \varepsilon_2)} = (\varepsilon_1 \cdot \varepsilon_2)^{\frac{n-1}{2}} \cdot \frac{(A_1\text{-}(1\text{-}\varepsilon_2)}{(1\text{-}\varepsilon_1 \cdot \varepsilon_2)}$$
$$A_n = (\varepsilon_1 \cdot \varepsilon_2)^{\frac{n-1}{2}} \cdot \frac{(A_1\text{-}(1\text{-}\varepsilon_2)}{(1\text{-}\varepsilon_1 \cdot \varepsilon_2)} + \frac{(1\text{-}\varepsilon_2)}{(1\text{-}\varepsilon_1 \cdot \varepsilon_2)}$$

Step 8: Use B_1 to find A_1, then get A_n, B_n

$$B_1 = \frac{\text{the threat point of the enterprise}}{\text{total benefit}}, A_1 = 1 - B_1$$

Then we can calculate A_n and B_n using the recursive relationship. To obtain the general formula of the proportion of farmers' interests in the nth round of bargaining, we introduce a knowledge point of the series – fixed point (ε) to help solve the general formula.
Case 1: n is an even number
1) n^{th} round:

In order to maximize their own interests, companies will allocate benefits to farmers that are exactly at the threat point:

$$A_1 = \frac{\text{Threat Point for farmer}}{\text{Total Return}}, B_1 = 1 - A_1$$

2) (n-1)th round:

$$B_2 = B_1 \cdot \varepsilon_2, A_1 = 1 - B_1 = 1 - B_1 \cdot \varepsilon_2$$

3) (n-2)th round:

$$A_3 = A_2 \cdot \varepsilon_1, B_3 = 1 - A_3 = 1 - \varepsilon_1(1 - B_1 \cdot \varepsilon_2)$$

4) Subsequent rounds:
Similarly, we find the recursive relationship:

$$B_n = B_{n-1} \cdot \varepsilon_2, A_n = 1 - B_n$$

And

$$B_{n-1} \text{ recursively relates to } B_{n-3}$$

General formula for B_n (even n)
1) Set up geometric progression:

$$B_{n-1} - k_2 = r \cdot (B_{n-3} - k_2)$$

2) Find the fixed point k_2:

$$k_2 = 1 - \varepsilon_1(1 - \varepsilon_2 \cdot k_2)$$
$$k_2 \cdot (1 - \varepsilon_1 \cdot \varepsilon_2) = 1 - \varepsilon_1$$
$$k_2 = \frac{1-\varepsilon_1}{1-\varepsilon_1 \cdot \varepsilon_2}$$

3) Subtract the fixed point to form a geometric progression:

$$B_{n-1} - k_2 = 1 - \varepsilon_1 \cdot (1 - B_{n-3} \cdot \varepsilon_2) - k_2$$
$$B_{n-1} - k_2 = \varepsilon_1 \cdot \varepsilon_2 \cdot (B_{n-3} - k_2)$$
$$B_{n-1} - k_2 = (\varepsilon_1 \cdot \varepsilon_2)^{\frac{n-2}{2}} \cdot (B_1 - k_2)$$

Express B_1 in terms of the farmer's threat point and total profit:

$$B_1 = \frac{(\text{total profit} - \text{farmer threat point})}{\text{total profit}}$$

Case 2: n is an odd number

1) n^{th} round:

In order to maximize their own interests, companies will allocate benefits to farmers that are exactly at the threat point:

$$A_1 = \frac{\text{Threat Point for farmer}}{\text{Total Return}}, B_1 = 1 - A_1$$

2) $(n\text{-}1)^{th}$ round:

$$B_2 = B_1 \cdot \varepsilon_2, A_2 = 1 - B_2 = 1 - B_1 \cdot \varepsilon_2$$

3) $(n\text{-}2)^{th}$ round:

$$A_3 = A_2 \cdot \varepsilon_1, B_3 = 1 - A_3 = 1 - \varepsilon_1(1 - B_1 \cdot \varepsilon_2)$$

4) Subsequent rounds:

Similarly, we find the recursive relationship:

$$B_n = B_{n-1} \cdot \varepsilon_2, A_n = 1 - B_n$$

And

$$B_{n-1} \text{ recursively relates to } B_{n-3}$$

General formula for B_n (odd n)

1) Set up geometric progression:

$$B_n - k_3 = r \cdot (B_{n-2} - k_3)$$

2) Find the fixed point k_3:

$$k_3 = 1 - \varepsilon_1(1 - \varepsilon_2 \cdot k_3)$$
$$k_3 \cdot (1 - \varepsilon_1 \cdot \varepsilon_2) = 1 - \varepsilon_1$$
$$k_3 = \frac{1 - \varepsilon_1}{1 - \varepsilon_1 \cdot \varepsilon_2}$$

3) Subtract the fixed point to form a geometric progression:

$$B_n - k_3 = 1 - \varepsilon_1 \cdot (1 - B_{n-2} \cdot \varepsilon_2) - k_3$$
$$B_n - k_3 = \varepsilon_1 \cdot \varepsilon_2 \cdot (B_{n-2} - k_3)$$

$$B_n - k_3 = (\varepsilon_1 \cdot \varepsilon_2)^{\frac{n-2}{2}} \cdot (B_1 - k_3)$$

Express B_1 in terms of the farmer's threat point and total profit:

$$B_1 = \frac{(\text{total profit} - \text{farmer threat point})}{\text{total profit}}$$

After analyzing the situation where the farmer first proposed the plan, we also consider the scenario in which the grain processing enterprise bargains first. Similarly, the equilibrium shares of both parties depend on the parity of n. Finally, the shares in the nnnth round can be expressed as:

$$A_n = 1 - B_n, B_n = (\varepsilon_1 \cdot \varepsilon_2)^{\frac{n-2}{2}} \cdot (B_1 - k) + k$$

where B_n is a function of the initial threat points and bargaining factors $\varepsilon_1 \cdot \varepsilon_2$.
Recurrence Relation and Geometric Progression Summary
1) (n-1)th round:

$$A_2 = A_1 \cdot \varepsilon_1, B_2 = 1 - A_2$$

2) (n-2)th round:

$$B_3 = B_2 \cdot \varepsilon_2 = (1 - A_2) \cdot \varepsilon_2, A_3 = 1 - B_3 = 1 - \varepsilon_2(1 - A_1 \cdot \varepsilon_1)$$

3) Recurrence relation between A_{n-1} and A_{n-3}:

$$A_{n-1} = 1 - \varepsilon_2(1 - \varepsilon_1 \cdot A_{n-3})$$

General formula for A_n
1) Set up geometric progression:

$$A_{n-1} - k_4 = r \cdot (A_{n-3} - k_4)$$

2) Finding the Fixed Point k_4:

$$A_n = A_{n-2} = k_4$$
$$k_4 = 1 - \varepsilon_2(1 - \varepsilon_1 \cdot k_4)$$
$$k_4 \cdot (1 - \varepsilon_1 \cdot \varepsilon_2) = 1 - \varepsilon_2$$
$$k_4 = \frac{1-\varepsilon_1}{1-\varepsilon_1 \cdot \varepsilon_2}$$

3) Subtract the fixed point to form a geometric progression:

$$A_{n-1} - k_4 = 1 - k_4 - \varepsilon_2(1 - \varepsilon_1 \cdot A_{n-3})$$

$$A_{n-1} - k_4 = 1 - \varepsilon_2 + \varepsilon_1 \cdot \varepsilon_2 \cdot A_{n-3} - 1 - k_4$$
$$A_{n-1} - k_4 = \varepsilon_1 \cdot \varepsilon_2 \cdot (A_{n-3} - k_4)$$
$$A_{n-1} - k_4 = (\varepsilon_1 \cdot \varepsilon_2)^{\frac{n-1}{2}} \cdot (A_1 - k_4)$$
$$A_{n-1} = (\varepsilon_1 \cdot \varepsilon_2)^{\frac{n-1}{2}} \cdot (A_1 - k_4) + k_4$$
$$A_n = A_{n-1} \cdot \varepsilon_1$$
$$B_n = 1 - A_n$$

Finally, substitute B_1 as the company's threat point/total profits to get:

$$A_1 (A_1 = 1 - B_1)$$

Then calculate A_n and Bn accordingly.

3.7. Crucial Principles

The crucial principles underlying the bargaining model can be summarized as follows. First, rational constraints for acceptable conditions: that is, each price respondent grain farmer or enterprise will only accept no less than their secondary choice - (for grain farmers, the secondary choice is to sell in the market, and for enterprise, its secondary choice is the income obtained from the purchase of low-quality grain from other channels). Second, the bidder will propose the optimal strategy for himself: The bidder will propose a proposal that the respondent has no objection to, that is, just meets the respondent's acceptable conditions to maximize his share. Third, the discount factor ε measures the patience of one party during the bargaining: the larger the ε in one party, the smaller the loss of one party after rejecting the other party's price proposal, vice versa. Finally, first-mover advantage: As the first bidder, one party can take advantage of the time cost of the other party, so the bidder will propose the best strategy for itself (second principle). Even if the value of the discount factor of both parties is equal, meaning they have the same patience in bargaining, the first bidder still has an advantage.

4. Conclusion

This study utilizes game-theoretic models, including Nash equilibrium and bargaining theory, to analyze the possible decision-making of Heilongjiang grain farmers. Our mathematical framework demonstrates that a Nash equilibrium exists where farmers and enterprises reach stable agreements through strategic bargaining, assuming rational preferences. Computational simulations in Python calculate the equilibrium outcomes, revealing how grain moisture content influences optimal strategies. The findings suggest that cooperative models can enhance farmers' profits, but their viability depends on equitable profit-sharing mechanisms. The bargaining model we used discusses various situations. There are several advantages. Firstly, we considered the situations where farmers and enterprises are given priority in bidding separately. This model not only considers the interests of grain farmers, but also provides incentives and constraints through a balance. Moreover, the optimized governance model of the raw grain supply chain in the grain processing center demonstrates that despite the conflict of interests between grain farmers and the grain processing center, there are long-term mutual benefits between the two parties. Only through coordinated efforts can high-quality raw grains be obtained, thereby achieving higher selling prices for the grains. The governance model of the raw grain supply chain in the grain processing center, derived from the incentive contract of the bargaining game, highlights the mutual benefits of both parties. There are some limitations to the paper. We assume that the discount factors of enterprises and farmers remain unchanged, respectively, while the discount factor cannot remain unchanged and can be influenced by production characteristics, financial pressure, and risk appetite. Improvements can be made by considering the subjective discount rate of farmers and studying their behavioral characteristics.

References

[1] Nash, J. F. (1950). Equilibrium points in n-person games. Proceedings of the National Academy of Sciences of the United States of America, 36(1), 48–49. https://doi.org/10.1073/pnas.36.1.48

[2] Leng, Z. J., Li, M., & Zhang, W. (2015). Governance model of raw grain supply chain based on bargaining game theory in grain processing centers. Systems Engineering-Theory & Practice, 35(5), 1167–1175.

[3] Zhao, Y., Jiang, Q., & Wang, Z. (2019). The system evaluation of grain production efficiency and analysis of driving factors in Heilongjiang Province. Water, 11(5), 1073. https://doi.org/10.3390/w11051073

[4] Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. Econometrica, 50(1), 97–109. https://doi.org/10.2307/1912531