

Multi-factor Quantitative Portfolio Risk-return Collaborative Optimization Model

Zihaochen Zhou^a

Beijing No.4 High School International Campus, Beijing 100031, P.R.China

^azihaochen.zhou@bhsfic.com

Abstract

In the traditional quantitative investment model, the separation of risk and return will lead to tail risk exposure and parameter sensitivity. This paper proposes a multi-factor quantitative portfolio risk-return collaborative optimization model. Construct a risk-benefit cooperative objective function, introduce a gain term based on risk contribution entropy, and upgrade risk diversification from passive constraint to active incentive; The dynamic IC_IR time-varying weighting mechanism is designed, and the factor information coefficient (IC) and information ratio (IR) are calculated by rolling window, and the failure factors are eliminated in real time and the weights are adjusted adaptively. Ledoit-Wolf covariance matrix compression estimation is used to reduce the influence of high-dimensional data noise on the optimization results. Hard constraints such as industry deviation, upper limit of stock weight and maximum retracement warning are embedded, and endogenous optimization of transaction costs is combined to ensure the enforceability of the strategy. The empirical results show that the model has an annualized return of 18.75%, a Sharpe ratio of 1.02, and a maximum drawdown of -22.48% in the Chinese A-share market, which is superior to mean variance optimization (MVO) and IC-IR equal weight combinations. The inter-partition test shows that the model has a robust income and risk control ability in bull-bear conversion, unilateral downside and structural bull market, and the sortino ratio-Kumar ratio advantage highlights its precise management of downside risks.

Keywords

Multi-factor Quantitative Investment; Risk-return; Collaborative Optimization; Dynamic IC_IR Weighting.

1. Introduction

Quantitative investment is not a linear game of "the more complex the model, the higher the income". In practice, the stability and risk control ability of strategic income often determine the long-term vitality of strategy more than the absolute income level [1]. Especially in the environment of intense market style rotation and frequent black swan events, the portfolio that simply pursues the maximization of income is extremely vulnerable to the fatal blow of tail risk. Therefore, how to build a portfolio optimization mechanism with dynamic synergy between risk and return within a multi-factor framework, instead of splitting them into a master-slave relationship of "constraint conditions" and "objective functions" [2], has become a key proposition that needs to be broken through in current quantitative investment research.

The classical mean-variance model lays the foundation of modern portfolio theory, but it is extremely sensitive to input parameters, and a small estimation deviation can lead to a violent shock in weight distribution. Black-Litterman model alleviates the sensitivity of parameters by integrating investors' subjective views, but its ability to model the complex correlation between multiple factors is still limited [3]. The risk parity model redefines the logic of asset allocation

from the perspective of risk contribution equalization. However, this method lacks the explicit inclusion of income expectation and is prone to income sacrifice in the bull market environment [4]. In recent years, dynamic weighting methods based on machine learning have gradually emerged, such as the regularization method proposed by Doukali M to optimize large-scale portfolio under the condition of asymmetric income and risk attitude [5]; Liu Y and others put forward the instrumental principal component analysis factor model of China stock option return [6]. Although these methods improve the adaptability, most of them still optimize the risk and return as independent modules, lacking the coupling and collaborative design of them. In the multi-factor field, Barra risk model and Aptos multi-factor framework provide standardized tools for factor risk decomposition [7], but the exploration of the unified objective function of "risk-return synergy" in their combination optimization level is still insufficient.

The existing research is mature in the extreme optimization of single dimension, but there is still an obvious method gap in the core link of collaborative coupling optimization of risk and benefit, which is the starting point of this paper. This paper focuses on the multi-factor quantitative portfolio optimization scenario, and proposes a multi-factor quantitative portfolio risk-return collaborative optimization model. Core innovations include:

(1) Build a risk-benefit synergy objective function. Breaking through the traditional mean-variance binary opposition paradigm of "maximizing returns+minimizing risks", a synergistic gain term is introduced to bring the risk-adjusted excess returns and risk budget into a unified optimization framework, so that the model can actively seek the positive contribution of risk dimension while pursuing returns.

(2) Design a dynamic IC-IR time-varying weighting mechanism. Based on the rolling window factor information coefficient (IC) and information ratio (IR), an adaptive adjustment module for factor weights is constructed to enable the model to perceive market style preferences in real time and dynamically adjust factor exposure directions.

(3) Integration of collaborative constraints and risk management module. Multi-dimensional constraints such as industry deviation, upper limit of individual stock weight and maximum retracement are embedded in the portfolio level, and the robustness of high-dimensional covariance matrix is improved by Ledoit-Wolf compression estimation.

(4) The end-to-end architecture is connected with the executive layer. The transaction cost model is embedded in the optimization closed loop, so that the output weight directly reflects the executable optimal solution, rather than just staying at the theoretical level.

2. Method Model

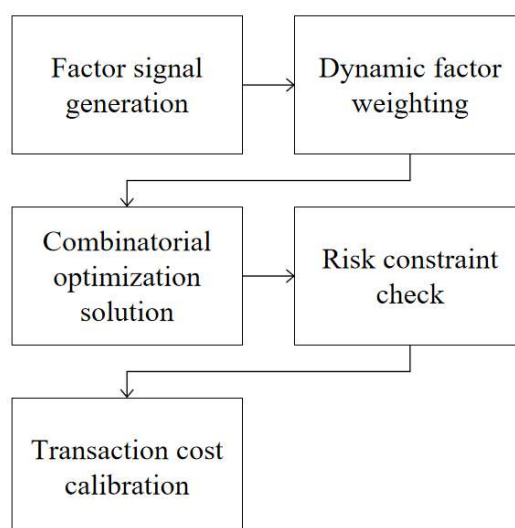


Figure 1. Overall process of model

The model adopts a five-tier end-to-end architecture (Figure 1). Each layer is connected in series by rolling window mechanism, and the whole process is re-executed every warehouse adjustment day to ensure that the model can continuously perceive the changes of market style and adjust itself adaptively.

2.1. Risk-benefit Cooperative Objective Function

The traditional mean-variance framework regards the maximization of income and the minimization of risk as two independent goals, which is essentially a serial logic of "income first, risk later" [8]. The core breakthrough of this model is to incorporate the two into the same objective function and introduce the synergistic gain term, so that the risk dimension can actively contribute value in the optimization process.

Suppose there are n stocks in the portfolio, and the weight vector is $\mathbf{w} \in R^n$, which satisfies $\sum_{i=1}^n w_i = 1$. The objective function of collaborative optimization is defined as follows:

$$\max_{\mathbf{w}} U(\mathbf{w}) = \underbrace{\boldsymbol{\mu}^T \mathbf{w}}_{\text{Prospective earnings}} - \frac{\lambda}{2} \underbrace{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}}_{\text{Combined variance}} + \underbrace{\gamma \phi(\mathbf{w}, \boldsymbol{\Sigma})}_{\text{Collaborative gain term}} \quad (1)$$

Among them: $\boldsymbol{\mu} \in R^n$ is the expected excess return vector of individual stocks, which is synthesized by multiple factors; $\boldsymbol{\Sigma} \in R^{n \times n}$ is the covariance matrix of individual stock return rate, which is estimated by Ledoit-Wolf compression; $\lambda > 0$ is the risk aversion coefficient, which controls investors' tolerance to risk; $\gamma \geq 0$ is the synergy strength coefficient, which controls the positive contribution weight of risk dimension; $\phi(\mathbf{w}, \boldsymbol{\Sigma})$ is a synergistic gain function, which measures the added value brought by risk diversification.

The design of synergetic gain function ϕ is the key to distinguish this model from the traditional framework. This paper adopts the measurement method based on risk contribution entropy:

$$\phi(\mathbf{w}, \boldsymbol{\Sigma}) = - \sum_{i=1}^n p_i \ln p_i \quad (2)$$

Among them,

$$p_i = \frac{w_i (\boldsymbol{\Sigma} \mathbf{w})}{\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}} \quad (3)$$

p_i represents the contribution of the i stock to the total portfolio risk [9]. When the risk contribution of each stock tends to be balanced, the entropy value increases and the synergistic gain term rises, and the model will actively reward diversification, which means that the model not only passively "controls" the risk, but also actively "uses" the robust dividend brought by risk diversification.

Compared with the traditional mean-variance framework, which only contains the first two terms, this objective function upgrades the risk diversification from "constraint" to "positive incentive" through the $\gamma \phi$ term, and realizes the collaborative optimization of risk and return rather than the separation of master and slave.

2.2. Dynamic IC_IR Time-varying Weighting Mechanism

The effectiveness of the factor is not constant, and the predictive ability of the factor may decline sharply when the market style switches [10]. In this model, the rolling window is used to calculate the IC and IR (IC_IR) of each factor, and the factor weights are dynamically adjusted accordingly.

Step 1: Calculate the factor IC sequence.

On each position adjustment day t , calculate the cross-section IC value of the factor f :

$$IC_{f,t} = \text{Corr}(\text{FactorExposure}_{f,t}, \text{ForwardReturn}_{t+1}) \quad (4)$$

That is, the cross-sectional correlation coefficient between the factor exposure vector and the next stock return vector [11].

Step 2: calculate IC_IR.

$$ICIR_{f,t} = \frac{\overline{IC}_{f,t-L+1:t}}{\sigma(IC_{f,t-L+1:t})} \quad (5)$$

Where L is the length of the rolling window, the numerator is the average value of IC in the window, and the denominator is the standard deviation of IC.

Step 3: Determine the dynamic weight of factors.

$$\eta_{f,t} = \frac{\max(ICIR_{f,t}, 0)}{\sum_g \max(ICIR_{g,t}, 0)} \quad (6)$$

Only the factors with positive ICIR are retained, that is, the factors with stable prediction ability in history, and the weights are allocated according to the ICIR ratio [12]. When the ICIR of a certain factor drops below zero, the model automatically eliminates or reduces its weight, which is equivalent to installing a "style awareness radar" for the model, so that it can quickly adjust the exposure direction of the factor when the market style switches.

2.3. Improved Covariance Matrix of Ledoit-Wolf Compression Estimation

Estimation of high-dimensional covariance matrix is a classical problem in combinatorial optimization. When the number of stocks n approaches or exceeds the length of time series T , the sample covariance matrix presents a serious deviation, and a lot of noise is mistaken for signals, which leads to the over-dependence of optimization weights on false correlation structures.

The model adopts the Ledoit-Wolf compression estimation method:

$$\Sigma_{\text{shrink}} = \alpha \mathbf{F} + (1 - \alpha) \mathbf{S} \quad (7)$$

Where: $\mathbf{S}$ is the sample covariance matrix, $\mathbf{F}$ is the target matrix, and $\alpha \in [0, 1]$ is the compression strength coefficient, which is determined by data-driven optimization.

The essence of compression estimation is to find the optimal balance between deviation and variance [13-14]. The optimal value of α is solved analytically by minimizing the mean square error between the estimator and the true covariance matrix, without subjective setting.

Compared with the sample covariance matrix, compressed estimation significantly improves the robustness of the model in the following two aspects. When $n > T$, the matrix can still be positive definite and the optimization problem can be solved; Greatly reduce the noise of extreme correlation estimation and avoid the excessive concentration of portfolio weight on a few stocks.

2.4. Collaborative Constraint and Risk Management Module

In addition to the objective function, the model embeds the following hard constraints to ensure that the solution results are feasible in practice:

(1) the upper and lower weight constraints

$$w_{\min} \leq w_i \leq w_{\max}, \forall i \quad (8)$$

Usually set $w_{\min} = 0$ (no short selling) and $w_{\max} = 5\%$ or 10% .

(2) Industry deviation constraint

$$|\mathbf{h}^T \mathbf{w} - h_{\text{bench}}| \leq \delta_h \quad (9)$$

Where $\mathbf{h}$ is the industry attribution vector, h_{bench} is the industry weight in the benchmark index, and δ_h is the maximum allowable deviation range.

(3) Maximum withdrawal warning constraint

Conditional constraints are introduced into rolling optimization. If the combined withdrawal of K period exceeds the threshold $MDD_{\max}$ in the past, the risk exposure will be automatically lowered, that is, λ will be increased or the position will be lowered.

2.5. End-to-end Transaction Cost Embedding

Traditional models often consider transaction costs after optimization, which leads to systematic deviation between theoretical weight and actual execution weight [15]. In this model, the transaction cost is directly embedded into the optimization objective function, so that the output weight is the executable optimal solution.

Let the previous position weight be $\mathbf{w}_{t-1}$ and the current optimization weight be $\mathbf{w}_t$, then the turnover rate is $\tau = \sum_i |w_{t,i} - w_{t-1,i}|$. After considering the transaction cost, the revised objective function is:

$$\tilde{U}(\mathbf{w}_t) = U(\mathbf{w}_t) - \kappa \sum_{i=1}^n c_i |w_{t,i} - w_{t-1,i}| \quad (10)$$

Where c_i is the unit transaction cost of stock i and κ is the cost penalty coefficient.

The advantage of this design is that when the optimization algorithm tries to adjust the position significantly, the transaction cost will automatically restrain excessive trading in the form of penalty term, so that the model can achieve an endogenous balance between "capturing new signals" and "saving transaction costs".

3. Experimental Verification

3.1. Data and Factor Setting

This paper takes China A-share market as the back test object, and the sample interval is from January 2015 to December 2025, with a total of 11 years. The stock pool excludes ST, *ST and stocks listed for less than 60 trading days. The data frequency is monthly, and positions are adjusted at the end of each month. Data sources include Wind and CSMAR databases. The factor system covers five dimensions, including 12 basic factors (as shown in Table 1).

Table 1. Factor system setting

Factor category	Specific factors	Calculation Instructions
Valuation	Book to market ratio, inverse P/E ratio	Seasonal frequency update, retrieve the latest financial report data
Growth	Revenue growth rate, net profit growth rate	Seasonal frequency year-on-year growth rate
Quality	ROE, total assets turnover rate, operating cash flow/total assets	Seasonal frequency data
Momentum	Accumulated income over the past 12 months (excluding the most recent month)	Monthly frequency calculation
Low volatility	Standard deviation of daily returns over the past 12 months	Monthly frequency calculation

Each factor is subjected to MAD (Mean Absolute Deviation) extreme value processing in each cross-sectional period, and market value and industry neutrality are implemented, finally converted into Z-score standardization. The factor synthesis adopts a rolling 24 month IC-IR dynamic weighting mechanism, and the factor weights are recalculated for each period.

3.2. Backtesting Results

Table 2 reports the overall performance of the equal weight combination (EW), mean variance optimization (MVO), IC-IR equal weight combination, and collaborative optimization model (Hereinafter referred to as "CO model") during the sample period from January 2015 to December 2025.

Table 2. Comparison of overall performance of various models (2015.01-2025.12)

Indicator	EW	MVO	IC_IR equal weight	CO model
Annual rate of return (%)	11.24	14.67	16.32	18.75
Annualized volatility (%)	23.41	18.92	19.87	16.34
Sharpe Ratio	0.41	0.67	0.72	1.02
Maximum drawdown (%)	-48.32	-32.15	-34.67	-22.48
Sortino ratio	0.52	0.89	0.95	1.38
Calmar Ratio	0.19	0.39	0.41	0.74
Annualized excess return (%)	—	3.43	5.08	7.51
IR	—	0.68	0.89	1.42
Monthly average turnover rate (%)	8.33	35.67	28.94	22.16

Note: Both excess returns and IR are based on the Shanghai and Shenzhen 300 Index.

The annualized rate of return of CO model is 18.75%, which is about 4.08 percentage points higher than that of MVO model and 2.43 percentage points higher than that of IC_IR equal

weight combination. This gap stems from the role of the $\gamma\phi(\mathbf{w}, \Sigma)$ term in the collaborative objective function. When the portfolio risk contribution tends to be dispersed, the collaborative gain term actively rewards the model, so that it does not spontaneously concentrate its risk exposure while chasing high-yield stocks, thus gaining a more stable accumulation at the income end.

The annualized volatility of the CO model is the lowest among the four groups, and the maximum retracement is nearly 10 percentage points lower than that of the MVO model. This improvement can be attributed to two aspects: first, the Ledoit-Wolf compressed estimation reduces the estimation noise of covariance matrix, so that the optimizer will not misjudge the false correlation in the sample as the real risk structure; Secondly, the positive incentive of the synergistic gain term to the risk contribution entropy leads to a more balanced risk distribution among stocks and avoids the risk of individual stocks leading the portfolio.

Sharp ratio of CO model is superior to MVO and IC_IR, and sortino ratio and Kama ratio are also the first. It is worth noting that the difference between Sotino's ratio and Sharp's ratio of CO model is smaller than that of other models, which indicates that CO model is more effective in controlling downside risks than upside fluctuations.

The average monthly turnover rate of CO model is lower than MVO and slightly lower than IC_IR. This is because the transaction cost term $\kappa \sum_i c_i |w_{t,i} - w_{t-1,i}|$ is directly embedded in the optimization objective, which makes the model endogenous to the turnover constraint when pursuing the optimal risk-return allocation, and avoids the common defect of drastic fluctuation of weight under the traditional MVO framework.

In order to further investigate the performance of the models in extreme market environment, Table 3 reports the performance decomposition of each model in three typical subintervals.

Table 3. Performance comparison among different model partitions (annualized rate of return/maximum retracement,%)

Interval	Market characteristics	EW	MVO	IC_IR equal weight	CO model
2015.01–2016.12	Bull Bear Conversion	5.32 / -48.32	8.91 / -32.15	10.24 / -34.67	12.87 / -22.48
2018.01–2018.12	Unilateral downward trend	-22.47 / -32.18	-16.83 / -25.43	-14.92 / -26.71	-10.56 / -19.34
2019.01–2021.12	Structural bull market	22.18 / -15.67	26.34 / -12.89	28.71 / -14.23	32.46 / -10.82

In the unilateral downward market in 2018, the annualized loss of the CO model is -10.56%, which is about 6.27 percentage points less than that of the MVO model, and the maximum retracement is also significantly narrowed. This shows that the dynamic IC_IR weighting mechanism can quickly reduce the weight of failure factors in the market downturn, and avoid excessive exposure of portfolios to factors that have lost their predictive ability. In the structural bull market in 2019-2021, the CO model captured the upward income, and the maximum retreat was controlled at -10.82%, which reflected the asymmetric advantage of the collaborative optimization framework of "upward participation and downward protection".

3.3. Robustness Test

In order to ensure that the above results are not caused by specific parameter selection or sample interval, this paper implements three robustness tests:

(1) Sensitivity analysis of different backtesting windows.

The starting point of backtesting was moved forward to January 2010 and then to January 2017, respectively. The Sharp ratio of the CO model was 0.98 and 1.04, and the maximum retracement was -23.71% and -21.82%, respectively, which was not significantly different from the main results, indicating that the model was insensitive to the selection of sample intervals.

(2) Comparison of different compression estimation methods.

The linear compression of Ledoit-Wolf is replaced by nonlinear compression estimation and sample covariance matrix, and the Sharp ratio of the CO model is 0.98 and 0.79 respectively. The nonlinear compression results are close to the main results, while the sample covariance results deteriorate significantly, which confirms the key role of compression estimation in improving the robustness of high-dimensional covariance matrix.

(3) Sensitivity of different synergistic strength coefficients γ .

Adjusting γ from the benchmark value of 0.5 to 0.2 and 0.8, respectively, the Sharp ratio of the CO model is 0.87 and 0.96, respectively, which are both above the MVO model. When it is $\gamma = 0$, the Sharp ratio drops to 0.67, which further verifies the independent contribution of synergistic gain term to performance improvement.

3.4. Dynamic Evolution of Factor Weight and Analysis of Cumulative Income Curve

In order to visually show the actual operation effect of the dynamic IC_IR weighting mechanism, Figure 2 draws the time series evolution of the weight of each factor during the sample period.

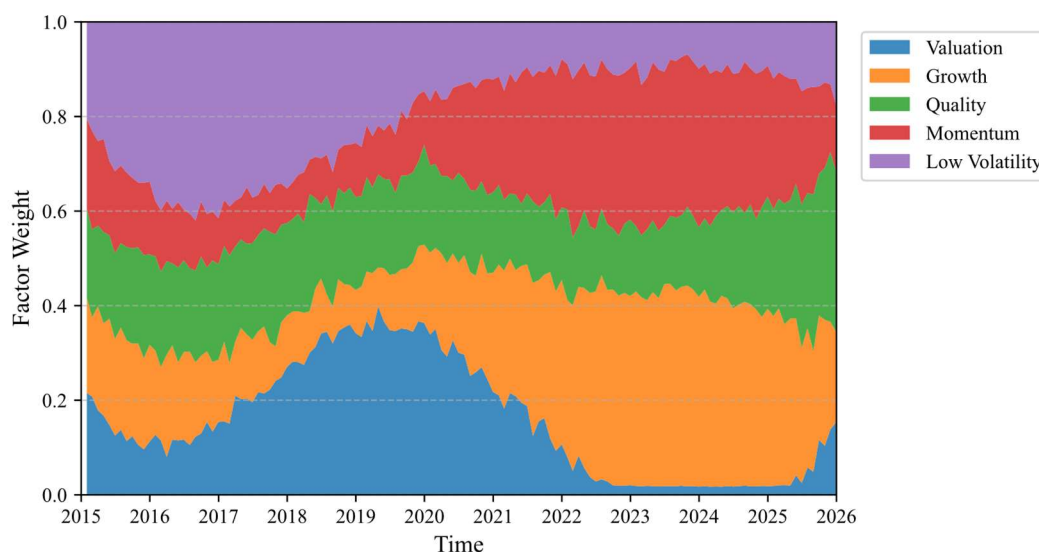


Figure 2. Dynamic weight evolution of each factor (2015–2025)

From Figure 2, it can be observed that during the bull-bear transition period in 2015-2016, the weight of low volatility factors increased significantly, reflecting that the model actively tilted towards defensive factors when market volatility intensified; During the dominant period of value style in 2017-2018, the weight of valuation factors continued to rise; During the dominant period of growth style from 2019 to 2021, the weight of growth and momentum factors dominated; Since 2022, the weight of quality factors has steadily increased. This dynamic evolution track shows that the time-varying weighting mechanism of IC_IR enables the model to perceive the switching of market style and adaptively adjust the exposure direction of factors. Figure 3 shows the cumulative net value curve of each model during the sample period. Based on January 2015, the initial net value is normalized to 1.

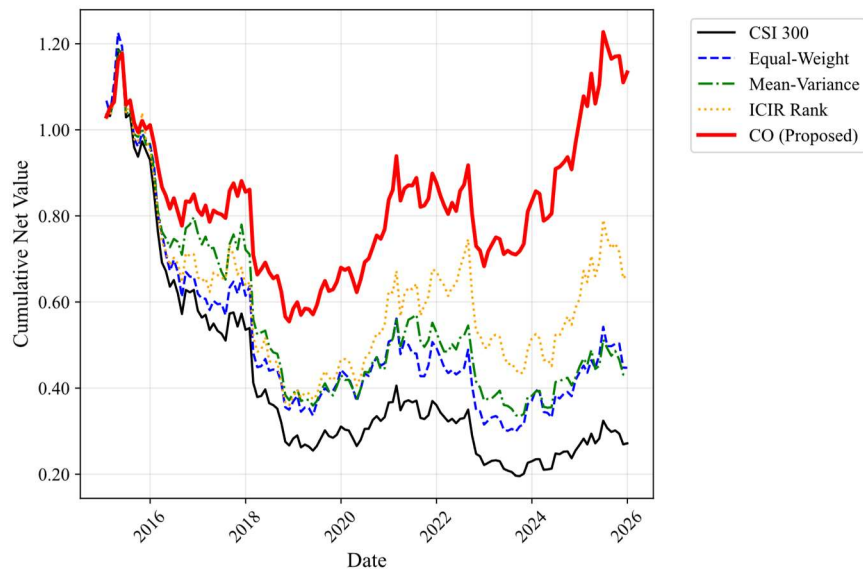


Figure 3. Cumulative net value curve of each model (January 2015–December 2025)

The cumulative net value curve of the CO model shows two remarkable characteristics: (1) In most market environments, it continuously outperforms three sets of benchmarks, and the excess returns accumulate monotonously over time instead of mean regression; (2) In the second half of 2015, the whole year of 2018 and 2022, when the market fluctuated greatly, the retracement range of the curve of the CO model was significantly smaller than that of other models, and the recovery speed was faster. This further proves that the collaborative optimization framework has achieved positive contributions in both revenue and risk dimensions, rather than sacrificing revenue for risk control.

4. Conclusion

The multi-factor quantitative portfolio risk-return CO model proposed in this paper is superior to benchmark methods such as equal weight portfolio, traditional mean-variance optimization and IC_IR factor scoring stock selection in core performance indicators such as annualized rate of return, Sharp ratio and maximum retracement. Inter-partition test and a number of robustness tests further confirmed the robustness of the model performance, indicating that the organic combination of the three core modules of collaborative objective function, dynamic IC_IR weighting and Ledoit-Wolf compression estimation can improve the risk-adjusted income of multi-factor quantitative combination.

References

- [1] Pang, T., Tian, D., & Tian, W. (2026). Optimal portfolio choice with comfortable consumption. *SIAM Journal on Financial Mathematics*, 17(3), 801–840. <https://doi.org/10.1137/25M1732222>
- [2] Bouteska, A., Harasheh, M., & Esposito, G. (2025). Climate transition risk, ESG rating divergence and portfolio performance: Evidence from composite scores and climate-adjusted factor models. *International Journal of Finance & Economics*, 31(2), 2208–2228. <https://doi.org/10.1002/ijfe.70038>
- [3] Găman, Ș., Tak, R., & Jheng, S. L. (2026). ESG-aware asset selection and portfolio optimization: A comparative study of multi-criteria decision-making methods. *Proceedings of the International Conference on Business Excellence*, 20(1), 133–147. <https://doi.org/10.2478/picbe-2026-0016>
- [4] Gong, Z., & Feng, J. (2025). A novel multifactor type-2 fuzzy time series model based on improved fuzzy C-means algorithm and justifiable granularity for stock index forecasting. *Soft Computing*, 29(7), 1–14. <https://doi.org/10.1007/s00500-025-10651-7>

- [5] Doukali, M., Kone, N., & Taamouti, A. (2026). A regularization approach to optimizing large portfolios under asymmetries in returns and risk attitudes. *Econometric Reviews*, 45(6), 826–855. <https://doi.org/10.1080/07474938.2026.2621658>
- [6] Liu, Y., Zhou, H., & Li, X. (2025). An instrumented principal component analysis factor model for Chinese equity options returns. *Accounting & Finance*, 65(5), 4370–4390. <https://doi.org/10.1111/acfi.70082>
- [7] Hafeez, A., Jagirani, S. T., & Hunt, K. A. (2025). How stock prices behave in response to institutional development: A four-factor asset pricing model. *The World Economy*, 48(4), 847–861. <https://doi.org/10.1111/twec.13676>
- [8] Rysz, M. (2026). Network portfolio optimization with dynamic lower bounds on asset weights. *European Journal of Operational Research*, 335(1), 216–227. <https://doi.org/10.1016/j.ejor.2026.02.020>
- [9] Ansgar, S. (2024). Are minimum variance portfolios in multi-factor models long in low-beta assets? *Mathematics and Financial Economics*, 18(1), 151–170. <https://doi.org/10.1007/s11579-024-00366-y>
- [10] Jin, X. (2024). Extreme risk connectedness in China's stock market: Fresh insights from time-varying general dynamic factor models. *Computational Economics*, 66(3), 1–33. <https://doi.org/10.1007/s10614-024-10779-y>
- [11] Kim, I. S. (2024). Unveiling mispricing risks: Nonlarge homogeneous portfolio factor copula models for enhanced valuation of subordinated loan securitization. *Journal of Futures Markets*, 44(10), 1710–1732. <https://doi.org/10.1002/fut.22535>
- [12] Fletcher, J. (2024). An examination of linear factor models in U.K. stock returns in the presence of dynamic trading. *Review of Quantitative Finance and Accounting*, 63(3), 1121–1147. <https://doi.org/10.1007/s11156-024-01286-0>
- [13] Yao, H., Xia, S., & Liu, H. (2024). Return predictability via an long short-term memory-based cross-section factor model: Evidence from Chinese stock market. *Journal of Forecasting*, 43(6), 1770–1794. <https://doi.org/10.1002/for.3096>
- [14] Dang, M. D., & Chen, C. (2026). Multiperiod mean-buffered probability of exceedance in defined contribution portfolio optimization. *SIAM Journal on Financial Mathematics*, 17(3), 1220–1278. <https://doi.org/10.1137/25M1764773>
- [15] Chen, T., Y. Q., & L, X. (2023). Application of DeepForest-CQP multi-factor model in quantitative stock selection. *Journal of Intelligent & Fuzzy Systems*, 44(3), 5425–5436. <https://doi.org/10.3233/jifs-222328>