Consideration of Harmonic Resonance Analysis and Harmonic Load Analysis for Harmonic Gearbox Modal Analysis

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Abstract

As a key component of industrial robots, the study of the dynamic characteristics of harmonic gear reducers and the determination of resonance positions are of significant importance. To avoid the occurrence of resonance problems in the operation of harmonic gear reducers, this study conducted modal analysis and harmonic response analysis on the harmonic gear reducer. Particularly, considering that the flexible wheel is the part of the reducer most likely to fail, it is the key component of vibration analysis. Here, we selected the SHG-25100 harmonic gear reducer as the research object and constructed its three-dimensional model under load as well as the finite element analysis model after the cam assembly bearing deformation. Considering its periodic input characteristics, the finite element method was used to conduct harmonic response analysis under constant torque. The acceleration response curve of the flexible wheel and rigid wheel contact surface in the x, y, and z directions, as well as key dynamic parameters such as natural frequency and response amplitude, were successfully obtained. The results indicate that the reducer exhibits significant vibration in the y direction at the resonance frequency of 95.5 Hz, and vibration in the x and y directions at the resonance frequencies of 40 Hz and 45 Hz respectively. When operating the reducer, these resonance frequencies should be avoided.

Keywords

Harmonic Gear Reducer, Finite Element Method, Harmonic Response Analysis, Modal Analysis.

1. Introduction

In the field of industrial robotics, the harmonic gear reducer is the most critical component in the robot joints. Furthermore, harmonic gear reducers possess the advantages of large transmission ratio and wide range, simple structure, small size, light weight, high transmission accuracy, smooth transmission, and high transmission efficiency. They are widely applicable in fields such as aerospace, automobile manufacturing, part processing, and humanoid robots, offering unmatched advantages and a broad prospect. Harmonic gear reducers have the following advantages: simple structure, few components, small size, and light weight. Compared to conventional gearboxes with the same reduction ratio, the parts of this device are reduced by approximately 50%, the volume and weight are reduced by more than 1/3. Its high reduction ratio and wide range of applications make it capable of carrying high loads and transmitting power accurately and quietly. The transmission is also highly stable due to the simultaneous engagement of multiple teeth and the low surface speed of the teeth. The device is suitable for transmission in high-vacuum, corrosive, and other special environments. It operates efficiently even under large reduction ratios, thanks to its high transmission efficiency. The performance of the industrial robot depends heavily on the harmonic gear reduction, and

as the requirements for precision in robot positioning and grasping are raised, the demands for the precision of the harmonic gear reduction are also increased. Among these, the resonance issue of the harmonic gear reduction has the most significant impact on the precision of the robot. Given the high reduction ratio and extensive working speed range of the harmonic gear reduction, conducting an analysis on the harmonic response in the vibration frequency zone is especially crucial. Ma J, Li C, Luo Y. study investigates the time-varying stiffness of harmonic drive gears by analyzing meshing teeth and torque variations, establishing a mathematical model and calculation method for torque under different stiffness, and validating the findings through experiments, offering theoretical support for analyzing load distribution and stress-strain relationships in flexible wheels[1]. Huang D, Zong P, Gu J. This study details a torsional vibration test method and signal analysis for harmonic reducers in industrial robots, assessing mechanical performance through characteristic frequency identification and defect diagnosis, and highlights the significance of these methods for improving online quality assurance in harmonic reducers. [2]. In the field of harmonic gear reducer's inherent characteristics study, Folega established finite element analysis models for various structural forms of harmonic gear reducers, and performed modal analysis on them [3].

Piotr Folęga and colleagues proposed a novel dynamic harmonic drive model that considers the nonlinear changes in stiffness and damping, and conducted research on this model in the Matlab-Simulink environment[4]. Jieliang Zhao studied the coupled vibration of a harmonic drive motor and flexible arm in a mechanical arm system, finding that the frequency of vibration significantly increases with an increase in the motor speed, indicating that a large range of motion can enhance system stiffness and provide a new approach for effective research on the coupled vibration of mechanical systems[5]. Chuang established a model that considers geometric shape, internal interactions, and assembly errors, revealing the dynamic behavior of harmonic reduction gears[6]. Xian Zhang and colleagues established a harmonic drive model that considers non-linear stiffness, motion errors, and friction. Experimental validation of the model was conducted, and the influence of different component stiffness on system dynamic response was analyzed. The results indicate that, at high speeds, an increase in component stiffness will decrease dynamic transmission precision. Specifically, the radial stiffness of the bearing has the most significant impact on system response [7]. Folęga P, Wojnar G, Burdzik R. study employs finite element modal analysis to investigate the vibration and transmission characteristics of RV reducers, analyzing natural frequencies and vibration modes of key components, validating simulation accuracy through experimental tests, and offering insights for the design and optimization of RV reducers in precision transmissions[8].Ya Chen through analyzing the impact of flexible bearing clearance on the reverse clearance of harmonic gear reducers, and establishing vibration equations, revealed its significant influence on natural frequency and nonlinear characteristics[9]. Rukang Hu, established a nonlinear torsional dynamic model considering nonlinear torsional stiffness and transmission errors in harmonic gear reducers, analyzed the impact of factors such as rotational speed, inertia, and torsional stiffness on vibration, and provided theoretical basis for optimization design and vibration control[10]. Xu T, Tan J. study investigates the impact response of harmonic reducers on positioning and trajectory accuracy in industrial robots by analyzing and experimentally verifying the torsional vibration model with periodic variable stiffness, providing foundational data for optimizing harmonic reducer applications.[11]. Current research primarily focuses on modal analysis of single components, while the analysis of harmonic response of complete machinery harmonic gear reducers is insufficient. This paper, using the SHG25-100 type harmonic gear reducer as an example, conducts harmonic response analysis using the ANSYS finite element software, obtaining the amplitude-frequency response curves and resonance frequency of key components under different frequencies. In addition, it explores the influence of the rated speed on the accuracy of gear transmission of the whole machine by examining the

coupling wheel. The analysis provides theoretical basis for the structural design of the gear reducer, the setting of operating conditions, and the optimization of transmission accuracy.

2. Harmonic Gear Reducer Structure

2.1. Structural and Transmission Principles

A harmonic gear reducer is a device that transmits power through periodic elastic deformation of a flexible wheel and meshing with a rigid wheel. Based on the input-output selection of its three basic transmission components, it can form four forms: acceleration, deceleration, differential, and uniform speed. As shown in Figure 1, the transmission principle of the harmonic gear reducer is demonstrated. The wave generator causes the flexible wheel to exhibit theoretical deformation, which then initiates initial meshing with the rigid wheel. In contrast, the rotation of the wave generator causes the flexible wheel to rotate counterclockwise.The teeth of the flexible rotor at the long axis position form a meshing zone with the teeth of the rigid rotor, while the teeth at the short axis position of the flexible rotor separate from the teeth of the rigid rotor, forming an unmated zone. The transition state lies between the two. In this case, the SHG25-100 flexible rotor is fixed and driven by the acoustic source. The rigid rotor is the output.

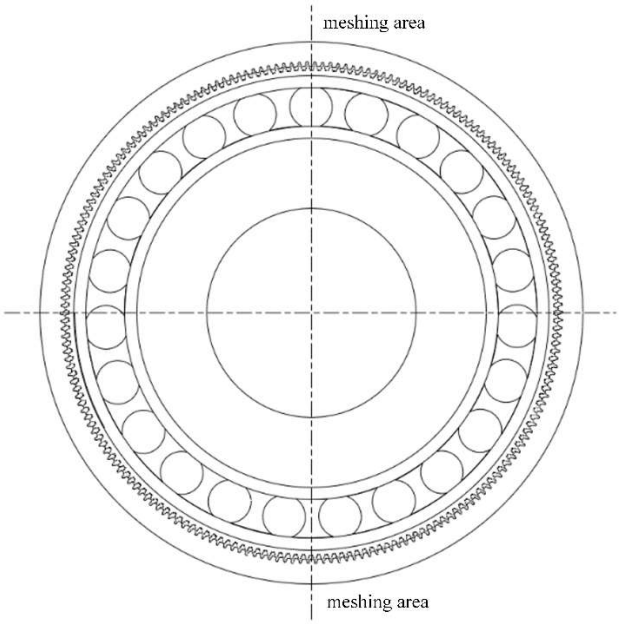
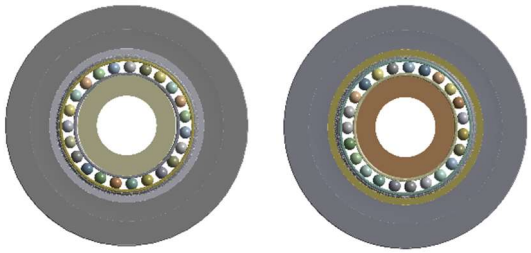


Figure 1. Schematic diagram of harmonic gear transmission

2.2. Establishment of Harmonic Reduction Gear Assembly Model

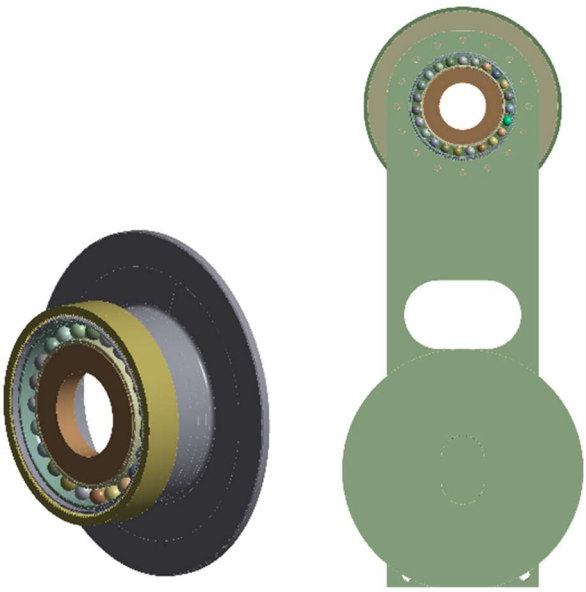
When constructing the harmonic gear model, all elements that do not involve the transmission structure should be removed, such as bearing retainer, fixed bolt hole, shell chamfer, and other complex features. At the same time, it is essential to ensure that these simplifications do not affect the final finite element analysis results. When assembling the harmonic gear in three-dimensional software, special attention should be paid to the installation depth of the wave generator and the possible interference of the cam during the assembly process. To ensure accurate calculation, a static analysis should be conducted in the workbench first, eliminating cam interference and forming a frictionless flexible wheel and flexible bearing assembly. The comparison between the figures 2.a and 2.b illustrates this.



(a) (b)

Figure 2. shows the assembly deformation of the harmonic gear reducer (a. without deformation b. with deformation).

To facilitate finite element analysis, the assembly model should be saved in step format. To improve the accuracy of rotating inertia analysis during the operation of a harmonic gear reducer, a complete three-dimensional model that includes the load and gear housing is required. Figure 3-a presents the harmonic gear reducer model without any load, while Figure 3-b shows the model with a load.



(a) (b)

Figure 3. presents the three-dimensional model (a. without load b. with load)

2.3. Finite Element Model Construction

The model is imported into ANSYS for automatic meshing, which enables an adaptive approach that both adjusts accuracy and maintains computational efficiency. However, this approach also presents challenges such as deformation on high-curvature surfaces and the potential for "tumbling" phenomena. To address these issues, we have adopted a combination of adaptive meshing and local refinement strategies. We have applied the Size and Refine methods for thread wheels and important shaft components to ensure proper refinement. Considering that dense mesh partitioning will increase computation load, while the main impact of load on the rotating inertia, the load mesh partition size can be appropriately increased. Finally, the finite element analysis model of the harmonic reducer with load includes 1001861 units and 1597594 nodes, as shown in Figure 4.

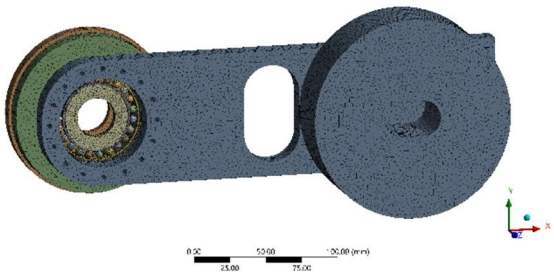


Figure 4. shows the finite element mesh partitioning for the harmonic gear reducer

3. Harmonic Response Analysis

3.1. Harmonic Response Analysis Theory

Based on rotor dynamics and finite element theory, we obtain the fundamental dynamic equations for harmonic response analysis structure:

$$M\ddot{u} + C\dot{u} + Ku = F \tag{1}$$

In the above equation, M structural quality matrix; C structural damping matrix; K structural stiffness matrix; $\ddot{u}$ the node acceleration vector; $\dot{u}$ node velocity vector; u node displacement vector; F vectorial force.

For the stable response induced by forced vibration, this study found that all nodes within the structure vibrate at a consistent frequency. Considering the influence of damping, the phase between different nodes may differ. Accordingly, the expression for nodal displacement can be described as follows:

$$u = u_{\max} e^{i\phi} e^{i\omega t} \tag{2}$$

In the equation, $u_{\max}$ the displacement amplitude; i imaginary numbers; ω rotational frequency ,rad/s; t time; ϕ the phase angle for displacement is rad; $e^{i\omega t}$, $e^{i\phi}$ the complex expression form of a simple harmonic vibration.

$$e^{i\phi} = \cos\phi + i\sin\phi \tag{3}$$

$$e^{i\omega t} = \cos\omega t + i\sin\omega t \tag{4}$$

Equation (2) can be written as:

$$u = (u_1 + iu_2)e^{i\omega t} \tag{5}$$

In the equation, u_1 the real part of displacement., $u_1 = u_{max}cos\phi$; u_2 the imaginary part of displacement, $u_2 = u_{max}sin\phi$.

Similarly, the force vector can also be expressed as:

$$F = (F_1 + iF_2)e^{i\omega t} \tag{6}$$

In the equation, F_1 is the real part, $F_1 = F_{max}\ cos\phi$; F_2 force is imaginary, $F_2 = F_{max}\ sin\phi$; F_{max} amplitude of the force; ϕ force angle, rad.

Substituting (4) and (5) into (1), the harmonic response analysis dynamic equation is obtained as follows:

$$(K - \omega^2 M + i\omega C)(u_1 + iu_2) = F_1 + iF_2 \tag{7}$$

Based on finite element analysis, we adopt modal superposition method to solve for displacement. Considering its advantages in metal structures, such as low friction, good lubrication, and minimal impact on amplitude-frequency characteristics, we also prioritize model efficiency and computational efficiency.

Suppose that the modal superposition method extracts the modal, then displacement $\{u\}$ can be represented by modal coordinates, that is:

$$u = \sum_{i=1}^{n} \phi_i y_i \tag{8}$$

In the equation, ϕ_i modal Synthesis; y_i the modal coordinates in the i-th order mode.

In the modal coordinate system, the structural dynamics equation can be represented as:

$$\dot{y}_i + 2\omega_i \xi_i \dot{y}_i + \omega_i^2 y_i = F_i \tag{9}$$

In the equation, ω_i the first-order natural frequency; ξ_i the i-th order damping ratio. F_i the force in the i-th order modal coordinate.

$$F_i = F_{ic} e^{i\omega t} \tag{10}$$

$$y_i = y_{ic} e^{i\omega t} \tag{11}$$

In the equation, F_{ic} the modal force amplitude at the i-th order. y_{ic} the amplitude of the i-th order modal coordinate.

Substituting Eqs. (10) and (11) into Eq. (8) enables us to solve for the displacement:

$$y_{ic} = \frac{F_{ic}}{\omega_i^2 - \omega^2 + i(2\omega\omega_i\xi_i)} \tag{12}$$

As can be seen from the above calculations, obtaining the displacement response at each prescribed frequency only requires a single calculation, without the need for Lanczos iterations and convergence processes. However, a million-node number will increase computational load. To balance computation accuracy and efficiency, a suitable increment in load steps should be made. Compared to modal Lanczos method, which requires multiple iterations and takes a longer time, it is advisable to reduce the frequency range to minimize the number of iterations.

3.2. Contact Pair and Boundary Condition Definition

The selection of contact types significantly impacts the accuracy of the computational results. The harmonic response equations derived from linear theory are unsuitable for nonlinear contacts, only applicable for Bond and No separations contact types. Bonds, as a binding constraint, prevent relative motion between contacting surfaces, while No separations allow for relative motion. Therefore, components with relative motion, such as flexible bearings and balls, should be set as No separations; components without relative motion, such as load and the upper shell, should be set as Bonds. The soft wheel of SHG-25-100 is set as fixed. Power is transmitted through the cam. The applied axial force is set to 67 Nm, the actual operating condition. The material parameters are listed in Table 1.

Determining the input frequency spectrum is the core of establishing boundary conditions. The main purpose is to specify the frequency interval and step size of dynamic loads to meet the frequency analysis requirements and expand the analysis range as much as possible. Considering the node number, contact number, and computational scale, we set the frequency of dynamic loads to 0~2000 Hz and the analysis step size to 10 Hz, resulting in a total number of load steps of 200.

Table 1. Material Properties

Part	Elastic modulus(Gpa)	Poisson's ratio	Density(10^3kg/m^3)
Flexible Wheel	199	0.3	7
Rigid Wheel	130	0.29	7.3
Generator	208	0.3	7.5
Load	210	0.3	7.8

4. Response Results Analysis

Modal superposition method is a commonly used method for harmonic response analysis. It achieves this by combining the modal analysis-derived modal shapes with the coefficients. As the study object, we select the contact surface between the flexible wheel and rigid wheel. We analyze the frequency displacement and acceleration curves in the x, y, and z directions to obtain the maximum displacement and amplitude frequency curves, as shown in Figure 5.

The torsional stiffness of the harmonic gear reducer is divided into K1, K2, K3, each corresponding to different torques, twisting angles, and operating conditions. It is often customary to use K1 as the torsional stiffness value for calculating the natural frequency of the reducer. The following table 2 provides reference values for the torsional stiffnesses K1, K2, K3 in the SHG25-100 model:

Table 2. Torsional Rigidity Parameters

Torsional stiffness	K_1	K_2	K_3
(10^4Nm/rad)	3.1	5	5.7

According to the calculation method of intrinsic frequency in the HD Manual, we can deduce that:

$$f = \frac{1}{2\pi}\sqrt{\frac{K_1}{J}} \tag{13}$$

The natural frequency (f, unit Hz) of the harmonic gear reducer is determined by its torsional stiffness (K1, unit Nm/rad) and the load rotating inertia (J, unit kgm2). According to the HD manual, the torsional stiffness of the SHG25-100 model is 31000 Nm/rad, and the load rotating inertia measured by solidworks software is 0.09352 kg/m2. Based on equation (1), the first-order natural frequency of the SHG25-100 is 91.63 Hz. The other-order natural frequencies are not considered due to their being far beyond the working speed range of the harmonic gear reducer. The value is close to the harmonic response finite element analysis result of 95.5 Hz, validating the accuracy of the analysis. Secondly, through the HD manual equation (2), the corresponding rotational speed can be calculated. When the natural frequency f = 95.5 Hz, the calculated rotational speed N is 2865 r/min.

$$N = \frac{f}{2} \cdot 60 \tag{14}$$

Where f is the intrinsic frequency, and N is the input speed.

The calculation results indicate that the natural frequency N is 2865r/min, which is within the operating range of SHG25-100. To reduce resonance, it is necessary to pay special attention to avoid this speed range.

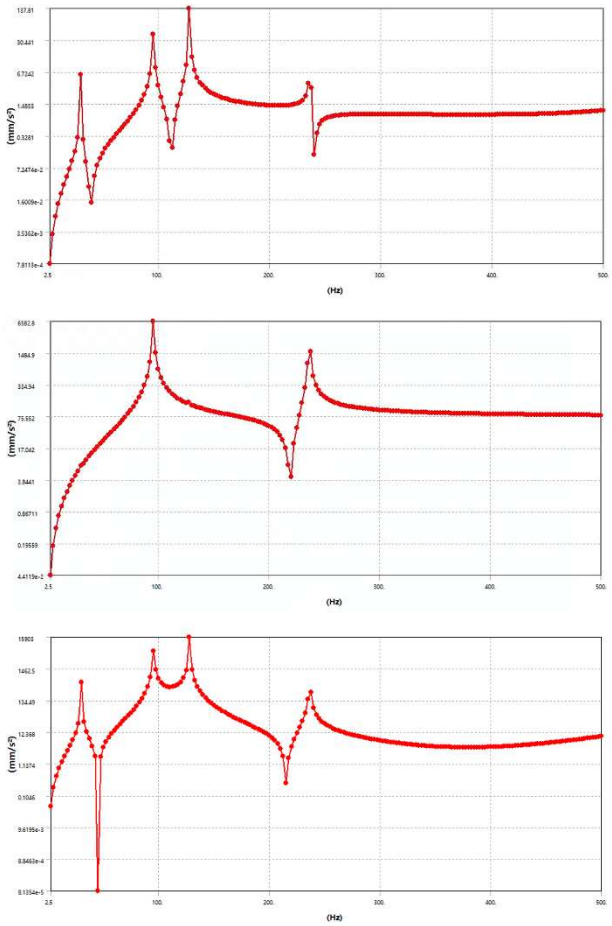


Figure 5. displays the Bode diagram of the acceleration of the flexible and rigid gear tooth contact surfaces (x, y, z)

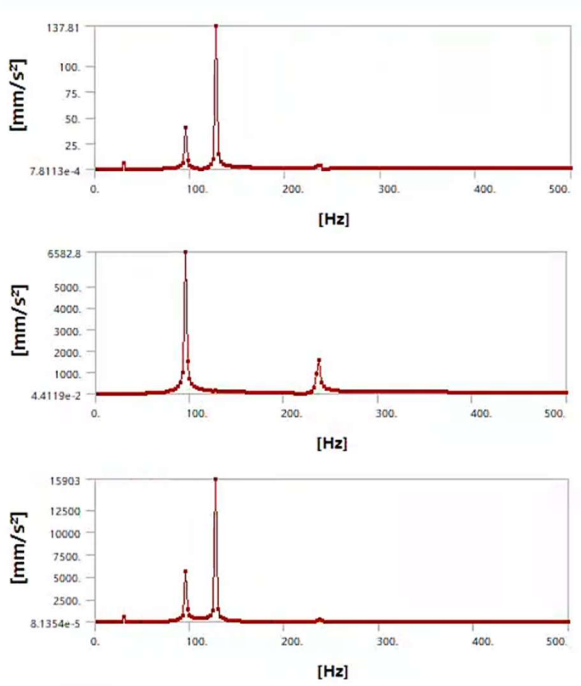


Figure 6. Frequency spectrum of the acceleration of the flexible wheel and rigid wheel contact point.

As shown in Figure 6, the contact surfaces of the flexible wheel and the rigid wheel during the operation of the whole machine exhibit significant vibration at the natural frequency of 95.5Hz. Among them, the y-axis represents the tangent direction of the harmonic gear reducer, which is the key direction of vibration during the operation of the gear reducer. By analyzing the vibration curves along the y-axis, it can be observed that the acceleration amplitude of vibration at 95.5Hz reaches 6582.8mm/s2, significantly higher than the amplitudes along the x and z directions. This is mainly because the harmonic gear reducer driven by the double wave generator not only generates one-harmonic vibration but also induces two-harmonic vibration, which becomes the dominant frequency of resonance. When the input speed reaches 2865r/min, the vibrations along the x and z directions primarily concentrate at 40Hz and 45Hz, approaching the input frequency one-time, while the y-axis vibrations primarily concentrate at 95.5Hz, which corresponds to the two-harmonic vibration of the harmonic gear reducer. Additionally, based on the experimental results of the previous work, it can be determined that the harmonic gear reducer two-harmonic vibration is the main source of vibration. Hence, it can be concluded that this harmonic response analysis is consistent with the experiment results.

5. Conclusion

The paper establishes the finite element analysis model of the cam follower installed in the bearing after assembly, as well as a three-dimensional model of the harmonic gear with load. The harmonic response analysis of the harmonic gear is conducted using the finite element method, which leads to the determination of the acceleration-frequency response curve and acceleration spectrum. The analysis of the results reveals that the angular acceleration value in the y direction is relatively large, and this is the two-fold frequency of the main vibration in the harmonic gear. Calculating from this frequency yields a rotational speed of 2865r/min. The x and z directions show the first appearance of vibration at 40Hz and 45Hz, respectively, corresponding to the one-fold frequency of the gear input rotational speed at 2865r/min. Thus, it is evident that to reduce vibration, people should avoid these rotational speed ranges.

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