

Obstacle Avoidance Trajectory Planning for SCARA Manipulators based on Analytical Kinematics and Improved APF

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Abstract

To address the issues of real-time performance and smoothness in obstacle avoidance trajectory planning for SCARA manipulators within complex industrial environments, a closed-loop planning scheme combining analytical kinematics, an improved Artificial Potential Field (APF) method, and Time-Optimal Trajectory Planning (TOTP) is proposed. Firstly, the forward and inverse kinematic models of the manipulator are established based on the D-H parameter method, utilizing closed-form analytical solutions to enhance spatial mapping efficiency. Secondly, aiming at the defects of traditional APF, such as trapping in local minima and goal unreachability, the repulsive field model is optimized by introducing a distance attenuation factor, and a safe geometric path is generated in Cartesian space combined with an obstacle expansion envelope strategy. On this basis, the TOTP algorithm is utilized to perform joint-space time-parameterization of the geometric path, ensuring the trajectory satisfies velocity and acceleration constraints. Simulation results demonstrate that the trajectory generated by this method is continuous and smooth without step phenomena, and the angular velocities of each joint exhibit a bell-shaped distribution, effectively solving interference problems in complex environments and meeting the requirements of high-intensity industrial assembly.

Keywords

SCARA Manipulator; Artificial Potential Field (APF); Trajectory Planning; TOTP; Obstacle Avoidance.

1. Introduction

In high-cycle scenarios such as industrial assembly, sorting, and handling, SCARA manipulators are widely adopted due to the high stiffness and high-speed characteristics of their two planar revolute joints. However, in confined workspaces, these manipulators often face interference issues from obstacles such as cylindrical columns and industrial fixtures. Trajectory planning must simultaneously satisfy multiple requirements: end-effector path feasibility, obstacle avoidance safety, smoothness of joint motion, and controllable online computational costs. The Artificial Potential Field (APF) method has garnered continuous attention in manipulator obstacle avoidance and online correction due to its intuitive structure, low computational overhead, and ease of online implementation; meanwhile, improved forms and engineering strategies targeting issues such as local minima and oscillations have emerged [1]. Z. Wang et al. proposed improvements to the potential function and hybrid optimization methods to address the local minima and oscillation problems of APF, thereby enhancing path stability and convergence performance [2]. However, obtaining a "geometrically feasible" path solely in Cartesian space is insufficient for direct execution: industrial manipulators must further undergo joint-space mapping and time-parameterization to ensure that constraints such as

velocity and acceleration can be reliably tracked by the drive and control systems. For the high-quality motion of industrial robots, time-optimal and jerk-related trajectory optimization and planning methods have been systematically studied in recent years, emphasizing a balance between cycle time and smoothness under constrained conditions, providing a practical time-scaling framework for actual execution [3].

Addressing a SCARA manipulator with known parameters, this paper proposes and implements an integrated trajectory planning procedure combining "kinematic modeling + APF obstacle avoidance + time scaling (TOTP-style)": First, the kinematic model is constructed using the structural characteristics of the SCARA manipulator to map the end-effector pose sequence into joint sequences rapidly and deterministically, avoiding the convergence uncertainty inherent in numerical iterations. Subsequently, using cylinders as obstacle models in Cartesian space, the linear interpolation path is corrected through online iterations of attractive-repulsive force superposition; an obstacle "expansion" strategy is introduced to form a safety envelope, reducing the engineering risk of "end-effector clearance but linkage interference." Finally, time-parameterization is performed on the resulting joint sequences to output joint angle, angular velocity, and angular acceleration curves, enabling the visual evaluation and comparison of motion smoothness and cycle time. Consistent with the research approach of industrial robot trajectories oriented toward "time-jerk" optimization, the procedure in this paper provides a complete closed loop from geometric obstacle avoidance to executable joint trajectories. Its advantages lie in its concise implementation, lightweight computation, and interpretable results, facilitating extension to more multi-obstacle dynamic scenarios in future work.

2. Algorithm Fundamentals

2.1. Kinematic Model of the SCARA Manipulator

The SCARA manipulator model used in this paper is shown in Fig. 1. It is an RRPR four-degree-of-freedom manipulator, consisting of two synergistic planar revolute joints, one vertical prismatic joint, and one terminal revolute joint. It can achieve positioning motion within the horizontal plane, lifting motion in the vertical direction, and rotation around the Z-axis at the end-effector [4]. Based on this, the joint coordinate systems are established as shown in Fig. 2.

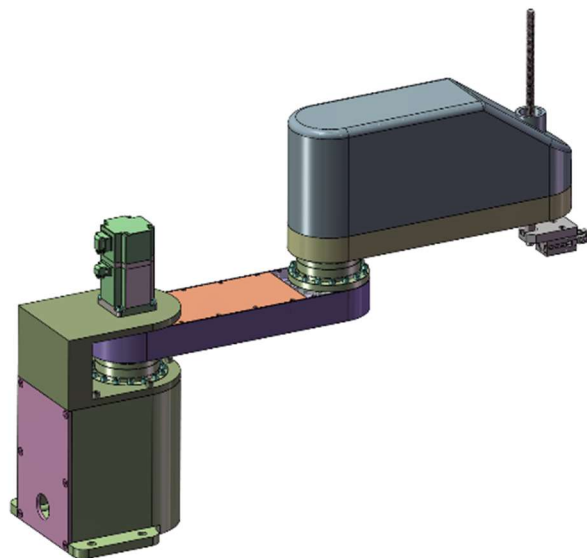


Fig. 1 Manipulator model diagram

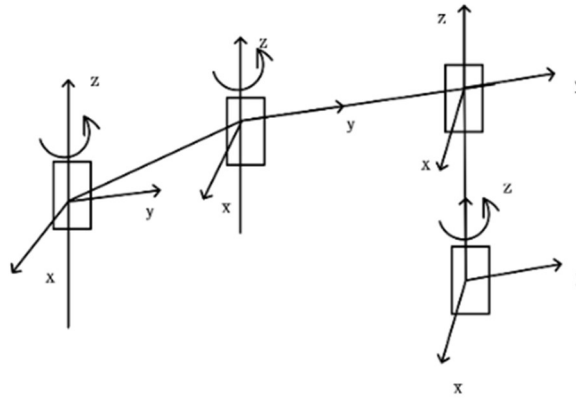


Fig. 2 Linkage coordinate system diagram

The D-H parameter table (Denavit-Hartenberg parameter table) is a standard method in robotics used to describe the geometric relationships between the links of a manipulator. By establishing coordinate systems for adjacent links and utilizing four parameters-link length, link twist, link offset, and joint angle-the position and orientation relationships between adjacent coordinate systems are described. Listing these parameters sequentially by joint results in the D-H parameter table, which is commonly used to establish the kinematic model of the manipulator and derive the forward and inverse kinematic equations [5]. The D-H parameters of this manipulator are shown in Table 1.

Table 1. D-H Parameter Table

Joint i	a_i/mm	$\alpha_i/^\circ$	d_i/mm	θ_i/rad
1	300	0	0	θ_1
2	300	0	0	θ_2
3	0	0	200	θ_3
4	0	0	0	θ_4

2.2. Forward Kinematics Modeling

The SCARA manipulator adopted in this paper features a typical four-degree-of-freedom serial structure (R-R-P-R), where the first, second, and fourth joints are revolute joints rotating about the Z-axis, and the third joint is a prismatic joint extending along the Z-axis. Let the joint variables be q_1, q_2, q_3, q_4 , where q_1, q_2, q_4 represents the angular variable and q_3 represents the vertical displacement variable. Based on the data provided in the D-H parameter table, calculations are performed using the homogeneous transformation matrices of forward kinematics.

The expression for the homogeneous transformation matrix is as follows:

$${}^{i-1}T_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Based on the homogeneous transformation matrix, the calculated value of T_0, T_1, T_2, T_3 and T_4 are obtained, followed by the calculation of T_4^0, T_4^0 , with the expression shown below:

$$T_4^0 = \begin{bmatrix} -\cos(q_1 + q_2) & \sin(q_1 + q_2) & 0 & 250 \cos(q_1 + q_2) + 300 \cos q_1 \\ \sin(q_1 + q_2) & -\cos(q_1 + q_2) & 0 & 250 \sin(q_1 + q_2) + 300 \sin q_1 \\ 0 & 0 & -1 & 150 - q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

Based on the homogeneous transformation matrix T_4^0 , the spatial position xyz and the orientation angle ϕ of the manipulator's end-effector in the base coordinate system can be obtained. The forward kinematics model provides the fundamental mapping relationship for the subsequent obstacle avoidance path planning in Cartesian space.

2.3. Inverse Kinematics Solution

To transform the obstacle avoidance planning path in Cartesian space into executable motor commands, an inverse kinematics solution must be performed. Due to the computational time requirements and convergence uncertainties associated with numerical iteration methods, this paper derives a closed-form analytical inverse kinematics solution based on the structural characteristics of the SCARA manipulator.

The target pose is set as $x=300\text{mm}$, $y=80\text{mm}$, $z=120\text{mm}$, and $\phi=0^\circ$. The analytical solution enables a rapid and deterministic mapping of the end-effector pose sequence, ensuring computational real-time performance while providing the foundational data for the time-parameterization of joint sequences.

Given the target pose matrix T_4^0 , of the manipulator end-effector in Cartesian space, with its position vector denoted as $P=[p_x, p_y, p_z]^T$, the objective of the inverse kinematics solution is to solve for each joint variable $\{\theta_1, \theta_2, d_3, \theta_4\}$ based on P .

The first two revolute joints of the SCARA manipulator constitute a planar two-link mechanism. According to the forward kinematics equations, the relationship between the projected coordinates of the end-effector on the horizontal plane and the joint angles is:

$$\begin{aligned} p_x &= a_1 \cos \theta_1 + a_2 \cos(\theta_1 + \theta_2) \\ p_y &= a_1 \sin \theta_1 + a_2 \sin(\theta_1 + \theta_2) \end{aligned} \quad (3)$$

Utilizing the law of cosines, the analytical solution for θ_2 is first obtained as follows:

$$\cos \theta_2 = \frac{p_x^2 + p_y^2 - a_1^2 - a_2^2}{2a_1a_2} \quad (4)$$

Let $D = \cos \theta_2$, then $\theta_2 = \text{atan2}(\pm\sqrt{1-D^2}, D)$ is obtained. The positive and negative signs correspond to the "left-hand" or "right-hand" configurations of the manipulator; this paper uniformly selects the right-hand rule.

Then, by using geometric relationships, we can determine that $\theta_1 = \text{atan2}(p_y, p_x) - \text{atan2}(a_2 \sin \theta_2, a_1 + a_2 \cos \theta_2)$.

Since the third joint of the SCARA is a prismatic joint perpendicular to the horizontal plane, its displacement is only affected by the z -axis coordinate:

$$d_3 = d_1 - p_z - d_4 \quad (5)$$

where d_1 is the base height and d_4 is the length of the end-effector. For the end-effector yaw angle θ_4 , which represents the total rotation of the end-effector relative to the base, it can be obtained from the link transformation relationship:

$$\theta_4 = \phi - (\theta_1 + \theta_2) \quad (6)$$

where ϕ is the preset orientation angle of the end-effector in Cartesian space. Through the above analytical derivation, real-time mapping from geometric path points to joint space commands can be achieved, avoiding the heavy computational load of numerical iteration and ensuring the real-time performance of obstacle avoidance control.

3. Cartesian Space Obstacle Avoidance Path Planning based on Improved APF

3.1. Traditional Artificial Potential Field Method

The traditional Artificial Potential Field (APF) method was proposed by Khatib et al. The fundamental idea is to construct an attractive field and a repulsive field within the workspace. The target point generates an attractive force to guide the manipulator's end-effector toward the goal, while obstacles generate repulsive forces to prevent the manipulator from colliding [6].

The traditional potential field function is typically represented as the superposition of an attractive field U_a and a repulsive field U_r , where the repulsive field is defined as:

$$U_r(x) = \frac{1}{2} k_r \left(\frac{1}{d(x)} - \frac{1}{d_0} \right)^2 \quad (7)$$

Where k_r is the repulsive gain coefficient, $d(x)$ is the distance from the manipulator end-effector to the obstacle, and d_0 is the influence radius of the obstacle's repulsive force.

Although the traditional APF calculation method is relatively simple, it is highly prone to falling into local minima in confined spaces, leading to immobility due to the balance between attractive and repulsive forces, as well as the problem of path oscillation near obstacles.

3.2. Improved Potential Field Function

To satisfy the operational requirements of the SCARA manipulator in complex working environments, this paper introduces improvements to the APF and a strategy to eliminate jitter and escape local optima in complex obstacle environments. In the repulsive field function, a term representing the distance $\rho(x, x_g)$ between the manipulator and the target point raised to the n -th power is introduced.

The improved repulsive function is:

$$U_r(x) = \frac{1}{2} k_r \left(\frac{1}{d(x)} - \frac{1}{d_0} \right)^2 \rho^n(x, x_g) \quad (8)$$

This improvement ensures that as the manipulator approaches the target point, $\rho(x, x_g) \rightarrow 0$, causing the repulsive field to decay to zero at the target. This eliminates the drawback in traditional APF where the repulsive force dominates near the goal, thereby ensuring the convergence of the algorithm.

In industrial assembly scenarios, both the SCARA manipulator's end-effector tools and the obstacles (such as columns and fixtures) possess specific physical geometric dimensions. To abstract complex solid obstacle avoidance into point-mass obstacle avoidance, this paper introduces an obstacle expansion strategy. Obstacles are simplified as cylinders, and the radius of column obstacles is expanded by R to form a safe envelope. This expands the collision-avoidance condition from simple end-effector point avoidance to anti-splitting (linkage interference prevention).

The improved total potential field U is the sum of the attractive field U_a and the improved repulsive field U_r , and its corresponding negative gradient direction represents the end-effector guiding force F : $F = -\nabla U_a - \nabla U_r$.

4. Experimental Simulation and Results Analysis

To verify the effectiveness of the analytical kinematics and the improved APF algorithm, simulation experiments were conducted using computer-aided simulation tools.

4.1. Analysis of Obstacle Avoidance Path Results

The initial end-effector position of the SCARA manipulator was set to $P_s=[300,80,120]$, and the target position was set to $P_g=[450,260,80]$. A cylindrical obstacle with a radius $r=60\text{mm}$ (including the expanded safety margin) was placed between the two points, as shown in Fig. 3.

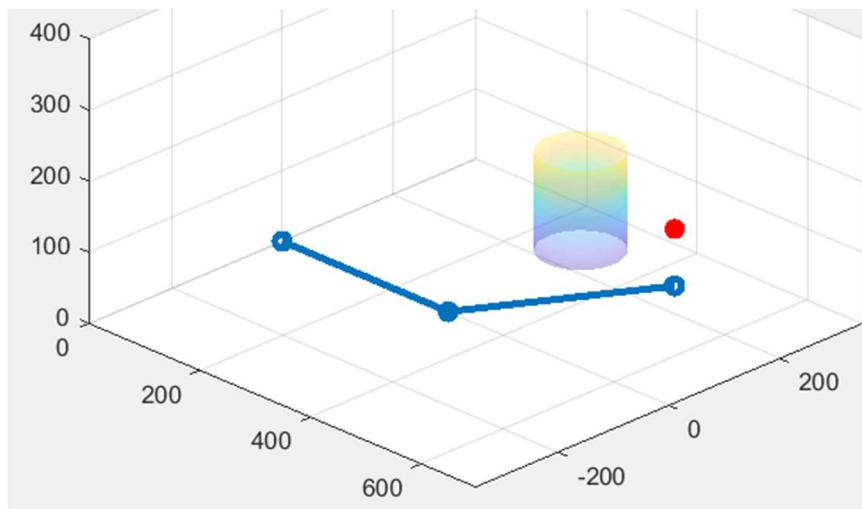


Fig. 3 Obstacle avoidance model diagram

As shown by the simulated Cartesian space path, the improved APF algorithm generates a smooth bypass trajectory when approaching the obstacle. Due to the introduction of the target distance factor, the manipulator is not disturbed by residual repulsive forces near the goal P_g upon arrival, ensuring absolute safety in geometric space.

4.2. Analysis of Joint-Space Operational Status

After transforming the geometric path through inverse solutions and performing time-parameterization, the operational curves for each joint over time are obtained, as shown in Fig. 4:

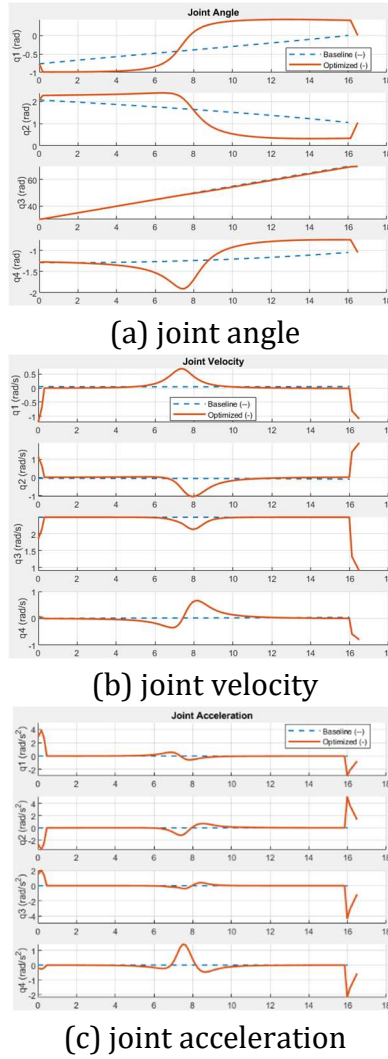


Fig. 4 Joint transformation operational curves

By mapping the Cartesian space geometric path generated by the improved Artificial Potential Field method into joint space and performing time-parameterization combined with the TOTP algorithm, the operational curves for each joint over time are obtained, as shown in Fig. 4. From the simulation results, it can be observed that the position curves of the four joints exhibit excellent C^2 continuity and monotonic smoothness. This not only verifies that no configurational singularities or multi-solution jumps occurred during the inverse kinematic transformation but also reflects the effective control of path curvature at the geometric level by the improved APF. Further analysis of the angular velocity curves for each joint reveals that, under the constrained optimization of the TOTP algorithm, the velocity profiles exhibit a regular, continuous bell-shaped distribution. This characteristic ensures that velocities strictly return to zero during the start and stop phases with no overshoot throughout the process, precisely limiting the motion states within the maximum speed range permitted by the motors. Simultaneously, because the time-parameterization process effectively suppresses acceleration steps, the acceleration curves remain steady without peak impacts. This high level of smoothness at the dynamical level essentially eliminates common mechanical vibrations in high-cycle operations. It satisfies the high-intensity and high-efficiency requirements of industrial assembly while significantly enhancing the stability of the control system and the service life of mechanical transmission components.

5. Conclusion

In summary, this paper successfully establishes a closed-loop planning system for SCARA robotic arms, addressing the demands for efficient obstacle avoidance and stable operation in complex fixture environments. This system bridges the gap between Cartesian geometric space and joint space dynamics. By constructing the D-H model and deriving a closed-form analytical inverse solution, computational efficiency for pose mapping in multi-constraint environments is significantly enhanced. For obstacle avoidance, introducing a distance decay factor to the traditional artificial potential field method and integrating an obstacle expansion envelope strategy not only completely resolves the inherent issue of unreachable target points but also ensures absolute path safety within the geometric space. Further combining this with the TOTP algorithm for time re-calibration enables the robot arm to achieve a continuous bell-shaped angular velocity distribution and complete suppression of acceleration steps while adhering to the physical limits of joint velocity and acceleration. Simulation validation demonstrates that this approach ensures smooth robotic arm operation and reduced mechanical vibration while balancing path compactness and motion real-time performance. It provides an engineering-ready solution for high-cycle obstacle-avoidance assembly tasks in industrial settings.

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