

# Cooperative Flight Control of Multi-UAV Systems under a Data-Driven Paradigm

Kexin Li\*

School of Aeronautics and Astronautics, Tiangong University, Tianjin, China

\*13102521856@163.com

## Abstract

**Aiming at the challenge of highly cooperative control of multi-UAV formation under nonlinear dynamics, communication restriction and malicious attacks, a data-driven Compact Format Dynamic Linearization based Model-Free Adaptive Control (CFDL-MFAC) method is adopted to transform the nonlinear system into a compact format dynamic linear model based on pseudo-partial derivatives, and a distributed control law and pseudo-partial derivative estimation algorithm are designed. Meanwhile, a reset mechanism is introduced to deal with data packet loss. Theoretical analysis verifies the tracking error convergence and the stability of bounded input and bounded output of the system under constant and time-varying trajectories. Simulation experiments indicate that the proposed method can effectively realize multi-UAV highly cooperative tracking with strong anti-jamming and robustness.**

## Keywords

**Multi-UAV System; Model-free Adaptation; Data Packet Loss; Pseudo-partial Derivative.**

## 1. Introduction

With the rapid development and wide application of UAV technology, the limitation of a single UAV in mission execution is increasingly apparent. Multi-UAV collaborative system has become a research hotspot and frontier in industrial application and academic research for its significant advantages in efficiency, robustness and mission coverage. In many research directions of multi-unmanned aerial vehicle systems, cooperative flight control is the core supporting technology for realizing complex tasks. However, traditional cooperative control methods highly rely on accurate dynamic models, while actual UAV systems often exhibit strong nonlinearity, coupling and uncertainty. Besides, its accurate mathematical models are difficult to obtain in advance, which brings fundamental challenges to model-based control design. Thus, the data-driven control method is becoming a vital research direction to overcome the limitations of traditional methods because it can achieve the control goal by only using the input-output data of the system[1-5].

For multi-UAV flight altitude control in UAV formation control, Wen et al. proposed a model-free adaptive control scheme with finite time prescribed performance to solve the leader-follower consistency of multi-robot systems under DoS attack. Liang et al. introduced the proportional correlation coefficient and relational correlation index for completely unknown nonlinear multi-agent systems to improve the finite-time performance accuracy. Chen et al. designed a model-free control strategy that features adaptive fault-tolerant iterative learning for networked nonlinear discrete systems with actuator failure and bidirectional data loss. Chi et al. proposed a data-driven dynamic internal model control scheme for unknown nonlinear non-affine systems. Huang et al. proposed a distributed predictive control scheme that is independent of the system model and can actively compensate the communication delay for the consistency and tracking of nonlinear multi-agent systems under communication delay. Li et al.

put forward a predictive control scheme of the secure distributed output feedback model for the leader-follower consistency of homogeneous linear perturbation multi-agent systems under multi-network attacks. Sun et al. reconsidered the consensus tracking of strongly connected nonlinear non-affine multi-agent systems from the perspective of communicable agents. Ma et al. proposed an improved dynamic linearized data model to solve the distributed model-free adaptive control for nonlinear multi-agent systems under DoS attacks. Pan et al. put forward a distributed model-free adaptive predictive control aiming at the nonlinear, non-affine characteristics and spoofing attacks of multi-input multi-output multi-agent systems.

The existing research has explored a variety of model-free adaptive control methods for multi-UAV flight altitude control in UAV formation control, but most of them have limitations on formation topology and flight scene constraints. Wen et al. studied the finite-time prescribed performance safety control of a vehicle queuing system under aperiodic DoS attack, but lacked the theoretical expansion and verification in the complex scenario of multi-UAV height control. Liang et al. studied the asymmetric bipartite learning consensus of non-affine nonlinear MAS under cooperative-adversarial interaction. However, when there are non-iterative repetitive nonlinear components in multi-UAV altitude systems, the dynamic parameter estimation is challenging, affecting the actual convergence. Chen et al. studied the tracking control of networked nonlinear systems under actuator failure and bidirectional data loss, but the controller design needs expansion. Hou et al. proposed the PFDL/CFDL-MFAC method for SISO discrete nonlinear systems, but it is currently only aimed at single UAV altitude control, with insufficient practical application verification. Chi et al. proposed the  $D^3$ IMC scheme. At present, the convergence analysis only aims at the constant height adjustment, and the convergence analysis of time-varying height tracking is still an open problem. Huang et al. studied the tracking control of HNMA under communication delay without considering the multi-attack superposition scenario in multi-UAV altitude control. Li et al. put forward a secure DOFMPC scheme under large-scale DoS and FDI attacks. The cost function depends on the accurate information of the pilot UAV, and the controller design under the unknown leader scenario needs to be studied. Sun et al. proposed CA-LDM and CA-MFAILC for non-affine nonlinear MAS with strongly connected topology, but they were not extended to non-strongly connected formation topology and competitive interaction scenarios. Ma et al. studied the DMFAC problem of nonlinear MAS under DoS attack, but did not take into account the multi-attack superposition scenario with multi-UAV height control. Pan et al. proposed a DMFAPC method for MIMO heterogeneous MAS under spoofing attack. But it did not involve the non-affine structure and non-uniform scenarios of multi-UAV height control, and the applicable scope of the algorithm needs to be expanded.

Based on the previous analysis, this paper not only focuses on the nonlinear dynamic coupling faced by multiple UAVs when performing highly cooperative tracking tasks in complex outdoor airspace, but also emphasizes the problem that the wireless communication in UAV formation is vulnerable to aperiodic DOS attacks, resulting in data packet loss and communication interruption. A model-free adaptive cooperative control scheme is proposed. The model-free adaptive control part only uses the real-time altitude and control input data of the UAV to design a distributed control law, while the real-time updating mechanism of the PPD estimate effectively repairs the neighbor information loss and data bias caused by aperiodic DOS attacks through dynamic reset and compensation.

1) Aiming at the direct data-driven paradigm for multi-UAV formation altitude control, the system characterization without model dependence is realized by dynamic linearization, and the altitude control algorithm and PPD estimation algorithm with control input constraints are constructed. Combined with real-time feedback, the CFDL-MFAC scheme is designed to dynamically correct PPD estimation and automatically reset abnormal parameters based on

input and output data, which smoothly controls the input while ensuring high tracking accuracy, and adapts to distributed formation scenarios.

2) The data packet loss forward compensation mechanism builds a forward data compensation mechanism for DOS attack and data packet loss scenarios. Besides, the data bias under attack is quantified based on the data packet state model, and the information gap with historical or current valid data is filled. It also relies on PPD dynamic estimation and judgement and reset mechanism to improve the tracking ability of the system for abnormal data.

The structure of the rest of the paper is as follows. The second section introduces the control algorithm and PPD estimation algorithm, then the third section proves the stability of the control system. Finally, the fourth section verifies the effectiveness of the proposed control algorithm through simulation experiments[6-10].

## 2. Preparatory Knowledge

In this section, core concepts such as weighted directed graph and PPD time-varying parameters are given, and then the UAV output control problem is restated.

### 2.1. Basic Graph Theory

$\zeta = (Z, \varphi, A)$  is a weighted directed graph, where  $Z = \{1, 2, \dots, N\}$  is the set of vertices,  $\varphi$  is the set of directed edges, and  $A = \{a_{ij} \mid i, j \in Z\}$  is the adjacency matrix. If  $(j, i) \in \varphi$ ,  $a_{ij} = 1$ .

Otherwise,  $a_{ij} = 0$ .  $N_i = \{j \mid j \in Z, (j, i) \in \varphi\}$  is the neighbor of agent  $i$ .  $E = \text{diag}(e_1, e_2, \dots, e_N)$  refers to a correlation matrix representing a leader and a follower. If the follower receives the leader signal directly, there is  $E_i = 1$ . Otherwise,  $E_i = 0$ .

### 2.2. Problem Description

A single-input single-output (SISO) discrete-time UAV system can be expressed as the following nonlinear mapping:

$$y(k-1) = f(y(k), \dots, y(k-n_y), u(k), \dots, u(k-n_u)) \tag{1}$$

Where  $y(k) \in \mathbb{R}$  indicates the position of the UAV at the time  $k$ , and  $u(k) \in \mathbb{R}$  indicates the height control input of the UAV at the time  $k$ . There are following hypotheses for the data mapping of the UAV altitude control system:

Hypothesis 1: Except for finite time, the partial derivative expression  $f(\dots)$  with respect to the  $(n_y + 2)$ -th variable is continuous. That is, the flight altitude of the UAV is continuous with respect to the partial derivative of the control input.

Hypothesis 2: Except for finite times, the UAV system (1) satisfies the generalized Lipschitz condition. In other words, for any  $k_1 \neq k_2$ ,  $k_1, k_2 \geq 0$  and  $u(k_1) \neq u(k_2)$ , there is

$$|y(k_1+1) - y(k_2+1)| \leq b |u(k_1) - u(k_2)| \tag{2}$$

Where  $y(k_i+1) = f(y(k_i), \dots, y(k_i - n_y), u(k_i), \dots, u(k_i - n_u))$ ,  $i = 1, 2$ ;  $b > 0$  is a constant.

Comment 1: Hypothesis 1 is the primary condition of controller design. Hypothesis 2 guarantees that the altitude control input with finite value variation will not lead to infinite output values of UAV flight altitude, which provides a basis for system stability analysis.

Theorem 1: When a multi-UAV system satisfies the above two hypotheses, and  $|\Delta u(k)| \neq 0$  at that time, there is a time-varying parameter  $\phi_c(k) \in R$  called PPD that transforms the system (1) into the following CFDL data model:

$$\Delta y(k+1) = \phi_c(k) \Delta u(k) \quad (3)$$

Where  $\phi_c(k)$  is the PPD of UAV system, which carries the nonlinear characteristics and dynamic response law of UAV high dynamics. After generalizing the dynamic linearization to heterogeneous UAV systems, the height dynamics of the  $i$ th UAV can be described as follows:

$$y_1(k-1) = f_i(y_1(\bar{k}), \dots, y_1(\bar{k} - n_y), u_1(\bar{k}), \dots, u_1(\bar{k} - n_u)) \quad (4)$$

Where  $y_i(k), \mu_i(k) \in R$  respectively represent the flight altitude and altitude control input and output of the  $i$ -th UAV at time  $k$ .  $n_y, n_u \in Z^+$  respectively refer to the unknown order of  $y_i(k), u_i(k)$ .

Lemma 1: The system also satisfies the above two hypotheses. When  $|\Delta u(k)| \neq 0$ , there is time-varying parameter  $\varphi_i(k)$ , which enables (4) to be rewritten as the following data model:

$$\Delta y_i(k+1) = \varphi_i(k) \Delta u_i(k) \quad (5)$$

Hypothesis 3: The UAV communication topology  $\zeta$  contains a spanning tree in which the leader UAV is designated as the root, specifying that the desired altitude trajectory can only be given a certain designated sub-direction by one designated UAV.

Comment 2: Hypothesis 3 proves that there are no isolated UAVs in the UAV system, which ensures the effective transmission of distributed control information. Hypothesis 4 predetermines a known height control direction.

The distributed altitude control output of the  $i$ -th UAV is:

$$\zeta_i(k) = \sum_{j \in N_i} \alpha_j (y_i(k) - y_j(k)) + \sigma_i (y^*(k) - y_i(k)) \quad (6)$$

Where  $N_i$  represents the communication neighbor set of the  $i$ -th UAV,  $y^*(k)$  is the desired altitude trajectory, and  $\alpha_{ij}$  refers to the neighbor order matrix element.

### 3. Control Algorithm Design and Stability Analysis

This section introduces the multi-UAV formation altitude control algorithm and its stability analysis.

#### 3.1. Control Algorithm Design

For the multi-UAV formation altitude control system, it is hoped that the actual flight altitude trajectory of each UAV can meet the expected altitude requirement of the formation to the greatest extent, which requires distributed control of the altitude tracking error of each UAV. Therefore, the following criterion function is proposed for the  $i$ -th UAV:

$$J(u(k)) = |y^*(k+1) - y(k+1)|^2 + \lambda |u(k) - u(k-1)|^2 \quad (7)$$

Where  $y^*(k+1)$  represents the expected height trajectory of the formation, and  $\lambda$  is the weight factor, which is used to balance the height tracking accuracy with the variation amplitude of the control input.

Comment 3: Equation (7) consists of two parts. The first part ensures the minimum altitude tracking error of the UAV, and the second part ensures the finiteness of altitude control input to avoid drastic changes in control quantity.

After deriving the criterion function  $u(k)$  and making it equal to 0, the altitude control algorithm of the  $i$ -th UAV can be obtained:

$$u(k) = u(k - 1) + \frac{\rho\phi_c(k)}{\lambda + |\phi_c|^2} (y^*(k + 1) - y(k)) \quad (8)$$

Where  $\rho$  is the step factor, which is added to make the algorithm more general. The actual value of PPD is difficult to obtain, so the estimation algorithm is designed for it. The following estimation criterion function is proposed, with  $\mu$  as the weight factor.

$$J(\phi_c(k)) = |y(k) - y(k - 1) - \hat{\phi}_c(k)\Delta u(k - 1)|^2 + \mu|\hat{\phi}_c(k) - \hat{\phi}_c(k - 1)|^2 \quad (9)$$

By finding the extreme value  $\hat{\phi}_c(k)$  of the estimation criterion function, we can obtain the PPD estimation algorithm of the  $i$ -th UAV:

$$\hat{\phi}_c(\hat{k}) = \hat{\phi}_c(\hat{k} - 1) + \frac{\eta_{\Delta U}(\hat{k}-1)}{\mu + \Delta U(\hat{k}-1)^2} (\Delta y(\hat{k}) - \hat{\phi}_c(\hat{k} - 1)\Delta U(\hat{k} - 1)) \quad (10)$$

Where  $\hat{\phi}_c(k)$  is the estimated value of PPD  $\phi_c(k)$ . In the case of UAV attack or data loss, the output data collected by the UAV deviates from the real value, and the system will continuously correct PPD according to the latest input and output data.

Combining Equations (8) and (10), the following CFDL-MFAC scheme for multi-UAV formation altitude control is given:

$$\hat{\phi}_c(\hat{k}) = \hat{\phi}_c(\hat{k} - 1) + \frac{\eta_{\Delta U}(\hat{k}-1)}{\mu + \Delta U(\hat{k}-1)^2} (\Delta y(\hat{k}) - \hat{\phi}_c(\hat{k} - 1)\Delta U(\hat{k} - 1)) \quad (11)$$

If  $|\hat{\phi}_c(k)| \leq \varepsilon$  or  $|\Delta u(k-1)| \leq \varepsilon$  or  $sign(\hat{\phi}_c(k)) \neq sign(\hat{\phi}_c(1))$

$$\hat{\phi}_c(k) = \hat{\phi}_c(1) \quad (12)$$

$$u(k) = u(k - 1) + \frac{\rho\hat{\phi}_c(k)}{\lambda + |\hat{\phi}_c(k)|^2} (y^*(k + 1) - y(k)) \quad (13)$$

Where  $\lambda > 0, \mu > 0, \rho \in (0, 1], \eta \in (0, 1]$ . Equation (12) indicates that when  $\phi_c(k)$  deviates from the expectation, it is updated and re-estimated, so that the PPD estimation algorithm has stronger tracking ability for time-varying parameters.

In the DOS attack, the data packet received by the  $i$ -th UAV is described as follows:

$$\zeta_{ai}(\vec{k}) = \ell_i(\vec{k})\zeta_i(\vec{k}) + (1 - \ell_i(\vec{k}))\zeta_i(\vec{k} - 1) \quad (14)$$

Where  $\ell_i(k) = 0$  indicates that the attack is successful, and the data packet of the  $i$ -th UAV at this time is equal to the output data of the previous moment when it was attacked.  $\ell_i(k) = 1$  indicates that the attack fails, and the output data of the  $i$ -th UAV after being attacked is equal to the output data at time  $k$ .

### 3.2. Stability Analysis

For the multi-UAV formation altitude control system, the following hypotheses are proposed:

Hypothesis 4: For a given bounded desired output signal  $y^*(k+1)$ , there is always a bounded  $u^*(k)$ , so that the output of the system under the control of the control input signal is equal to  $y^*(k+1)$ .

Theorem 2: For a multi-UAV formation altitude control system, if hypotheses 1-2 and 5-6 are satisfied and when  $y^*(k+1) = y^* = const$ , using the CFDL-MFAC scheme, there is a positive number  $\lambda_{min} > 0$ , and the following can be achieved when  $\lambda > \lambda_{min}$ :

(1) The system output tracking error is monotonically convergent, and  $\lim_{k \rightarrow \infty} |y^* - y(k+1)| = 0$ .

(2) The closed-loop system is BIBO stable, i.e. the output sequence  $\{y(k)\}$  and the input sequence  $\{u(k)\}$  are bounded.

(1) Boundedness of PPD Estimation Under Constant Trajectory

Define  $\tilde{\phi}_c(k) = \hat{\phi}_c(k) - \phi_c(k)$  as the PPD estimation error, and  $\phi_c(k)$  is subtracted simultaneously on both sides of Equation (8).

$$\begin{aligned} \hat{\phi}_c(k) - \phi_c(k) &= \hat{\phi}_c(k-1) - \phi_c(k-1) + \phi_c(k-1) - \phi_c(k) \\ &\quad + \frac{\eta \Delta u(k-1)}{\mu + \Delta u(k-1)^2} (\Delta y(k) - \hat{\phi}_c(k-1) \Delta u(k-1)) \\ \tilde{\phi}_c(k) &= \phi_c(k-1) - \phi_c(k) + (1 - \frac{\eta \Delta u(k-1)^2}{\mu + \Delta u(k-1)^2}) \tilde{\phi}_c(k-1) \end{aligned} \quad (15)$$

Taking absolute values for both sides of Equation (11) generates:

$$|\tilde{\phi}_c(k)| \leq \left| 1 - \frac{\eta \Delta u(k-1)^2}{\mu + \Delta u(k-1)^2} \right| |\tilde{\phi}_c(k-1) + \phi_c(k-1) - \phi_c(k)| \quad (16)$$

Notably, the function  $\frac{\eta \Delta u(k-1)^2}{\mu + \Delta u(k-1)^2}$  is monotonically increasing with respect to the variable  $|\Delta u(\vec{k} - 1)|^2$ , with  $\frac{\eta \varepsilon^2}{\mu + \varepsilon^2}$  as its minimum. When  $0 < \eta \leq 1$  and  $\mu > 0$ ,  $d_1$  exists,

$$0 \leq \left| 1 - \frac{\eta \Delta u(\vec{k}-1)^2}{\mu + \Delta u(\vec{k}-1)^2} \right| \leq 1 - \frac{\eta \varepsilon^2}{\mu + \varepsilon^2} = d_1 \leq 1 \quad (17)$$

According to  $|\phi_c(k)| \leq \bar{b}$ , there is  $|\phi_c(k-1) - \phi_c(k)| \leq 2\bar{b}$ . The following inequality is obtained by using Inequalities (16) and (17):

$$\begin{aligned} |\tilde{\phi}_c(k)| &\leq d_1 |\tilde{\phi}_c(k-1)| + 2\bar{B} \leq d_1^2 |\phi_c(k-2)| + 2\bar{B}d_1 + 2\bar{B} \\ &\leq \dots \leq d_1^{k-1} |\tilde{\phi}_c(1)| + \frac{2\bar{b}(1-d_1^{k-1})}{1-d_1} \end{aligned} \quad (18)$$

Inequality (18) means  $\tilde{\phi}_c(k)$  is bounded. Because  $\phi_c(k)$  is bounded, there is  $\hat{\phi}_c(k)$ .

### (2) Convergence of Tracking Error Under Constant Trajectory

Define the altitude tracking error of the i-th UAV as

$$e(k+1) = y^* - y(k+1) \quad (19)$$

Substituting CDFL data model (3) into Equation (19) and taking absolute values for both sides generates:

$$|\varepsilon(\hat{k}+1)| \leq |y^* - y(\hat{k}+1)| = |y^* - \phi_c(\hat{k})\Delta u(\hat{k})| \leq \left| 1 - \frac{\rho\phi_c(\hat{k})\hat{\phi}_c(\hat{k})}{\lambda + |\hat{\phi}_c(\hat{k})|^2} \right| \varepsilon(\hat{k}) \quad (20)$$

If  $\lambda_{\min} = \frac{\bar{b}^2}{4}$ , by using the inequality  $\alpha^2 + \beta^2 \geq 2\alpha\beta$ , the boundedness of  $\hat{\phi}_c(k)$  is obtained. If  $\lambda > \lambda_{\min}$ , there must be a constant  $0 < M_1 < 1$ , realizing the following equation.

$$0 < M_1 \leq \frac{\phi_c(k)\hat{\phi}_c(k)}{\lambda + |\hat{\phi}_c(k)|^2} \leq \frac{\bar{b}\hat{\phi}_c(k)}{\lambda + |\hat{\phi}_c(k)|^2} \leq \frac{\bar{b}\hat{\phi}_c(k)}{2\sqrt{\lambda}\phi_c(k)} < \frac{\bar{b}}{2\sqrt{\lambda_{\min}}} = 1 \quad (21)$$

According to Equation (17),  $0 < \rho \leq 1$  and  $\lambda > \lambda_{\min}$ , there must be a constant  $d_2 < 1$ .

$$\left| 1 - \frac{\rho\phi_c(k)\hat{\phi}_c(k)}{\lambda + |\hat{\phi}_c(k)|^2} \right| = 1 - \frac{\rho\phi_c(k)\hat{\phi}_c(k)}{\lambda + |\hat{\phi}_c(k)|^2} \leq 1 - \rho M_1 = d_2 < 1 \quad (22)$$

Combined Inequalities (20) and (22), we have

$$|e(k+1)| \leq d_2 |e(k)| \leq d_2^2 |e(k-1)| \leq \dots \leq d_2^k |e(1)| \quad (23)$$

Equation (23) means that the tracking error is bounded. Since  $y^*(k)$  is a constant, the boundedness of the tracking error means that  $y(k)$  is bounded.

### (3) BIBO Stability of System Under Constant Trajectory

Using inequalities  $(\sqrt{\lambda})^2 + |\hat{\phi}_c(k)|^2 \geq 2\sqrt{\lambda}\hat{\phi}_c(k)$  and  $\lambda > \lambda_{\min}$ , the following equation can be obtained from Equation (13):

$$\begin{aligned} \Delta u(k) &= \left| \frac{\rho\hat{\phi}_c(k)(y^* - y(k))}{\lambda + |\hat{\phi}_c(k)|^2} \right| \leq \left| \frac{\rho\hat{\phi}_c(k)}{\lambda + |\hat{\phi}_c(k)|^2} \right| |e(k)| \\ &\leq \frac{|\rho\hat{\phi}_c(k)|}{2\sqrt{\lambda}\hat{\phi}_c(k)} |\varepsilon(k)| \leq \frac{\rho}{2\sqrt{\lambda_{\min}}} |\varepsilon(k)| = \mathcal{M}_2 |\varepsilon(k)| \end{aligned} \quad (24)$$

Where  $M_2 = \rho / (2\sqrt{\lambda_{\min}})$  is a bounded constant.

Using Equations (23) and (24), we have:

$$\begin{aligned}
 |\mu(\hat{k})| &\leq |\mu(\hat{k}) - \mu(\hat{k} - 1)| + |\mu(\hat{k} - 1)| \\
 &\leq |u(k) - u(k - 1)| + |u(k - 1) - u(k - 2)| + |u(k - 2)| \\
 &\leq \mathcal{M}_2(|e(k)| + |e(k - 1)| + \dots + |e(2)| + |u(1)|) \\
 &\leq M_2(d_2^{k-1}|\varepsilon(1)| + d_2^{k-2}|\varepsilon(1)| + \dots + d_2|\varepsilon(1)|) + |u(1)| \\
 &< \mathcal{M}_2 \frac{d_2}{1-d_2} |e(1)| + |u(1)|
 \end{aligned} \tag{25}$$

So the input sequence  $u(k)$  is bounded.

#### (4) Convergence of Tracking Error Under Time-varying Trajectory

For the formation time-varying trajectory  $y_i^*(k + 1) \neq y_i^*(k)$ , the height tracking error  $e_i(k + 1) = y_i^*(k) - y_i(k)$  is defined.

Substituting the CFDL data model and control law, the absolute value of tracking error is taken and scaled:

$$\begin{aligned}
 |e(k + 1)| &= |y^* - y(k + 1)| = |y^*(k + 1) - y(k) - \phi_c(k)\Delta u(k)| \\
 &= |y^*(k + 1) - y^*(k) + y^*(k) - y(k) - \phi_c(k)\Delta u(k)| \\
 &= \left| \Delta y^*(k + 1) + e(k) - \phi_c(k) \cdot \frac{\rho\phi_c(k)}{\lambda + |\phi_c(k)|^2} e(k) \right| \\
 &\leq |\Delta y^*(k + 1)| + |e(k)| \cdot \left| 1 - \phi_c(k) \cdot \frac{\rho\phi_c(k)}{\lambda + |\phi_c(k)|^2} \right|
 \end{aligned} \tag{26}$$

Given that time-varying trajectory increment  $|\Delta y^*(k + 1)|$  is bounded, there is  $|\Delta y^*(k + 1)| \leq b$ . According to the conclusion of the constant trajectory stability, there is  $0 < d_2 < 1$  and

$$\left| 1 - \phi_c(k) \cdot \frac{\rho\phi_c(k)}{\lambda + |\phi_c(k)|^2} \right| = d_2.$$

Recursive scaling for tracking error:

$$\begin{aligned}
 |e(k + 1)| &\leq d_2|e(k)| + b \leq d_2[d_2|e(k - 1)| + b] + b = d_2^2|e(k - 1)| + bd_2 + b \\
 &\leq d_2^2[d_2|e(k - 2)| + b] + bd_2 + b = d_2^3|e(k - 2)| + bd_2^2 + bd_2 + b \\
 &\leq d_2^k \cdot |e(0)| + b \cdot [1 + d_2 + d_2^2 + \dots + d_2^{k-1}] \\
 &\leq d_2^k|e(0)| + b \frac{1-d_2^k}{1-d_2}
 \end{aligned} \tag{27}$$

When  $k \rightarrow \infty$ ,  $d_2^k \rightarrow 0$ , there is  $\limsup_{k \rightarrow \infty} |e(k)| \leq \frac{b}{1-d_2}$ .

The tracking error is ultimately uniformly bounded and keeps converging into the bounded neighborhood under time-varying trajectories.

## 4. Simulation Experiment

### 4.1. Parameter Tuning Experiment

Under time-varying desired trajectory conditions, the following desired trajectories are considered:

$$y^*(k+1) = \begin{cases} 0.5 \times (-1)^{\text{round}(k/500)}, & k \leq 300 \\ 0.5 \sin(k\pi/100) + 0.3 \cos(k\pi/50), & 300 < k \leq 700 \\ 0.5 \times (-1)^{\text{round}(k/500)}, & k > 700 \end{cases}$$

The impact of  $\lambda$  on tracking performance is investigated.  $\mu = 1, \rho = 0.6, \eta = 1$  are fixed. Different values of  $\lambda$  are selected. The control input, PPD estimation and tracking error are given in Figure 1-2 respectively.

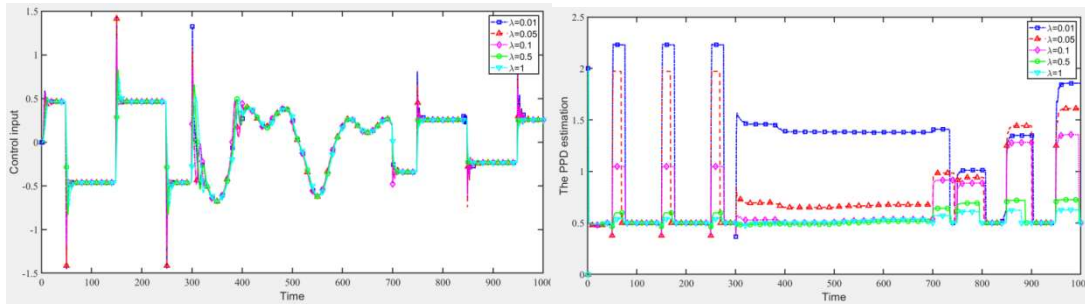


Figure 1. Control Inputs and Tracking Performance for Different Values of  $\lambda$

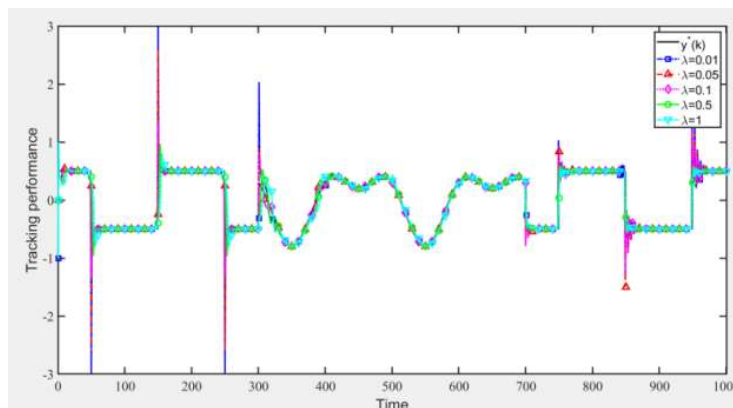


Figure 2. PPD Estimates for Different Values of  $\lambda$

The impact of  $\rho$  on tracking performance is studied.  $\mu = 1, \lambda = 0.1$  and  $\eta = 1$  are fixed. Different values of  $\rho$  are selected. Figure 3-4 indicate the control input, PPD estimate and tracking error respectively.

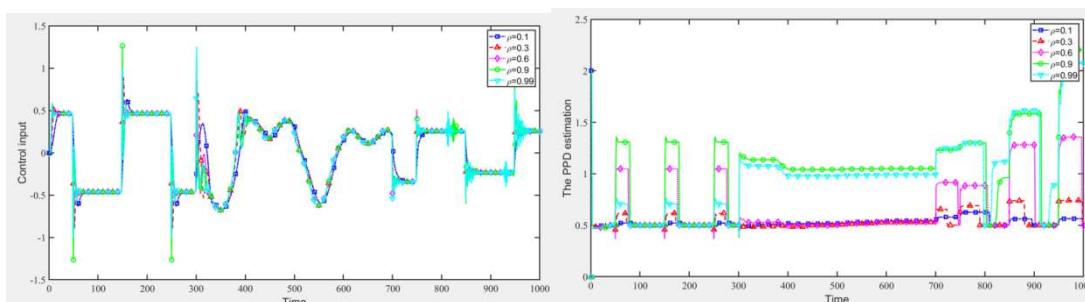


Figure 3. Control Input and Tracking Performance for Different Values of  $\rho$

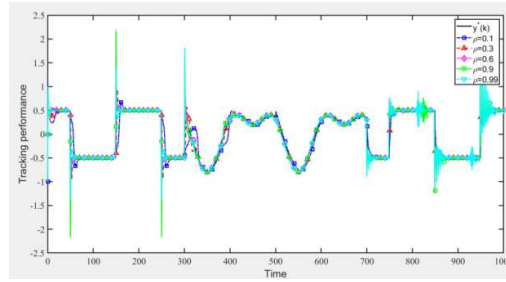


Figure 4. PPD Estimates for Different Values of  $\rho$

### 4.2. Multi-agent Collaboration

For multi-agent systems, it is proved that the leader follows the consistency. The following desired trajectories are considered for leaders:

$$y^* = \begin{cases} 2 & t \leq 200 \\ 0.5 & t > 200 \end{cases}$$

The initial conditions are  $u_1(1) = 0, \hat{\varphi}_t(1) = 2, y_1(1) = 0.5, y_2(1) = 2.5, y_3(1) = 3.5, y_4(1) = 4$ . There are  $\mu = 0.5, \eta = 1, \lambda = 0.5$  and  $\varepsilon = 10^{-5}$ . Figure 6-7 present the variation of the input, tracking performance, PPD estimation and tracking error of the system.

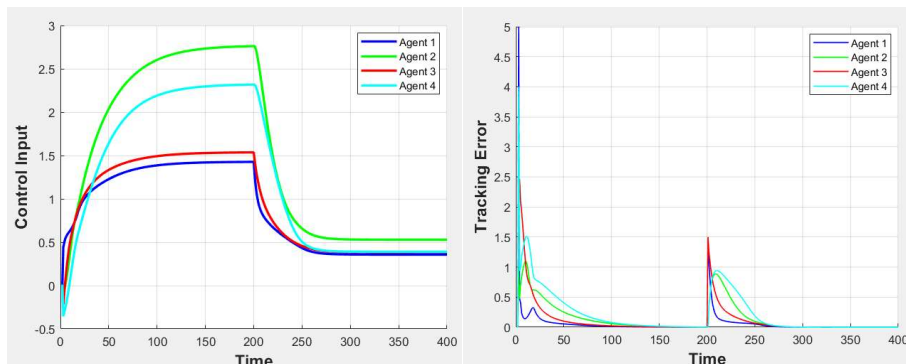


Figure 5. Changes of Tracking Error and Input of the System

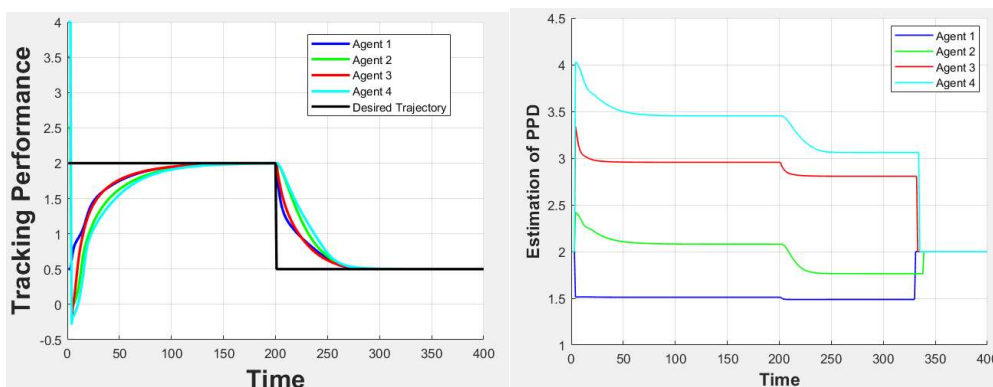


Figure 6. Changes of PPD Estimation and Tracking Performance

### 4.3. Random Data Packet Loss

Aiming at the cooperative control of multi-agents under complete interruption of communication link, a complete packet loss in a continuous interval is set, and a zero-order

hold compensation mechanism is introduced. The robustness of the system is analyzed by comparing four sets of simulation curves, as shown in Figure 8-9.

When there is no compensation for complete packet loss, the communication is completely interrupted at  $k \in [500,600]$ , and the control input is set to zero as  $u(k) = 0, 500 \leq k \leq 600$ . When complete packet loss is compensated, the control input at the previous moment is maintained during packet loss, and system oscillation is suppressed as  $u(k) = u(k-1), 500 \leq k \leq 600$ .

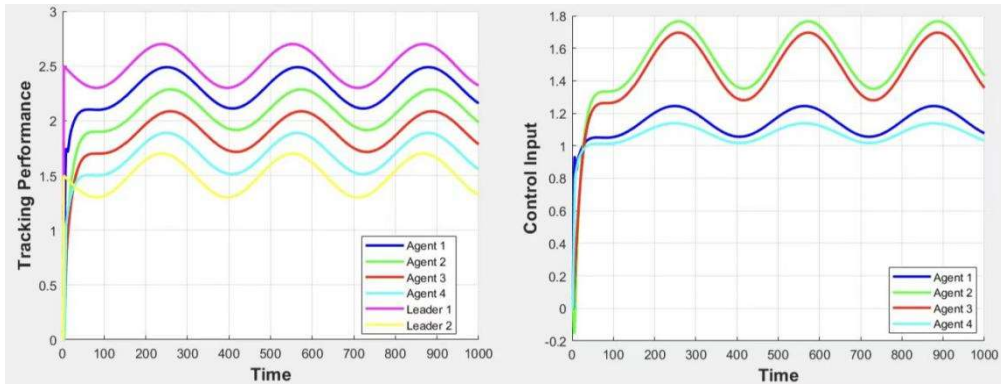


Figure 7. Comparison of Tracking Performance and Control Input

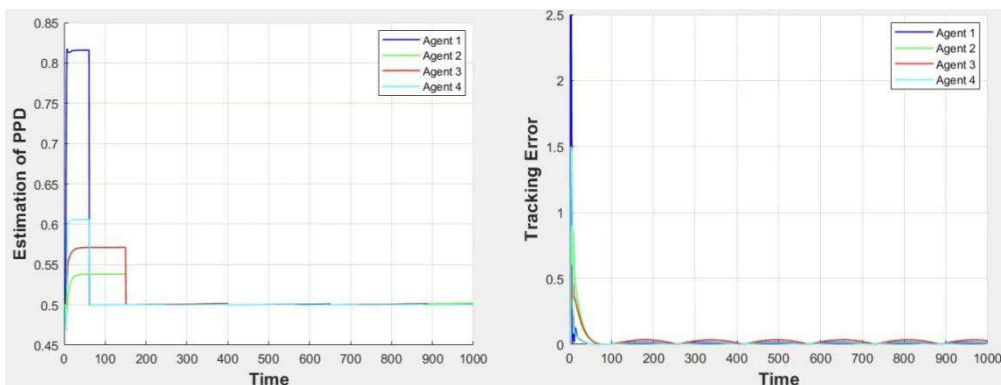


Figure 8. Comparison of PPD Estimation and Tracking Error

## 5. Conclusion

Aiming at the difficulty of highly cooperative control of multi-UAV formation under nonlinear dynamics, communication limitation and malicious attacks, the data-driven CFDL-MFAC is adopted to transform the nonlinear system into a compact dynamic linear model based on pseudo-partial derivatives, and a distributed control law and pseudo-partial derivative estimation algorithm are designed. In addition, a reset mechanism is introduced to deal with data packet loss. The theoretical analysis verifies the tracking error convergence and bounded input and bounded output stability of the system under constant and time-varying trajectories. In the future, the compensation mechanism can be constructed by weighted fusion of effective data and historical information for partial information loss and actuator fault expansion research. Meanwhile, hardware faults such as UAV wings and sensors are included in the analysis, which further improves the adaptability of the algorithm under complex actual working conditions.

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