Models Comparison and Strategy Analysis of Futures Option Pricing and Arbitrage

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Abstract

Futures options are important derivative instruments in modern financial markets, playing a crucial role in risk management, price discovery, speculation and arbitrage. Scientific and accurate pricing is not merely related to investors' trading decisions, but directly affects the effectiveness of arbitrage strategies and the operational efficiency of the market. After reviewing relevant theories, this paper conducts a comprehensive comparison of several typical futures option pricing models, including the Black-Scholes model, binomial tree model and Monte Carlo method, stochastic volatility model (Heston model), GARCH-type models, and machine learning models. These models have both advantages and disadvantages in terms of pricing accuracy, applicable scenarios, and complexity. The operation of Black-Scholes model is simple and effective, but it neglects the dynamic characteristics of volatility. Stochastic volatility models and GARCH-type models can well explain the volatility smile, while machine learning methods have potential in prediction and nonlinear modeling. Furthermore, this paper introduces common arbitrage strategies based on the no-arbitrage principle, analyzes the arbitrage opportunities brought by model errors, and discusses the impact of transaction costs and liquidity constraints on arbitrage strategies. Through theoretical comparison and strategy analysis, it is argued that the rational selection and combination of pricing models is the key to improving the feasibility of arbitrage strategies, which provides reference significance for both the research and practice of the futures option market.

Keywords

Futures options, pricing models, no-arbitrage principle, arbitrage strategies, volatility.

1. Introduction

Futures options are important derivative instruments in modern financial markets. They play a vital role in risk management and asset allocation, also serving as key tools for speculation and arbitrage[1]. With the continuous development of China's futures market, the variety of futures options has gradually increased, and market participation has steadily risen. The method to properly fix the price of futures options has become a major concern shared by both the academic and practical communities. Whether a pricing model is scientific and reasonable goes beyond investors' trading decisions, to the effectiveness of arbitrage strategies and the operational efficiency of the market. In terms of pricing research, the Black-Scholes model is a representative classic model. Its formula is simple and clear, providing a theoretical foundation for the development of the derivatives market[2]. However, due to its overly idealized assumptions, such as constant volatility and a market without friction, it can hardly fully align with actual market conditions. To address the deficiency, scholars have proposed methods including the binomial tree model, stochastic volatility models (e.g. the Heston model), and GARCH models. In recent years, new approaches like machine learning have also been attempted to improve pricing accuracy. These models both have their own advantages and disadvantages in coping with issues such as volatility smiles, jump risks, and nonlinear characteristics. Conducting a systematic comparison of the application value of different models in futures option pricing, with further exploring their role in the construction of arbitrage strategies, extends beyond significant theoretical value to providing practical guidance for investment operations.

2. Common Futures Option Pricing Models

2.1. Black-Scholes Model

The Black-Scholes model (B-S model), proposed by Fischer Black and Myron Scholes in 1973, is the most classic model in option pricing theory [3]. The core idea of this model is based on the no-arbitrage principle and risk-neutral pricing, which derives the theoretical price of European options by constructing a replicating portfolio. The main assumptions of the B-S model include: ①The financial market is perfectly efficient and frictionless, meaning that there are no transaction costs or taxes; ②The price of the underlying asset follows a geometric Brownian motion, and its dynamics satisfy

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

Among them, S_t denotes the price of the underlying asset, μ represents the expected return rate of the asset, σ stands for the constant volatility, and W_t is the standard Brownian motion; ③ The risk-free interest rate r remains constant; ④Early exercise is not allowed, and the model is only applicable to European options.

Under the above conditions, the pricing formula for a European call option is as follows:

$$C = S_0 N(d_1) - K e^{-rT} N(d_2) \tag{2}$$

Among which:

$$d_1 = \frac{\ln\frac{S_0}{K} + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T} \tag{3}$$

While the price of put option is:

$$P = K e^{-rT} N(-d_2) - S_0 N(-d_1) \tag{4}$$

Among them, S_0 denotes the initial price of the underlying asset, K represents the strike price, T stands for the time to maturity, $N(.)$is the standard normal distribution function.

The advantages of the Black-Scholes (B-S) model lie in its rigorous derivation, concise formulas, and wide application, as well as providing a theoretical foundation for the development of the derivatives market. However, its flaws can also be seen obviously [3]. For instance, it assumes constant volatility, which fails to explain the "volatility smile" in the market. Additionally, the model does not take jump risks and liquidity constraints into account, so it may lead to significant pricing errors in complex market environments. Nevertheless, the B-S model is still regarded as the cornerstone of option pricing theory and serves as an important reference for comparing and improving other models.

2.2. Binomial Tree Model and Monte Carlo Simulation

The Binomial Tree Model, proposed by Cox, Ross, and Rubinstein in 1979, stands for a discrete-time pricing method. The model describes the future movement of the underlying asset price using two scenarios, which are "upward movement" and "downward movement". It treats such movements as a binomial branch and then calculates the option value step by step [4]. The advantage of this model is intuitive and flexible to operate, making it well-suited for valuing American options in that the model can account for the possibility of early exercise at each node. However, when the option's time to maturity is lengthened, the binomial tree requires more iterations to ensure accuracy, leading to a sharp increase in computational load. Monte Carlo Simulation is a numerical method that relies on random number generation and probability statistics. It is particularly suitable for handling derivatives with strong path dependence or complex structures. Its main content is to simulate a large number of potential underlying asset price paths, then to calculate the expected value of the option's payoff at maturity, finally to derive the present value through discounting. While Monte Carlo Simulation offers high flexibility, it requires a sufficiently large sample size to reduce errors, resulting in relatively low computational speed. Overall, though both the Binomial Tree Model and Monte Carlo Simulation are numerical solution methods that provide significant support for practical option pricing, there still exists a trade-off between their efficiency and accuracy.

2.3. Stochastic Volatility Model (Heston Model)

To overcome the limitation of "constant volatility" in the Black-Scholes model, Steven Heston proposed the stochastic volatility model (the Heston Model), which has turned into a well-known model in financial market [5]. The core idea of this model is to model the underlying asset price and volatility simultaneously. It assumes that the underlying asset price still follows a stochastic process, while volatility itself also undergoes a random evolution, which enables the model to explain the "volatility smile" phenomenon that is widely observed in the market.

The basic form of the Heston Model is:

$$\mathrm{dS_t = \mu S_t dt + \sqrt{v_t} S_t dW_t^S} \tag{5}$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma_v \sqrt{v_t} dW_t^v \tag{6}$$

Among them, S_t represents the asset price at time t, v_t stands for the instantaneous variance; κ is the mean reversion speed, θ means the long-term variance level, σ_v denotes the volatility of volatility, W_t^S and W_t^v are two correlated Brownian motions. This specification implies that volatility is not merely stochastic but with property of mean-reverting.

Under the risk-neutral measure, the pricing formula for a European call option is:

$$\mathrm{C(S_0, v_0, t) = S_0 P_1 - Ke^{-r}\ \ P_2} \tag{7}$$

Among which,

$$\mathrm{P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left[\frac{e^{-i\phi ln}\ \ f_j(\phi)}{i\phi}\right] d\phi, \quad j = 1{,}2} \tag{8}$$

$f_j(\phi)$ stands for the characteristic function, which allows for a semi-analytical solution meanwhile avoids the complexity of full numerical simulation.

There are three main advantages of the Heston Model: ① It can explain volatility smiles and skews; ② It retains a certain degree of analytical solution, resulting in relatively high computational efficiency; ③ It has clear economic implications, with parameters that carry financial interpretations [5]. However, it has several drawbacks. It involves a relatively large number of parameters, making estimation difficult, especially the "volatility of volatility" and correlation coefficient, which are hard to estimate stably in empirical studies. Additionally, although the model can describe volatility dynamics, it still cannot fully capture jump risks and extreme events in the market. In a word, the Heston Model is a crucial extension of the Black-Scholes Model [2]. It occupies a crucial position in both academic research and practical applications, laying a theoretical foundation for the subsequent pricing of complicated derivatives and volatility modeling.

3. Arbitrage Mechanism and Basic Strategies

3.1. No-arbitrage Principle

The no-arbitrage principle is the basis of the pricing theory for financial derivatives. In an efficient market, if there are asset portfolios with equal risk and equal cash flows, their prices should be the same. Otherwise, investors can obtain risk-free returns through buying low and selling high [6]. Pricing models for futures options are derived based on the no-arbitrage principle. A typical example is the risk-neutral pricing concept in the Black-Scholes model, which involves constructing a replicating portfolio to equate the option price to the value of a combination of the underlying asset and the risk-free asset [2]. When the market price deviates from the theoretical value, arbitrage opportunities arise, driving the price back to a reasonable level. The no-arbitrage principle serves not just as a theoretical constraint but as a crucial mechanism for fair market competition and price stability. Different arbitrage strategies are usually designed to identify and exploit such price deviations, thereby generating profits as the market corrects itself. Therefore, understanding and applying the no-arbitrage principle constitutes the theoretical basis for studying futures option arbitrage.

3.2. Model Errors and Arbitrage Opportunities

Different option pricing models show pricing errors due to differences in their underlying assumptions and parameter estimation methods. The deviation between the theoretical price and the market price is the source of arbitrage opportunities. For example, if the Black-Scholes model overestimates the price of an option, while the Heston model or the actual market price indicates that the option should be cheaper, investors can sell out the overvalued option and buy a hedging portfolio to realize arbitrage. Similarly, inadequate volatility modeling can also lead to biases in option pricing, thereby enabling straddle or calendar arbitrage [7]. When capturing complex nonlinear features, machine learning models might identify pricing discrepancies that traditional models fail to detect, and these discrepancies can also serve as arbitrage signals. However, real-world markets involve transaction costs, liquidity constraints, and risk exposure, all of which reduce the likelihood of converting model errors into arbitrage profits. Thus, when applying arbitrage strategies in practice, apart from relying on the price-discovery function of models, investors should also adjust the strategies according to market frictions.

4. The Impact of Different Models on Arbitrage Strategies

4.1. Arbitrage under the Black-Scholes Model

Under the Black-Scholes model, option prices are derived from a series of strict assumptions. Theoretically, the calculated option price should equal to the market price. However, real-world markets often deviate from these idealized conditions, which creates arbitrage opportunities.

When volatility smile appears in the market, the option prices computed by the Black-Scholes model typically show systematic differences from actual prices. Arbitrageurs can exploit such discrepancies by constructing straddle or butterfly strategies to make profits when market volatility reverts to normal levels [8]. The Black-Scholes model fails to take issues like early exercise and transaction frictions into account, leading to larger pricing biases in American options and complex market environments. Under this model, arbitrage opportunities are usually identified by comparing differences between "theoretical prices" and "market prices". Its greater value is presented in providing benchmark pricing and revealing market inefficiencies.

4.2. Arbitrage under Stochastic Volatility and GARCH Models

Stochastic volatility models (e.g. Heston) and GARCH-type models characterize the dynamic properties of volatility more properly, enabling them to better explain market phenomena such as volatility smiles and skews. These models allow arbitrageurs to more precisely identify deviations between option prices and their market values, thereby enhancing the effectiveness of arbitrage strategies. For instance, if the Heston model indicates certain options are overvalued and the conditional volatility predicted by GARCH model supports this conclusion, investors can sell the overvalued options and implement hedging to achieve arbitrage. As these models better reflect the time-varying nature of market volatility, arbitrage strategies that are based on them are more dependent on market conditions and better suited for short-term volatility-driven trading opportunities. Whereas the models themselves are relatively complex, and parameter estimation errors might bring new risks. Therefore, in practical application, arbitrage strategies must balance model accuracy and complexity to avoid false arbitrage signals caused by "overfitting".

4.3. Arbitrage under Machine Learning Models

With the development of big data and artificial intelligence, machine learning methods have been applied to option pricing and arbitrage research. Machine learning models are able to capture nonlinear relationships and hidden features in the market, identifying price discrepancies that traditional models ignore. For example, neural network or random forest-based methods can predict implied volatility, using the difference between predicted values and actual market values as arbitrage signals. Compared to classic models, machine learning offers advantages of flexibility and higher accuracy, particularly in handling high-dimensional data and complex market structures. However, there are also some drawbacks, such as poor interpretability and over-reliance on data, which could lead to a "black box" problem. In terms of arbitrage, machine learning models is able to help dynamic trading strategies identify opportunities in real time but require cross-validation with traditional models to improve strategy stability.

4.4. Transaction Costs and Liquidity Constraints

Even if different models successfully identify arbitrage opportunities, real-world transaction costs and liquidity constraints significantly impact strategy execution. Firstly, transaction costs, including bid-ask spreads, commissions, and slippage, would erode much of the arbitrage profit, rendering several theoretically viable opportunities unprofitable in practice. Secondly, liquidity constraints would possibly prevent large trades from executing at expected prices, potentially resulting in losses throughout the arbitrage process. For high-frequency arbitrage strategies, market impact costs are particularly severe. Most models assume a "frictionless market" when identifying arbitrage opportunities, which does not align with the reality. Therefore, when using arbitrage models, investors should not merely evaluate pricing errors but consider actual market conditions and costs to ensure strategy feasibility. Transaction costs and liquidity constraints are critical factors in transforming arbitrage from "theoretically feasible" to "practically feasible".

5. Conclusion

This paper compares different futures option pricing models and provides a theoretical analysis by integrating arbitrage mechanisms and strategies. The study yields several key findings. First, the Black–Scholes (BS) model, as a classical framework, remains the most concise and widely applied pricing tool; however, its reliance on idealized assumptions often leads to significant deviations under volatile market conditions or when frictions occur. Second, the binomial tree model and Monte Carlo simulation offer greater flexibility and are well suited to valuing American options and complex derivatives, though they demand extensive computational resources due to repeated simulations. Third, stochastic volatility models, such as the Heston and GARCH frameworks, demonstrate stronger capabilities in capturing the dynamic nature of volatility, effectively explaining phenomena like volatility smiles and providing a richer foundation for identifying arbitrage opportunities. In addition, emerging machine learning approaches show unique advantages in detecting nonlinear and high-dimensional patterns, thereby uncovering pricing inefficiencies that traditional models may overlook.

Overall, the research indicates that no single model is universally optimal; the effectiveness of arbitrage strategies depends heavily on the chosen pricing methodology and the broader market environment. Nonetheless, the study's main limitation lies in its reliance on theoretical discussion and case-based evidence, with limited large-scale empirical validation. Future research should integrate advanced machine learning with reinforcement learning to enhance predictive accuracy and adaptability, while also incorporating high-frequency trading, order book dynamics, and liquidity shocks to build more robust and practical arbitrage models suitable for increasingly globalized and complex financial markets.

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