

A New Optimal Water Level Evaluation Strategy of the Great Lakes

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Abstract

With the change of environment and the change of social supply and demand, the current optimal water level evaluation methods for the Great Lakes in North America are outdated in many aspects. Therefore, proposing new optimal water level evaluation strategies is crucial for water level management and control of the Great Lakes. The determination of the optimal water level for the Great Lakes is obviously a complex optimization evaluation problem. Therefore, we have developed a multi-objective optimization model based on the idea of multi-objective nonlinear programming algorithm. We first use the AHP method to comprehensively analyze the various factors that affect the water level of the Great Lakes, and then substitute them into the Monte Carlo Simulations model to find the approximate range of optimal water level for each lake. Subsequently, taking this range as the initial value, we employed the multi-objective nonlinear programming algorithm to further solve the problem, ultimately obtaining the optimal water level situation for the Great Lakes that can balance the current social demand.

Keywords

The Great Lakes; optimal water level evaluation strategies; Monte Carlo Simulations; AHP; Multi-Objective Nonlinear Programming.

1. INTRODUCTION

Composed primarily of Lake Superior, Lake Michigan, Lake Huron, Lake Erie, and Lake Ontario, the Great Lakes are the largest freshwater lake group in the world, accounting for approximately 20% of the world's total freshwater reserves. Throughout history, these lakes and their adjoining watersheds have been vital sources of various natural resources, including water, fisheries, and shipping. The shores of the Great Lakes are home to numerous significant cities and harbors, supporting nearly 400,000 people. Consequently, monitoring and timely regulation of the water level, water flow, and other factors in the Great Lakes are crucial for sustaining the productive lives of the people in the region.

There are certain differences in water levels between different lakes in the Great Lakes. The determination of the optimal water levels at any time of the year requires a comprehensive consideration of various factors. Beyond basic navigation safety, it is also influenced by economic, social, and environmental factors.

Therefore, establishing a multi-objective optimization model that comprehensively reflects the interests of all parties is a suitable choice. Maximizing the interests of various stakeholders

is set as the primary objective, with considerations for seasonality, precipitation, water flow balance, and other factors as constraint conditions.

2. MODEL DESCRIPTION

The selection of initial values has a significant impact on the iterative process and results of the multi-objective optimization algorithms, selecting an appropriate initial value for iteration can effectively shorten the number of iterations. Therefore, it is necessary and reasonable to first use Monte Carlo simulation to preliminarily determine the approximate range of the optimal depth of the lake that satisfies the constraints of the multi-objective optimization model, and then use multi-objective Nonlinear programming algorithms to find the optimal depth of the Great Lakes. Based on this, we ultimately determined the optimal average water level for each lake in the Great Lakes.

To maximize the interests of all stakeholders, we have considered the economic benefits, social benefits, environmental benefits from three aspects and ultimately established a multi-objective optimization model that can maximize overall benefits.

2.1. Notations

The primary notations used are listed in **Table 1**.

Table 1. Index weight matrix

Symbol	Description
<i>i</i>	the <i>i</i> th lake in the Great Lakes
<i>j</i>	the <i>j</i> th river in the Great Lakes
<i>h_i</i>	Mean Water Level of <i>i</i> _{th} Lake
<i>S_i</i>	Surface area of <i>i</i> _{th} Lake
<i>r_j</i>	Average Flow Rate of <i>j</i> _{th} River
<i>m</i>	month
<i>k</i>	Hydropower station number
<i>P_g</i>	The power generation of hydropower stations
<i>S_q</i>	The shipping benefits of the Great Lakes
<i>sc</i>	Seasonal factors of shipping
<i>tp</i>	Transportation costs of shipping
<i>cr</i>	Economic benefits of shipping

2.2. The Objective

(1) Maximization of economic benefits

As of now, the water flow in the North American Great Lakes system is primarily controlled by two mechanisms—the compensating works at Sault Ste. Marie (consisting of three hydroelectric plants, five locks, and a dam located at the head of the rapids) and the Moses-Saunders Dam in Cornwall. The electricity generation of a hydroelectric power station is directly related to the water level of the reservoir. The specific formula is as follows:

$$P_g = h_i \cdot r_j \cdot \eta \cdot g \tag{1}$$

In the formula, h_i represents the mean water level of the i th lake, r_j is the average water flow of the j th river, η is the efficiency of the hydroelectric power station, and g is the local acceleration due to gravity.

For hydropower stations in the Great Lakes region, the goal is to maximize the total annual power generation to maximize economic benefits.

$$\max \left(\sum_{m=1}^{12} \sum_{k=1}^n P g_{mk} = h_i \cdot r_j \cdot \eta_{mk} \cdot g_{mk} \right) \quad (2)$$

In the formula, m represents the month, k represents the number of the hydropower station.

(2) Maximization of shipping benefits

The shipping efficiency in the Great Lakes region is closely related to the depth of the lakes in this area, and is also affected by other factors such as seasonal variations, transportation costs and local economic development.

$$Sg = w_1 (h_i - h_{min}) \cdot sc - w_2 \cdot tp + w_3 \cdot cr \quad (3)$$

In the formula, w_1 , w_2 and w_3 are weight coefficients, sc is the seasonal factor, tp is the shipping benefit, and cr is the economic benefit.

By changing sc_m in different months, we can achieve the impact of seasonal factors. The significant effects of global climate change on international commercial navigation in the Great Lakes may include changes in water levels and a shorter duration of ice cover. Ice formation closes the Great Lakes-St. Lawrence River system for at least two months each year. Therefore, we have set a seasonal factor to enhance the robustness of the model.

$$\max \left(\sum_{m=1}^{12} Sg_m = w_1 (h_i - h_{min}) \cdot sc_m - w_2 \cdot tp_m + w_3 \cdot cr_m \right) \quad (4)$$

2.3. The Constraints

(1) Climate conditions and weather factors

The weather conditions in the Great Lakes region significantly impact the determination of the optimal water level. For instance, excessive evaporation or in-adequate discharge can lead to a decrease in water levels, hindering navigation. Conversely, low evaporation or excessive discharge can result in rising water levels, affecting the daily lives of coastal residents and personal security.

Additionally, weather factors themselves are unpredictable. In order to reasonably reflect the influence of weather conditions on the water level of the Great Lakes, it is a suitable choice to introduce a random disturbance factor representing weather conditions. Using MATLAB to configure this factor to generate random disturbances, simulating the influence of different weather conditions on the water level of the Great Lakes which reflects the impact of the weather on the objective function.

$$unifrnd(i, m) \quad (5)$$

The output of the *unifrnd* function can be positive or negative. Therefore, by configuring random disturbance factors that represent the impact of weather conditions, we can simulate the effects of different weather conditions on the water levels of the Great Lakes.

(2) The minimum navigable water level

When sailing in the low-water areas of the Great Lakes, vessels may collide or run aground. For safety reasons, it is essential to establish a lower water level limit h_{safety} for shipping to ensure the secure passage of vessels through the basin. During other navigable times, it should be ensured that

$$h \geq h_{safety} \tag{6}$$

(3) The preferred water level for residents

Unlike the high-water levels anticipated by the shipping industry, individuals overseeing shipping terminals or residing in the vicinity of the Port of Montreal, homeowners along the shores of Lake Ontario, as well as recreational boaters and fishing boats on Lake Ontario, prefer mid-level and stable water levels.

To ensure the safety of these groups, it is imperative to establish an upper limit on the water level. Excessively high-water levels pose a significant risk of triggering serious natural disasters, such as floods, which can result in substantial damage to nearby residents, homes, and infrastructure. Therefore, we define

$$h \leq h_{max} \tag{7}$$

(4) Water resource conservation

The Great Lakes are the largest freshwater lake group in the world, accounting for approximately 20% of the global total volume of freshwater lakes. In order to prevent excessive exploitation of water resources from the Great Lakes, the optimal expected water level from the model should better protect the freshwater resources of the Great Lakes.

$$\sum_{i=1}^5 WP_{hi} \geq \sum_{i=1}^5 WP_{Hi} \tag{8}$$

In the formula, WP_{hi} is the water resource protection for the i_{th} lake, and WP_{Hi} is the previous water resource protection for the i_{th} lake.

(5) Water flow balance

The inflow and outflow of water in the Great Lakes are crucial indicators directly reflecting changes in the water levels of the Great Lakes. The Great Lakes water system is complex but exhibits certain regularities, with water levels gradually decreasing from west to east. This implies that the overall flow direction of the major rivers is also from west to east (**Figure 1**).

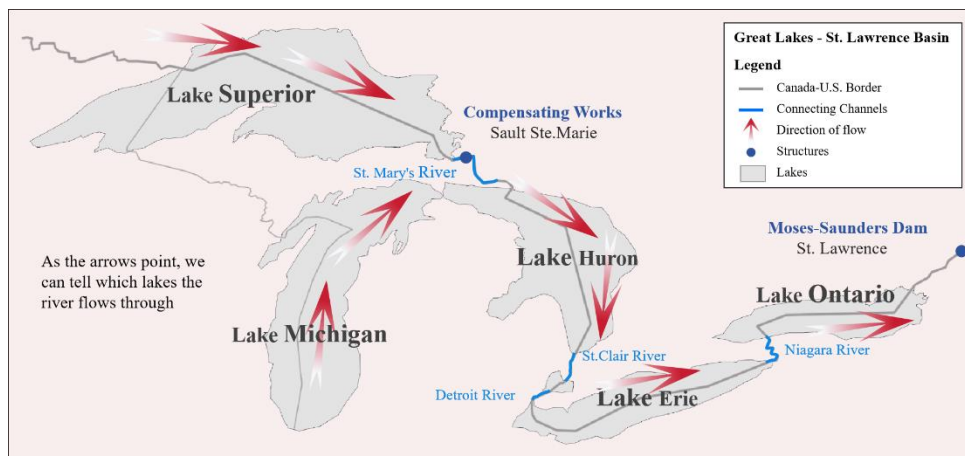


Figure 1. River Flow Diagram

Therefore, we can formulate a balance equation for the water flow in the Great Lakes to constrain changes in their water levels.

$$W(i, m) = W(i, m-1) + \sum_{m-1}^m [WR(j-1, m) - WR(j, m)] \tag{9}$$

In the formula, $W(i,m)$ and $W(i,m-1)$ represent the water flow of the j th river in the m th month and the $(m-1)$ th month, respectively. $WR(j-1,m)$ and $WR(j,m)$ represent the water flow rates in the upstream and downstream rivers of the i th lake, respectively.

Based on the above discussion, we have comprehensively considered various influencing factors such as economic benefits, social benefits, environmental protection, safety, and weather impact. We have ultimately established a multi-objective optimization model, and the formulas included in the model are as follows:

$$\begin{aligned}
 & \max \left(\sum_{m=1}^{12} \sum_{k=1}^n P g_{mk} = h_i \cdot r_j \cdot \eta_{mk} \cdot g_{mk} \right) \\
 & \max \left(\sum_{m=1}^{12} S q_m = w_1 (h_i - h_{min}) \cdot s c_m - w_2 \cdot t p_m + w_3 \cdot c r_m \right) \tag{10} \\
 & St. \begin{cases} unifrnd(i,m) \\ h_{safety} \leq h \leq h_{max} \\ \sum_{i=1}^5 W P_{hi} \geq \sum_{i=1}^5 W P_{Hi} \\ W(i,m) = W(i,m-1) + \sum_{m-1}^m [WR(j-1,m) - WR(j,m)] \end{cases}
 \end{aligned}$$

3. MODEL IMPLEMENTATION AND RESULTS

The optimal water level of any lake will be affected by seasonal factors; However, over-refinement of the date will greatly increase the calculation and the result will be unsatisfactory. Both, we calculate the twelve months of a year as twelve time periods.

3.1. Determination of Weights of Each Objective Function

Our ultimate goal is to determine the optimal monthly average level of the Great Lakes. Overall, we can consider it from three aspects: economic benefits, social benefits, environmental benefits, it can be divided into the following factors specifically:

1. Economic benefits: Power generation, Shipping, Water supply, Irrigation
2. Social benefits: Recreation, Fishing, Construction
3. Environmental benefits: flood and drought protection, survival and reproduction of plants and animals

Based on this, we obtained the Analytic Hierarchy Process (AHP) diagram as shown below (Figure 2).

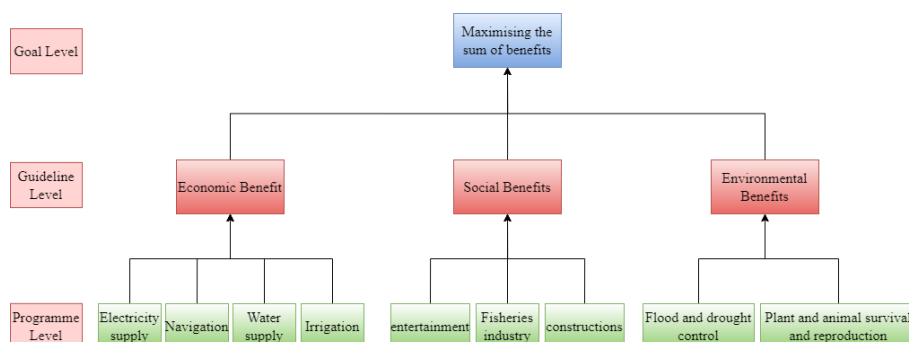


Figure 2. Hierarchy diagram

Finally, the weight matrix obtained through the AHP method is shown in the **Table 2**.

Table 2. Index weight matrix

Comprehensive impact	Economic benefits	Social benefits	Environmental benefits	Weight (%)
Economic benefits	1	5	2	58.126
Social benefits	1/2	1	3	30.915
Environmental benefits	1/5	1/3	1	10.959

Consistency Ratio: $0.004 < 0.1$

3.2. Determination of the approximate range of optimal lake depth

The basic idea of Monte Carlo simulation is to approximate the solution to a problem by generating a large number of random numbers. By repeatedly executing a random process, it aims to infer the possible optimal solution of the problem. As its definition suggests, the high level of randomness in Monte Carlo also means that it cannot provide a highly accurate solution to the problem. Instead, it gives an approximate range of the optimal solution, which is often sufficient for practical problem-solving.

The code logic of Monte Carlo simulation is shown in the **Table 3**.

Table 3. The code logic of Monte Carlo simulation

Algorithm: Monte Carlo method
Input: Arrays of random depths for each month in the Great Lakes and related river water levels
Output: Optimal values for the depths of the Great Lakes and the corresponding river flow data at that time
1: For $j = 1:12$
2: Generate random arrays for the depths of the Great Lakes and the related river flow data for the j^{th} month.
3: end
4: for $i = 1: 100000$
5: for $j = 1:12$
6: Import the lake depth and river flow data (x_{ij}, y_{ij}) for the j^{th} month.
7: If x_{ij} and y_{ij} satisfy the constraints (st.).
8: For $k = 1:5$
9: Calculate the values of the objective functions and sum them, then record the result in the variable <i>result</i> .
10: end
11: end
12: If $result > result_max$
13: $result_max = result;$
14: Record the lake depths and the flow data of relevant rivers for the current state of the Great Lakes.

15: end
 16: end
 17: end

Based on the Monte Carlo algorithm we developed, we ran the program repeatedly and ultimately determined the optimal range of lake depths for each month of the year, as shown in the following **Figure 3**.

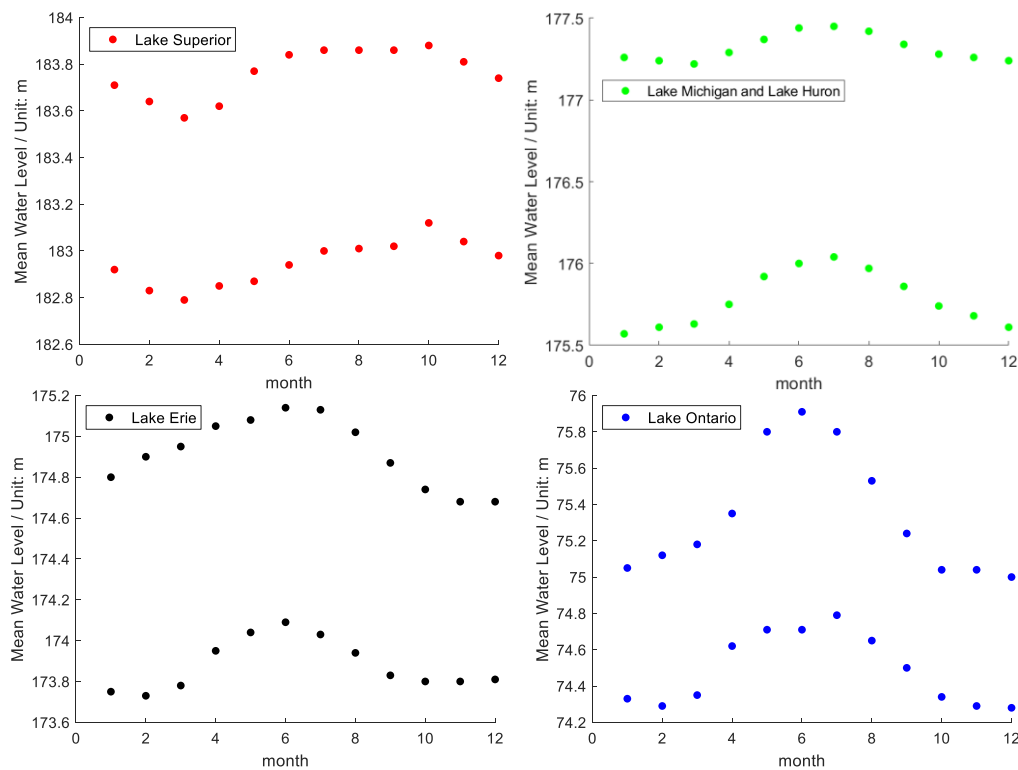


Figure 3. The water level range for the Great Lakes

3.3. The optimal solution for lake depths

The principle of multi-objective nonlinear programming requires that there can only be one final discriminating objective function. Therefore, based on the results of the Analytic Hierarchy Process (AHP) mentioned earlier, we calculate the weights of each objective function and accumulate the impacts caused by multiple objectives. The final weights of the objective functions are shown in the following **Table 4**.

Table 4. eights of objective functions

The Objective Function	<i>Pg</i>	<i>Sq</i>
Weight	0.7495	0.2505

Consistency Ratio: 0.021 < 0.1

We used the lake depth range which determined by Monte Carlo simulation as the initial value and substituted it into the multi-objective nonlinear programming algorithm. Through this, the problem of finding the optimal lake depth in a large scale can be transformed into an

optimization problem of the objective function on a small range. Finally, we obtained the reference value of the best water level of each lake in the Great Lakes for every month of a year (Figure 4).

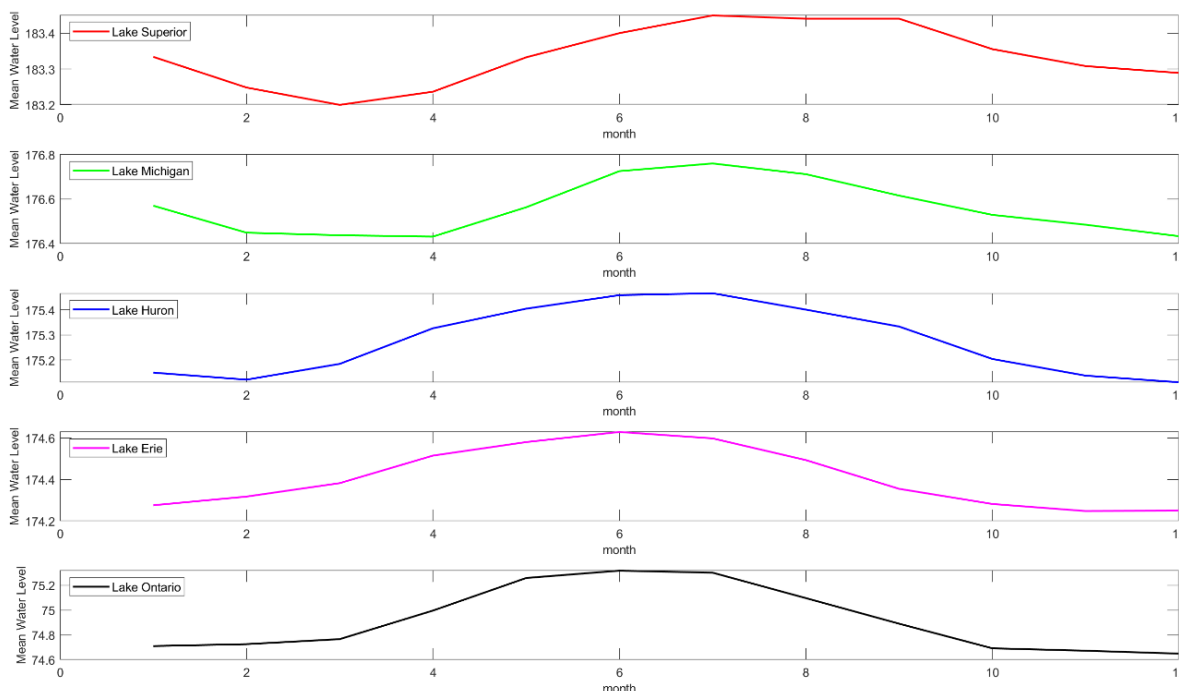


Figure 4. Optimal water levels for each month in the Great Lakes

According to the map, it is evident that Lake Michigan and Lake Huron are connected by a waterway, creating a connected water body geographically. Therefore, in the analysis of water flow, people often refer to Lake Michigan and Lake Huron as a single lake system, known as the Michigan-Huron. The water levels of these two lakes can influence each other, but they are not entirely identical. Thus, when analyzing the water levels of the lakes, we need to consider them as two independent bodies of water.

Based on relevant data, it is not difficult for us to draw a conclusion that level variations of the Great Lakes throughout the year are indeed within a very narrow range. This is consistent with our algorithm's determination of the optimal water levels.

To more intuitively depict the optimal water level variations of the Great Lakes, we calculated the annual averages of the water levels (Table 5). These allow us to obtain the difference between the monthly average optimal water level and the annual average optimal water level of each lake in the Great Lakes (Figure 5).

Table 5. Average annual optimal water level of the Great Lakes

Name	Lake Superior	Lake Michigan	Lake Huron	Lake Erie	Lake Ontario
Average	183.4633m	176.5843m	176.5634m	174.4253m	74.9247m

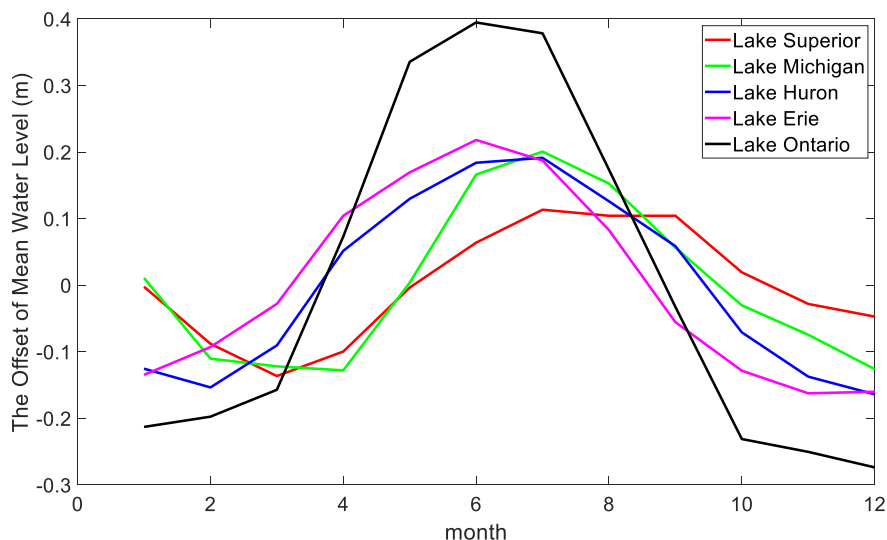


Figure 5. Monthly optimal water level offset for the each lake

4. CONCLUSION

Based on the results of the model implementation, we can conclude that the best water level data trends for the first four lakes of the Great Lakes (Lake Superior, Lake Michigan, Lake Huron, and Lake Erie) are almost identical, indicating that their water systems are more closely connected, and this conclusion is also supported by geographical maps. In addition, there are significant seasonal fluctuations in the optimal water level for each month of the year in the Great Lakes. From August to February of the following year, except for Lake Ontario, the optimal water levels of the other four lakes show a downward trend, the decline in the optimal water level of Lake Ontario only lasts until December. From February to July to August, the optimal water level of the Great Lakes shows varying degrees of increase, and this is consistent with the fact that lakes rise in water level during summer due to increased rainfall.

The North American Great Lakes have a typical temperate continental climate, with relatively warm summers and cold winters, and rainfall is distributed relatively evenly. In winter, the Great Lakes are also influenced by the lake-effect, where water vapor evaporates from the lakes and condenses into clouds, leading to snowfall when carried by the wind. Therefore, in winter, the optimal water levels of the Great Lakes exhibit a slight upward trend.

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