

An Improved Branch-and-Price Algorithm for Solving the Capacitated Vehicle Routing Problem

Yi Qu¹, Jing Fan²

¹School of Computer and Information Engineering, Shanghai Polytechnic University, Shanghai, China

²School of Mathematics, physic and Statistics, Shanghai Polytechnic University, China

Abstract

For the capacitated vehicle routing problem, this paper proposes an improved branch-and-price algorithm. This algorithm is based on the Dantzig-Wolfe decomposition principle, which decomposes the complex vehicle routing problem into a master problem and subproblems. The master problem is responsible for selecting the optimal transportation scheme from a set of candidate vehicle routes, while the subproblems generate new transportation schemes according to the feedback from the master problem. A special strategy which integrates hybrid greedy-simulated annealing and local search, is introduced to improve the generated partial routes, and to increase the solution speed. This approach provides higher-quality solutions for the master problem in each iteration. Furthermore, through small-scale and large-scale simulation experiments and comparative analysis, the accuracy and efficiency of the improved branch-and-price algorithm are verified.

Keywords

Vehicle Routing Problem; Branch-and-Price Algorithm; Column Generation Algorithm; Simulated Annealing; Local Search; Branch-and-Bound Algorithm.

1. INTRODUCTION

The development of autonomous driving holds great potential, and its technological advantages are reflected in several aspects^[1]. Its applications will have a positive impact on improving road safety, optimizing traffic flow, and lowering energy consumption. As one of the key elements, the routing programming plays a crucial role in ensuring the efficient and safe operation of autonomous vehicles^[2].

In fact, the routing programming of autonomous vehicle is essentially a well-known vehicle routing problem (VRP). This problem has been studied since the 1950s. In 1959, Dantzig and Ramser^[3] first introduced the Vehicle Routing Problem, in which some autonomous vehicles select paths to serve customers in order to minimize the total cost^[4]. The VRP is a classical NP-hard combinatorial optimization problem, and its computational complexity grows exponentially with the increasement of the number of customers.

The Capacitated Vehicle Routing Problem (CVRP) is a classical variant of VRP, where a load capacity is assigned to each autonomous vehicle^[5]. Under the condition that each customer is served only once, the objective of the CVRP is to determine the optimal delivery routes that minimize the total path length.

Algorithms for solving the CVRP can be classified into three categories^[6]: exact algorithms (e.g., the branch-and-bound methods), heuristic algorithms (e.g., the cluster-first-route-second strategy), and metaheuristic algorithms (e.g., the local search algorithm, the ant colony

optimization^[7-8], the genetic algorithm^[9-10]). Currently, the branch-and-price algorithm proposed by Barnhart et al.^[11], one of exact algorithm, is commonly used to solve the CVRP. This algorithm is based on an iterative approach, combining column generation and branch-and-bound technique to effectively determine the vehicle routes for autonomous vehicles.

According to the studies by Zhang Yuzhe et al.^[12], Li Decheng et al.^[13], Wang Sihan et al.^[14], Li Yanfeng et al.^[15], and Zhou Saiqi et al.^[16], some simple heuristic methods are generally employed to generate an initial solution. If the initial solution deviates large from the optimal solution, the convergence speed of the algorithm will decrease, and its running time will increase. To get rid of these limitations, this paper proposes an improved branch-and-price algorithm. In detail, it first uses a greedy algorithm to obtain the initial solution, ensuring that the solution satisfies the capacity constraints. Then, an integrated neighborhood search strategy is applied to improve the initial solution. Finally, an iterative combination of column generation and branch-and-bound method is used to determine the vehicle routes and their corresponding lengths.

2. PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

2.1. Description of CVRP

Consider a scenario where a batch of items needs to be delivered by an autonomous vehicle M from a distribution center 0 to various customers. After completing the deliveries, the autonomous vehicle must return to the distribution center. Let the set of customers be denoted as I , with each customer i having a demand q_i , and the distance between customers i and j is represented by d_{ij} . The capacity of the autonomous vehicle Q , i.e., its maximum load, is denoted by Q . This paper focuses on the case where the demand of each customer must be satisfied exactly once, and the total demand exceeds the capacity of the vehicle. Since the autonomous vehicle cannot complete all delivery tasks in one trip, the number of delivery trips must be more than one. And the total demands of the selected customers in each trip doesn't exceed the vehicle's maximum capacity Q . The objective of CVRP is to minimize the total trip length of the autonomous vehicle by the optimization of the customer set division and routing programming.

2.2. Mathematical Model

Table 1. Parameters and symbols

Parameters and Variables	Definition
V	The distribution center and the set of customers, $V = I \cup \{0\}$
Table 1. Parameters and symbols(continued)	
Parameters and Variables	Definition
k	The k-th set division of customers, including some customers, and a corresponding set of delivery routes
P	Set of customer divisions, i.e., $P = \{1, 2, 3, \dots, K, \dots\}$
x_{ij}^k	The 0-1 variable, which indicates whether the autonomous vehicle chooses the path from customer i to customer j in customer set k , $x_{ij}^k = 1$, Otherwise $x_{ij}^k = 0$
R_k	The set of routes corresponding to customer set k , which includes all possible routes that cover the customers involved in batch k
r	One route in R_k , $r \in R_k$
z_r^k	The 0-1 variable, when the route r is selected in batch k , $z_r^k = 1$, Otherwise $z_r^k = 0$
V_r	The subset of customers involved in the route r

The capacitated vehicle routing problem (CVRP) can be formulated as an integer programming. The relevant parameters and symbols are listed in Table 1.

The established mathematical programming model for CVRP is as follows:

$$\min Z = \sum_{k \in P} \sum_{i \in V} \sum_{j \in V \setminus \{i\}} d_{ij} \cdot x_{ij}^k \quad (1)$$

$$\text{s.t. } \sum_{k \in P} \sum_{j \in V \setminus \{i\}} x_{ij}^k = 1, \quad \forall i \in I \quad (2)$$

$$\sum_{i \in V_r} q_i \sum_{j \in V_r, j \neq i} x_{ij}^k \leq Q \cdot z_r^k \quad \forall k \in P \quad \forall r \in R_k \quad (3)$$

$$\sum_{j \in V \setminus \{0\}} x_{0j}^k - \sum_{j \in V \setminus \{0\}} x_{j0}^k = 1 \quad (4)$$

$$\sum_{r \in R_k} z_r^k = 1 \quad \forall k \in P \quad (5)$$

$$x_{ij}^k = 0 \text{ or } 1 \quad \forall i, j \in V, \forall k \in P \quad (6)$$

$$z_r^k = 0 \text{ or } 1 \quad \forall k \in P, \forall r \in R_k \quad (7)$$

Equation (1) is the objective function, which minimizes the total of autonomous vehicles. Equation (2) ensures that each customer is served exactly once. Equation (3) imposes a constraint that the total delivery quantity in each route r must not exceed the maximum capacity of the autonomous vehicle. Equation (4) stipulates that the autonomous vehicle must depart from the distribution center and then ultimately return to the distribution center. Equation (5) specifies that the delivery route of customer set k is uniquely selected from the set of candidate routes R . Equations (6) and (7) define two binary variables of CVRP.

2.3. Decomposition of the Model

To solve the CVRP by using the branch-and-price algorithm, the original problem needs to be decomposed. The decomposed master problem and subproblems are presented here, similarly with the descriptions in the references [11, 17–19].

2.3.1. The Master Problem

The parameters and variable required for formulating the mathematical model of the master problem are listed in Table 2.

Table 2. Master problems and symbol explanation

Parameters and Variables	Definition
d_r^k	The length of the route r in k , $d_r^k = \sum_{i,j \in V} d_{ij} \cdot x_{ij}^k$
a_{ir}^k	If the customer i is visited in route r of customer set k ; the value is 1 the value is 0 otherwise the value is 0, and $\sum_{j \in V, i \neq j} x_{ij}^k = \sum_{i \in V} \sum_{r \in R_k} a_{ir}^k$
Z_r	A 0-1 variable, which represents whether route r is selected in the solution.

The model of the master problem is:

$$\min Z = \sum_{k \in P} \sum_{r \in R_k} d_r^k \cdot Z_r \quad (8)$$

$$\text{s.t. } \sum_{r \in R_k} a_{ir}^k \cdot Z_r = 1 \quad \forall i \in V \setminus \{0\} \quad (9)$$

$$Z_r \in \{0,1\} \quad \forall r \in R_k \quad (10)$$

Equation (8) is the objective, just like Equation (1), which is to minimize of the total length of autonomous vehicles. Equation (9) aligns with Equation (2), to ensure that each customer is visited exactly once. Equation (10) specifies that the value is 1 when route r is selected in batch R_k , and 0 otherwise.

In general, the master problem is responsible for determining the customer set, under the constraint that the total load does not exceed the maximum capacity of the autonomous vehicle. Therefore, several routes are selected from the candidate route set R_k to construct a feasible solution. After the process of iterative column generation and branch-and-bound processes, the objective of minimizing the total routing length is achieved.

2.3.2. Subproblem

The subproblem model is formulated by minimizing the length of all feasible routes under the constraints (3) to (7) in the original problem.

$$\min \sum_{i \in V} \sum_{j \in V} \hat{d}_{ij} \cdot x_{ij}^k \quad (11)$$

$$\text{s.t. Constraints (3) } \sim (7) \quad (12)$$

where, $\hat{d}_{ij} = \hat{d}_{ij} - \pi_i$ represents the optimized cost from node i to node j , and π_i denotes the dual variable value for customer i . The adjustment mechanism of the variable coefficients in the objective function takes into account the influence of distance factors and customer-specific factors, which is the core of achieving route selection optimization. By adjusting the value of $\hat{d}_{ij}$ in response to changes in the dual variable π_i , we can iteratively optimize route selection. This process is known as the pricing process, which is a critical step in the column generation algorithm for identifying potentially optimal routes. Through continuous iterative updates, the algorithm will find the path with the lowest cost on the global optimization scale.

3. IMPROVED BRANCH-AND-PRICE ALGORITHM

Based on the branch-and-price algorithm^[11] proposed by Barnhart et al., this paper introduces an improved branch-and-price algorithm. The traditional branch-and-price algorithm is an optimization method that combines the branch-and-bound framework with column generation techniques. Branch-and-bound systematically partitions the solution space and progressively approaches the optimal solution, while column generation dynamically generates variables (columns) to efficiently solve the linear relaxation of large-scale problems.

Detailed explanations of the column generation algorithm, the improvement strategies, and the branch-and-bound algorithm are described as follows.

3.1. Column Generation Algorithm

The column generation algorithm is employed to solve the master problem. Its advantage lies in the ability to generate new route columns according to the local characteristics of the current solution, which can facilitate faster convergence to an improved solution.

The steps of the column generation algorithm are illustrated in Figure 1.

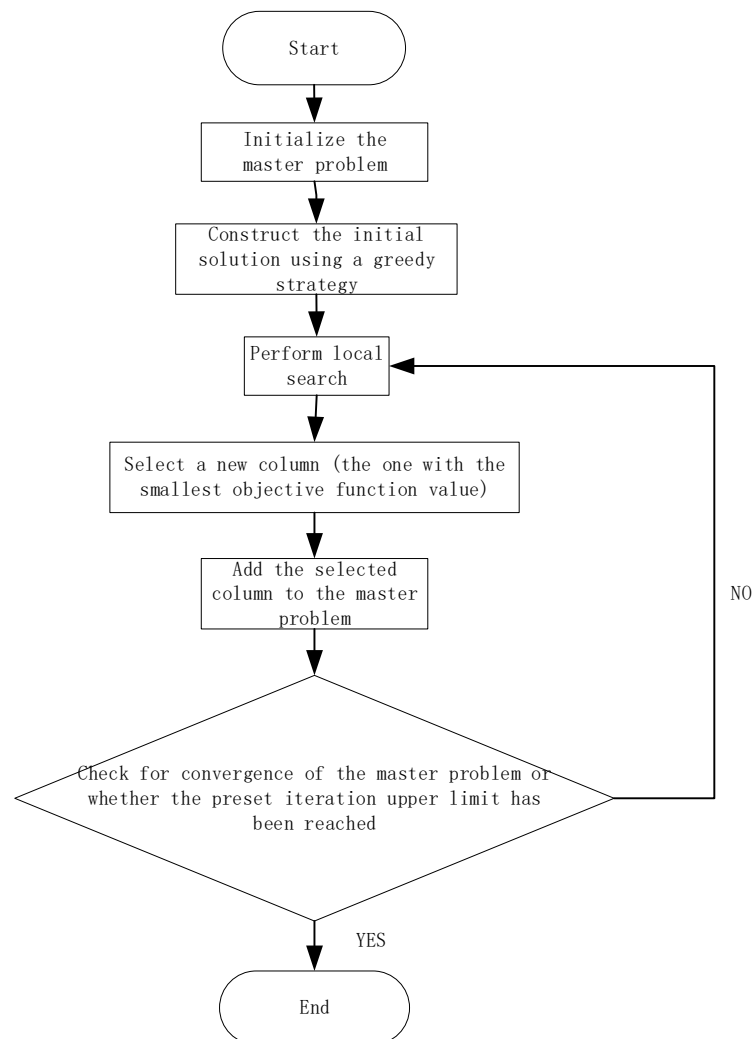


Figure 1. Flow chart of column generation algorithm solution

3.2. Improvement Strategy

During the process of the branch-and-price algorithm, the efficiency of the subproblem is regarded as the critical section to enhance the overall algorithm's performance. As a result, this paper proposes a Collaborative Hybrid Greedy-Simulated Annealing with Local Search (HGSA-LS) strategy.

First, a multi-starting greedy initialization method is used to generate multiple customer subsets, from which the best solution is selected as the initial solution. This process takes advantages of a greedy strategy to rapidly construct diverse sets of candidate solutions. Subsequently, the initial solution is refined through neighborhood search and local search, where the former search method neighboring solutions via exchanging operations to explore better solutions.

In order to prevent being trapped by local optimal solution, the simulated annealing mechanism is incorporated into the local search process. Simulated annealing regulates the balance between exploration and exploitation by gradually decreasing temperature parameter, to enable the algorithm to accept suboptimal solutions with a certain probability, such that the local optima can be broken. When neighborhood search fails to identify a better solution, perturbation operations are introduced to randomly adjust the current solution, to further enhance the algorithm's ability to break the local optima.

This strategy significantly accelerates the solution speed for the subproblems and provides higher-quality solutions for the master problem in each iteration.

3.3. Branch-and-Bound Algorithm

The branch-and-bound algorithm is a well-known method for solving discrete optimization problems by effectively reducing the number of candidate solutions through 'branch' and 'bound'.

The steps of the branch-and-bound algorithm are as follows:

- a. Branching: Starting from the depot, select a path to branch out and generate two child nodes.
- b. Bounding: Under capacity constraints, calculate the lower bound for each child node, which represents the minimum cost of completing the route from the current node to the depot via the child node.
- c. Pruning: Based on the current best-known solution, prune child nodes whose lower bounds exceed the current optimal solution or violate the vehicle's capacity constraints. Thus, ineffective searches are eliminated.
- d. Selection: Select the node with the lowest lower bound from the list of candidate nodes for further exploration, to enable a faster approach to the optimal solution.
- e. Iteration: Repeat the above steps until the optimal solution is found or all candidate nodes have been explored.

4. DATA SIMULATION EXPERIMENTS AND RESULT ANALYSIS

To validate the accuracy and efficiency of the improved branch-and-price algorithm, this paper conducts data simulation experiments and performs a comparative analysis with solutions obtained by using the traditional branch-and-price algorithm as well as the optimal solutions from standard benchmark datasets.

4.1. Operating Environment

The hardware and software configurations of the data experiment in this article are shown in Table 3.

Table 3. Hardware and software configuration

Hardware		Software	
CPU	i5-8250U	Programming Languages	Python 3.7.4
Memory	8192 MB	Operating System	Windows10
		Translation Tools	PyCharm 2019.1.2

4.2. Experimental Data

The experimental phase of this study utilized instances available from publicly accessible benchmark websites (<https://www.bernabe.dorronsoro.es/vrp/>, <http://vrp.atd-lab.inf.puc->

rio.br/index.php/en/) provided by Homberger and Gehring^[20] (abbreviated as H&G), Augerat et al.^[21], and Solomon^[22].

The first two files contain instances in varying scales, including customer location coordinates, demand volumes, and vehicle capacities, as well as the optimal solutions for these instances. The third file provides instances with 100 customers. These instances constitute classic benchmark datasets for the Capacitated Vehicle Routing Problem (CVRP) and are widely employed to evaluate algorithmic performance.

4.3. Experimental Results and Comparative Analysis

For several small-scale instances with fewer than 100 customers^[21], the improved branch-and-price algorithm demonstrates significant enhancements in solutions obtained at the first branching stage compared to the original branch-and-price algorithm (https://mp.weixin.qq.com/s/-C5zM8ClSeJR88xInoezjA) under the same branching conditions. For instance, in the SET-A example A-n44-k6, the objective value at the first branching stage using the improved algorithm is 1195.1, whereas the solution from the original algorithm is 4168.74.

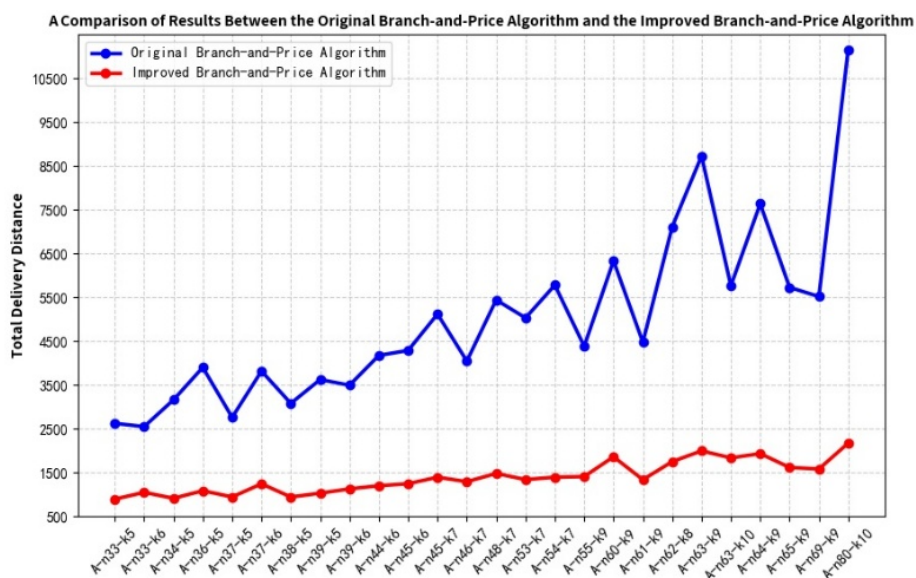


Figure 2. Comparison of SET-A solution results for each instance

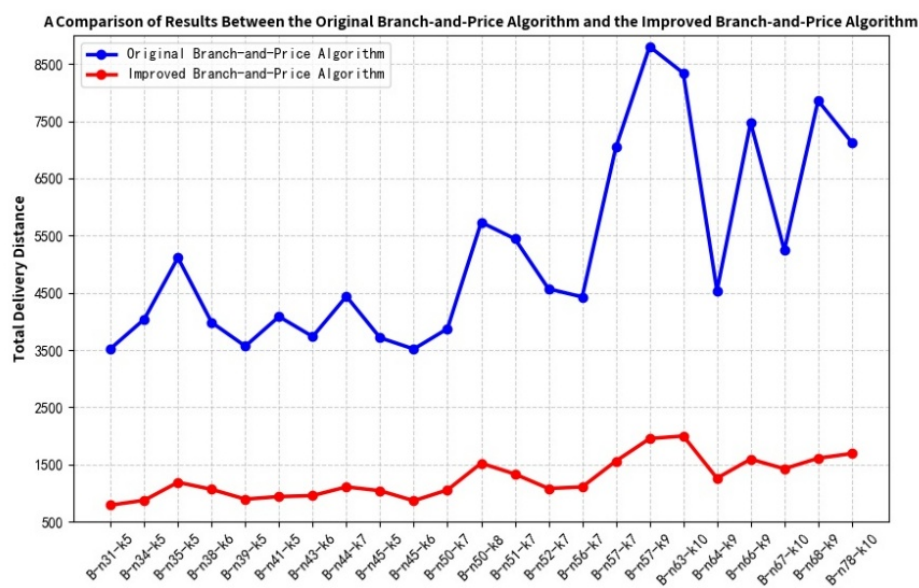


Figure 3. Comparison of SET-B solution results for each instance

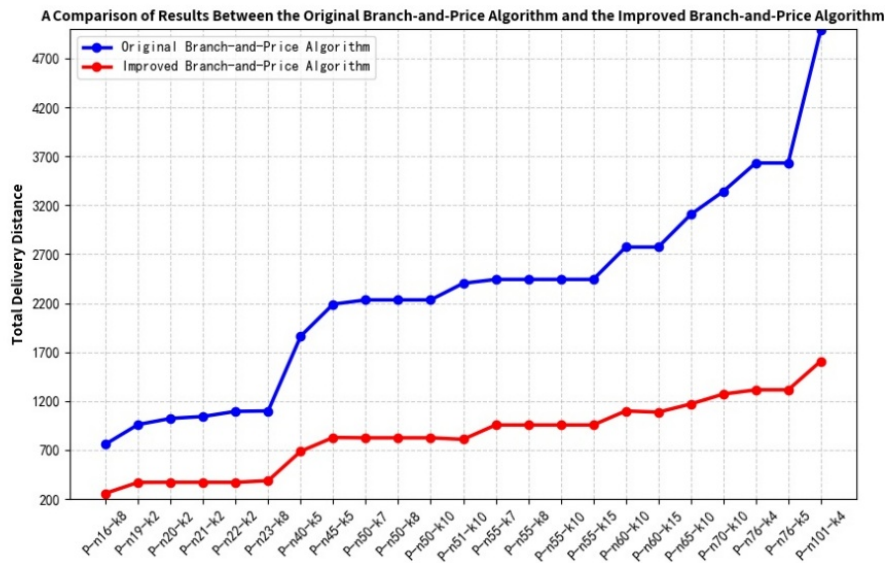


Figure 4. Comparison of SET-P solution results for each instance

Figure 2, 3, and 4 present the comparative results for instances in SET-A, SET-B, and SET-P, respectively. In these figures, the lower polyline represents the solutions from the improved algorithm, while the upper polyline corresponds to the original algorithm. The average optimization rates achieved by the improved branch-and-price algorithm are 70.95%, 75.65%, and 63.16% for SET-A, SET-B, and SET-P, respectively. These results underscore the improved algorithm's substantial advantages in enhancing both solution efficiency and quality.

Table 4. Comparison of CVRP standard calculation results

Instances		Vehicle Capacity	Number of Customers	Number of Iterations	Running time
Solo-mon	C101	200	100	2	3.62 s
	C201	700	100	1	2.74 s
	R101	200	100	5	8.68s
	R201	1000	100	12	12.22s
	RC101	200	100	4	5.92s
	RC201	1000	100	1	2.94s
H&G	C1_2_1	200	200	1	27.30 s
	C2_2_1	700	200	1	15.24 s
	R1_2_1	200	200	2	25.38 s
	R2_2_1	1000	200	2	22.01 s
	RC1_2_1	200	200	2	35.83 s
	RC2_2_1	1000	200	1	21.70 s
	C1_4_1	200	400	1	3.60min
	C2_4_1	700	400	2	3.04min
	R1_4_1	200	400	6	7.64min
	R2_4_1	1000	400	1	1.81min
	RC2_4_1	1000	400	1	2.44min
	C2_6_1	700	600	1	6.94min
	R1_6_1	200	600	2	13.23min
	R2_6_1	1000	600	1	6.76min
	RC1_6_1	200	600	3	22.40min

Large-scale instances can fully validate the performance of the algorithm. Therefore, instances with 200, 400, and 600 customers from the H&G dataset, as well as instances with 100 customers from the Solomon dataset, are selected for evaluation. Table 4 presents the number of iterations and the runtime of the improved branch-and-price algorithm for solving specific instances. It is notable that when the number of customers is fixed, an increase in vehicle capacity can reduce the runtime. With the exception of few cases where longer running times, the median time required to solve instances with 100, 200, 400, and 600 customers is 4.77s, 23.70s, 2.74 minutes, and 10.09 minutes, respectively. This fully demonstrates the improved algorithm's effectiveness and efficiency in handling large-scale instances.

5. CONCLUSION

The improved branch-and-price algorithm is designed for the vehicle routing problem with capacity constraints. This method introduces a multi-start greedy initialization, exchange neighborhood search, and local search, while integrating a hybrid strategy that combines simulated annealing within the local search process. This approach ensures that each search starts from a high-quality solution and escapes from local optima.

By improvements from the traditional branch-and-price algorithm, the effectiveness and efficiency of the proposed algorithm have been preliminarily validated. Moreover, data experiments on small-scale and large-scale instances demonstrate the accuracy of the improved algorithm.

Note that the algorithm's stability still requires further enhancement, and the model can be expanded by incorporating additional factors. Future research can build upon this foundation to explore and achieve more outcomes.

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